Quantitative Easing and Financial Stability*

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Abstract

The massive expansion of central-bank balance sheets in response to recent crises raises important questions about the effects of such "quantitative easing" policies, both their effects on financial conditions and on aggregate demand (the intended effects of the policies), and their possible collateral effects on financial stability. The present paper compares three alternative dimensions of central-bank policy — conventional interest-rate policy, increases in the central bank's supply of safe (monetary) liabilities, and macroprudential policy (possibly implemented through discretionary changes in reserve requirements) — showing in the context of a simple intertemporal general-equilibrium model why they are logically independent dimensions of variation in policy, and how they jointly determine financial conditions, aggregate demand, and the severity of the risks associated with a funding crisis in the banking sector. In the proposed model, each of the three dimensions of policy can be used independently to influence aggregate demand, and in each case a more stimulative policy also increases financial stability risk. However, the policies are not equivalent, and in particular the relative magnitudes of the two kinds of effects are not the same. Quantitative easing policies increase financial stability risk (in the absence of an offsetting tightening of macroprudential policy), but they actually increase such risk less than either of the other two policies, relative to the magnitude of aggregate demand stimulus; and a combination of expansion of the cental bank's balance sheet with a suitable tightening of macroprudential policy can have a net expansionary effect on aggregate demand with no increased risk to financial stability. This suggests that quantitative easing policies may be useful as an approach to aggregate demand management not only when the zero lower bound precludes further use of conventional interest-rate policy, but also when it is not desirable to further reduce interest rates because of financial stability concerns.

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Since the global financial crisis of 2008-09, many of the leading central banks have dramatically increased the size of their balance sheets, and also have shifted the composition of the assets that they hold, toward greater holdings of longer-term securities (as well as toward assets that are riskier in other respects). While many have hailed these policies as contributing significantly to contain the degree of damage to both the countries' financial systems and real economies resulting from the collapse of confidence in certain types of risky assets, the policies have also been and remain quite controversial. One of the concerns raised by skeptics has been the suggestion that such "quantitative easing" by central banks may have been supporting countries' banking systems and aggregate demand only by encouraging risk-taking by ultimate borrowers and by financial intermediaries of a kind that increases the risk of precisely the sort of destructive financial crisis that had led these policies to be introduced.

The most basic argument for suspecting that such policies create risks to financial stability is simply that, according to proponents of these policies in the central banks (e.g., Bernanke, 2012), they represent alternative means of achieving the same kind of relaxation of financial conditions that would under more ordinary circumstances be achieved by lowering the central bank's operating target for short-term interest rates — but a means that continues to be available even when short-term nominal interest rates have already reached their effective lower bound, and so cannot be lowered to provide further stimulus. If one believes that cuts in short-term interest rates have as a collateral effect — or perhaps even as the main channel through which they affect aggregate demand, as argued by Adrian and Shin (2010) — an increase in the degree to which intermediaries take more highly leveraged positions in risky assets, increasing the likelihood of and/or severity of a potential financial crisis, then one might suppose that to the extent that quantitative easing policies are effective in relaxing financial conditions in order to stimulate aggregate demand, they should similarly increase risks to financial stability.

One might go further and argue that such policies relax financial conditions by increasing the supply of central-bank reserves,[1] and one might suppose that such an increase in the availability of reserves matters for financial conditions precisely because it relaxes a constraint on the extent to which private financial intermediaries

[1]The term "quantitative easing," originally introduced by the Bank of Japan to describe the policy that it adopted in 2001 in attempt to stem the deflationary slump that Japan had suffered in the aftermath of the collapse of an asset bubble in the early 1990s, refers precisely to the intention to increase the monetary base (and hence, it was hoped, the money supply more broadly) by increasing the supply of reserves.

can issue money-like liabilities (that are subject to reserve requirements) as a way of financing their acquisition of more risky and less liquid assets, as in the model of Stein (2012). Under this view of the mechanism by which quantitative easing works, one might suppose that it should be even more inevitably linked to an increase in financial stability risk than expansionary interest-rate policy (which, after all, might also increase aggregate demand through channels that do not rely upon increased risk-taking by banks).

Finally, some may be particularly suspicious of quantitative easing policies on the ground that these policies, unlike conventional interest-rate policy, relax financial conditions primarily by reducing the risk premia earned by holding longer-term securities, rather than by lowering the expected path of the risk-free rate.[2] Such a departure from the normal historical pattern of risk premia as a result of massive central-bank purchases may seem a cause for alarm. If one thinks that the premia that exist when market pricing is not "distorted" by the central bank's intervention provide an important signal of the degree of risk that exists in the marketplace, one might fear that central-bank actions that suppress this signal — not by actually reducing the underlying risks, but only by preventing them from being reflected so fully in market prices — run the danger of distorting perceptions of risk in a way that will encourage excessive risk-taking.

The present paper considers the extent to which these are valid grounds for concern about the use of this policy tool by central banks, by analyzing further the mechanisms just sketched, in the context of an explicit model of the way in which quantitative easing policies influence financial conditions, and the way in which monetary policies more generally affect the incentives of financial intermediaries to engage in maturity and liquidity transformation of a kind that increases the risk of financial crisis. It argues, in fact, that the concerns just raised are of little merit. But it does not reach this conclusion by challenging the view that quantitative easing policies can indeed effectively relax financial conditions (and so achieve effects on aggregate demand that are similar to the effects of conventional interest-rate policy); nor does it deny that risks to financial stability are an appropriate concern of monetary policy deliberations, or that expansionary interest-rate policy tends to increase such risks (among other

[2] Again see Bernanke (2012) for discussion of this view of how the policies work, though he also discusses the possibility of effects of quantitative easing that result from central-bank actions being taken to signal different intentions regarding future interest-rate policy.

effects). The model developed here is one in which risk-taking by the financial sector can easily be excessive (in the sense that a restriction on banks' ability to engage in liquidity transformation to the degree that they choose to under laissez-faire would raise welfare); in which, when that is true, a reduction in short-term interest rates through central-bank action will worsen the problem by making it even more tempting for banks to finance acquisitions of risky, illiquid assets by issuing short-term safe liabilities; and in which the purchase of longer-term and/or risky assets by the central bank, financed by creating additional reserves (or other short-term safe liabilities, such as reverse repos or central-bank bills, that would also be useful in facilitating transactions), will indeed loosen financial conditions, with an effect on aggregate demand that is similar, though not identical to, the effect of a reduction in the central bank's operating target for its policy rate. Nonetheless, we show that quantitative easing policies should not increase risks to financial stability, and should instead tend to reduce them.

The reason for this different conclusion hinges on our conception of the sources of the kind of financial fragility that allowed a crisis of the kind just experienced to occur, and the way in which monetary policy can affect the incentives to create a more fragile financial structure. In our view, the fragility that led to the recent crisis was greatly enhanced by the notable increase in maturity and liquidity transformation in the financial sector in the years immediately prior to the crisis (Brunnermeier, 2009; Adrian and Shin, 2010) — in particular, the significant increase in funding of financial intermediaries by issuance of collateralized short-term debt, such as repos (financing investment banks) or asset-backed commercial paper (issued by SIVs). Such financing is relatively inexpensive, in the sense that investors will hold such instruments even when they promise a relatively low yield, because of the assurance they provide that the investor can be sure of payment and can withdraw their funds at any time on short notice if desired. But too much of it is dangerous, because it exposes the leveraged institution to funding risk, which may require abrupt de-leveraging through a "fire sale" of relatively illiquid assets. The sudden need to sell relatively illiquid assets in order to cover a shortfall of funding can substantially depress the price of those assets, requiring even more de-leveraging and leading to a "margin spiral" of the kind described by Shleifer and Vishny (1992, 2010) and Brunnermeier and Pederson (2009).

It is important to ask why such fragile financial structures should arise as an equilibrium phenomenon, in order to understand how monetary policy may increase or decrease the likely degree of fragility. According to the perspective that we adopt here, investors are attracted to the short-term safe liabilities created by banks or other financial intermediaries because assets with a value that is completely certain are more widely accepted as a means of payment.[3] If an insufficient quantity of such safe assets are supplied by the government (through means that we discuss further below), investors will pay a "money premium" for privately-issued short-term safe instruments with this feature, as documented by Greenwood *et al.* (2010), Krishnamurthy and Vissing-Jorgensen (2012), and Carlson *et al.* (2014). This provides banks with an incentive to obtain a larger fraction of their financing in this way. Moreover, they may choose an excessive amount of this kind of financing, despite the funding risk to which it exposes them, because each individual bank fails to internalize the effects of their collective financing decisions on the degree to which asset prices will be depressed in the event of a "fire sale." This gives rise to a pecuniary externality, as a result of which excessive risk is taken in equilibrium (Lorenzoni, 2008; Jeanne and Korinek, 2010; Stein, 2012).

Conventional monetary policy, which cuts short-term nominal interest rates in response to an aggregate demand shortfall, can arguably exacerbate this problem, as low market yields on short-term safe instruments will further increase the incentive for private issuance of liabilities of this kind (Adrian and Shin, 2010; Giavazzi and Giovannini, 2012). The question of primary concern in this paper is, do quantitative easing policies, pursued as a means of providing economic stimulus when conventional monetary policy is constrained by the lower bound on short-term nominal interest rates, increase financial stability risks for a similar reason?

In the model proposed here, quantitative easing policies lower the equilibrium real yield on longer-term and risky government liabilities, just as a cut in the central bank's target for the short-term riskless rate will, and this relaxation of financial conditions has a similar expansionary effect on aggregate demand in both cases. Nonetheless, the consequences for financial stability are not the same. In the case of conventional monetary policy, a reduction in the riskless rate lowers the equilibrium yield on risky assets as well because, if it did not, the increased spread between the two yields

[3] The role of non-state-contingent payoffs in allowing an asset to be widely acceptable as a means of payment is stressed in particular by Gorton and Pennacchi (2010), and in recent discussions such as Gorton (2010) and Gorton, Lewellen and Metrick (2012).

would provide an increased incentive for maturity and liquidity transformation on the part of banks, which they pursue until a point at which the spread has decreased (because of diminishing returns to further investment in risky assets) to where it is again balanced by the risks associated with overly leveraged investment. (This occurs, in equilibrium, partly through a reduction in the degree to which the spread increases — which means that the expected return on risky assets is reduced — and partly through an increase in the risk of a costly "fire sale" liquidation of assets.) In the case of quantitative easing, instead, the equilibrium return on risky assets is reduced, but in this case through a reduction, rather than an increase in the spread between the two yields. The "money premium," which results from a scarcity of safe assets, should be reduced if the central-bank asset purchases increase the supply of safe assets to the public, as argued by Caballero and Farhi (2013) and Carlson *et al.* (2014). Hence the incentives for creation of a more fragile financial structure are not increased as much by expansionary monetary policy of this kind.

The idea that quantitative easing policies, when pursued as an additional means of stimulus when the risk-free rate is at the zero lower bound, should increase risks to financial stability because they are analogous to an expansionary policy that relaxes reserve requirements on private issuers of money-like liabilities is also based on a flawed analogy. It is true, in the model of endogenous financial stability risk presented here, that a relaxation of a reserve requirement proportional to banks' issuance of short-term safe liabilities will (in the case that the constraint binds) increase the degree to which excessive liquidity transformation occurs. And it is also true that in a conventional textbook account of the way in which monetary policy affects financial conditions, an increase in the supply of reserves by the central bank relaxes the constraint on banks' issuance of additional money-like liabilities ("inside money") implied by the reserve requirement, so that the means through which the central bank implements a reduction in the riskless short-term interest rate is essentially equivalent to a reduction in the reduction in the reserve requirement. However, this is *not* a channel through which quantitative easing policies can be effective, when the risk-free rate has already fallen to zero (or more generally, to the level of interest paid on reserves). For in such a case, reserves are necessarily already in sufficiently great supply for banks to be satiated in reserves, so that the opportunity cost of holding them must fall to zero in order for the existing supply to be voluntarily held. Under such circumstances (which is to say, those existing in countries like the US since the end of 2008), banks' reserve requirements have already ceased to constrain their behavior. Hence, to the extent that quantitative easing policies are of any use at the zero lower bound on short-term interest rates, their effects cannot occur through this traditional channel.

In the model presented here, quantitative easing *is* effective at the zero lower bound (or more generally, even in the absence of reserve requirements, or under circumstances where there is already satiation in reserves); this is because an increase in the supply of safe assets (through issuance of additional short-term safe liabilities by the central bank, used to purchase assets that are not equally money-like) reduces the equilibrium "money premium." But whereas a relaxation of a binding reserve requirement would increase banks' issuance of short-term safe liabilities (and hence financial stability risk), a reduction in the "money premium" should reduce their issuance of such liabilities, so that financial stability risk should if anything be reduced.

The idea that a reduction in risk premia as a result of central-bank balance-sheet policy should imply a greater danger of excessive risk-taking is similarly mistaken. In the model presented here, quantitative easing achieves its effects (both on the equilibrium required return on risky assets and on aggregate demand) by lowering the equilibrium risk premium — that is, the spread between the required return on risky assets and the riskless rate. But this does not imply the creation of conditions under which it should be more tempting for banks to take on greater risk. To the contrary, the existence of a smaller spread between the expected return on risky assets and the risk-free rate makes it less tempting to finance purchases of risky assets by issuing safe, highly liquid short-term liabilities that need pay only the riskless rate. Hence again a correct analysis implies that quantitative easing policies should increase financial stability, rather than threatening it.

The remainder of the paper develops these points in the context of an explicit intertemporal monetary equilibrium model, in which it is possible to clearly trace the general-equilibrium determinants of risk premia, the way in which they are affected by both interest-rate policy and the central bank's balance sheet, and the consequences for the endogenous capital structure decisions of banks. Section 1 presents the structure of the model, and section 2 then derives the conditions that must link the various endogenous prices and quantities in an intertemporal equilibrium. Section 3 considers the effects of alternative balance-sheet policies on equilibrium variables, focusing on the case of a stationary long-run equilibrium with flexible prices. Section 4 compares the ways in which quantitative easing and adjustments of reserve requirements affect banks' financing decisions. Finally, section 5 compares (somewhat more briefly) the short-run effects of both conventional monetary policy, quantitative easing, and macroprudential policy in the presence of nominal rigidities that allow conventional monetary policy to affect the degree of real economic activity. Section 6 concludes.

1 A Monetary Equilibrium Model with Fire Sales

This section develops a simple model of monetary equilibrium, in which it is possible simultaneously to consider the effects of the central bank's balance sheet on financial conditions (most notably, the equilibrium spread between the expected rate of return on risky assets and the risk-free rate of interest) and the way in which private banks' financing decisions can increase risks to financial stability. An important goal of the analysis is to present a sufficiently explicit model of the objectives and constraints of individual actors to allow welfare analysis of the equilibria associated with alternative policies that is based on the degree of satisfaction of the individual objectives underlying the behavior assumed in the model, as in the modern theory of public finance, rather than judging alternative equilibria on the basis of some more ad hoc criterion.[4]

Risks to financial stability are modeled using a slightly adapted version of the model proposed by Stein (2012). The Stein model is a three-period model in which banks finance their investments in risky assets in the first period; a crisis may occur in the second period, in which banks are unable to roll over their short-term financing and as a result may have to sell illiquid risky assets in a "fire sale"; and in the third period, the ultimate value of the risky assets is determined. The present model incorporates this model of financial contracting and occasional fire sales of assets into a fairly standard intertemporal general-equilibrium model of the demand for money-like assets, the "cash-in-advance" model of Lucas and Stokey (1987). In this way, the premium earned by money-like assets, that is treated as an exogenous parameter in Stein (2012), can be endogenized, and the effects of central-bank policy on this

[4]The proposed framework is further developed in Sergeyev (2016), which considers the interaction between conventional monetary policy and country-specific macroprudential policies in a currency union.

variable can be analyzed, and through this the consequences for financial stability.

1.1 Elements of the Model

Like most general-equilibrium models of monetary exchange, the Lucas and Stokey (1987) model is an infinite-horizon model, in which the willingness of sellers to accept central-bank liabilities as payment for real goods and services in any period depends on the expectation of being able to use those instruments as a means of payment in further transactions in future periods. The state space of the model is kept small (allowing a straightforward characterization of equilibrium, despite random disturbances each period) by assuming a representative household structure; the two sides of each transaction involving payment using cash are assumed to be two members of a household unit with a common objective, that can be thought of as a "worker" and a "shopper." During each period, the worker and shopper from a given household have separate budget constraints (so that cash received by the worker as payment for the sale of produced goods cannot be immediately used by the shopper to purchase goods, in the same market), as is necessary for the "cash-in-advance" constraint to matter; but at the end of the period, their funds are again pooled in a single household budget constraint (so that only the asset positions of households, that are all identical, matter at this point).

We shall employ a similar device, but further increasing the number of distinct roles for different members of the household, in order to introduce additional kinds of financial constraints into the model, while retaining the convenience of a representative household. We suppose that each infinite-lived household is made of four members with different roles during the period: a "worker" who supplies the inputs used to produce all final goods, and receives the income from the sale of these goods; a "shopper" who purchases "regular goods" for consumption by the household, and who holds the household's cash balance, for use in such transactions; a "banker" who buys risky durable goods, and issues short-term safe liabilities in order to finance some of these purchases; and an "investor" who purchases "special" final goods, and can also bid for the risky durables sold by bankers in the event of a fire sale.[5] As

[5]The distinction between bankers, investors, and worker/shopper pairs corresponds to the distinction in the roles of "bankers," "patient investors," and "households" in the model of Stein (2012). In the Stein model, these three types of agents are distinct individuals with no sharing of resources among them, rather than members of a single (larger) household; the device of having them pool

in the Lucas-Stokey model, the different household members have separate budget constraints during the period (which is the significance of referring to them as different people), but pool their budgets at the end of each period in a single household budget constraint.

Four types of final goods are produced each period: durable goods and three types of non-durable goods, called “cash goods,” “credit goods,” and “special goods.” In addition, we suppose that workers also produce intermediate “investment goods” that are used as an input in the production of durable goods. Both “cash” and “credit” goods are purchased by shoppers; the distinction between the two types of goods is taken from Lucas and Stokey (1987), where the possibility of substitution by consumers between the two types of goods (one subject to the cash-in-advance constraint, the other not) allows the demand for real cash balances to vary with the size of the liquidity premium (opportunity cost of holding cash), for a given level of planned real expenditure. This margin of substitution also results in a distortion in the allocation of resources that depends on the size of the liquidity premium, and we wish to take this distortion into account when considering the welfare effects of changing the size of the central bank's balance sheet.

The introduction of “special goods” purchased only by the investor provides an alternative use for the funds available to the investor, so that the amount that investors will spend on risky durables in a fire sale depends on how low the price of the durables falls.[6] The produced “durable goods” in our model play the role of the risky investment projects in the model of Stein (2012): they require an initial outlay of resources, financed by bankers, in order to allow the production of something that may or may not yield a return later. The device of referring separately to investment goods and to the durable goods produced from them allows us to treat investment goods as perfect substitutes for cash or credit goods on the production side, allowing a simple specification of workers' disutility of supplying more output, without having also to treat durable goods as perfect substitutes for those goods, which would not allow the relative price of durables to rise in a credit boom.

assets at the end of each “period” is not needed to simplify the model dynamics, because the model simply ends when the end of the first and only “period” is reached (in the sense in which the term “period” is used in this model). Note that in the present model, the representative household device also allows more unambiguous welfare comparisons among equilibria.

[6]The opportunity of spending on purchases of special goods plays the same role in our model as the possibility of investment in “late-arriving projects” in the model of Stein (2012).

All of the members of a given household are assumed to act so as to maximize a common household objective. Looking forward from the beginning of any period t, the household objective is to maximize

$$\mathrm{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[u(c_{1\tau},\, c_{2\tau}) + \tilde{u}(c_{3\tau}) + \gamma \underline{s}_\tau - v(Y_\tau) - w(x_\tau)\right]. \tag{1.1}$$

Here c_{1t}, c_{2t}, c_{3t} denote the household's consumption of cash goods, credit goods, and special goods respectively in period t; $\underline{s}_t$ denotes the quantity of durables held by the household at the end of period t that have not proven to be worthless, and hence the flow of services in period t from such intact durables; Y_t denotes the household's supply of "normal goods" (a term used collectively for cash goods, credit goods, and investment goods, that are all perfect substitutes from the standpoint of a producer) in period t; and x_t denotes the household's supply of special goods in period t. The functions $u(\cdot,\cdot), \tilde{u}(\cdot), v(\cdot)$, and $w(\cdot)$ are all increasing functions of each of their arguments; the functions $u(\cdot,\cdot)$ and $\tilde{u}(\cdot)$ are strictly concave; and the functions $v(\cdot)$ and $w(\cdot)$ are at least weakly convex. We also assume that the function $u(\cdot,\cdot)$ implies that both cash and credit goods are normal goods, in the sense that it will be optimal to increase purchases of both types of goods if a household increases its expenditure on these types of goods in aggregate, while the (effective) relative price of the two types of goods remains the same.[7] In addition, the discount factor satisfies $0 < \beta < 1$, and $\gamma > 0$. The operator $\mathrm{E}_t[\cdot]$ indicates the expectation conditional on information at the beginning of period t.

Each of the infinite sequence of periods $t = 0, 1, 2, \ldots$ is subdivided into three subperiods, corresponding to the three periods in the model of Stein (2012). The sequence of events, and the set of alternative states that may be reached, within each period is indicated in Figure 1. In subperiod 1, a financial market is open in which bankers issue short-term safe liabilities and acquire risky durables, and households decide on the cash balances to hold for use by the shopper.[8] In subperiod 2, information is revealed about the possibility that the durable goods purchased by the banks will prove to be valueless. With probability p, the "no crisis" state is

[7] By the effective relative price we mean the relative price taking into account the cost to the household of having to hold cash in order to purchase cash goods, as discussed further below.

[8] This sub-period corresponds both to the first period of the Stein (2012) model, in which risky projects are financed, and to the securities-trading subperiod of the model in section 5 of Lucas and Stokey (1987), in which bonds are priced and hence the liquidity premium on cash is determined.

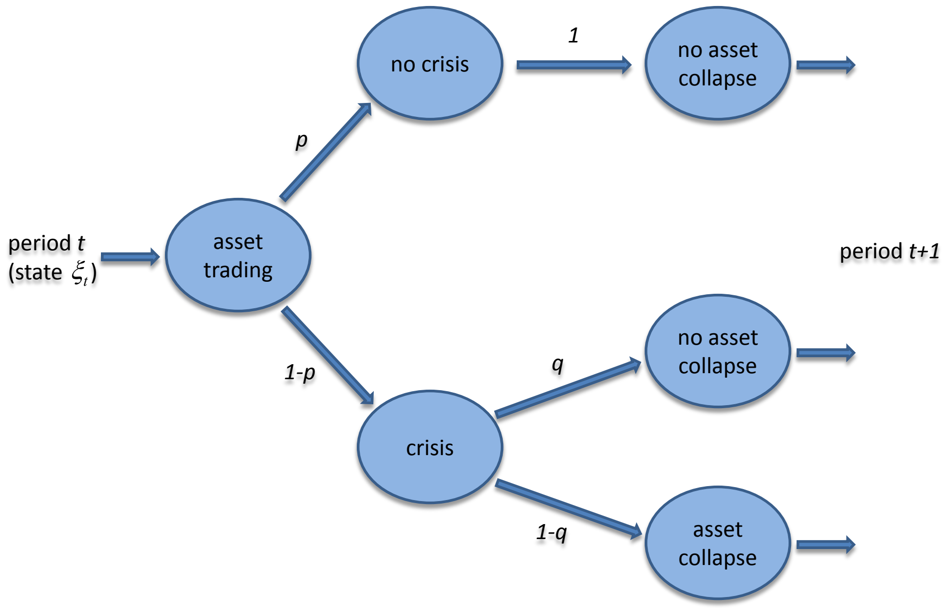

Figure 1: The sequential resolution of uncertainty within period t.

reached, in which it is known with certainty that the no collapse in the value of the assets will occur, but with probability $1 - p$, a "crisis" state is reached, in which it is understood to be possible (though not yet certain) that the assets will prove to be worthless. Finally, in subperiod 3, the value of the risky durables is learned. In both of the states labeled "no asset collapse," a unit of the durable good produces one unit of services, while in the "asset collapse" state (that occurs with probability $1-q$, conditional on the crisis state being reached), durables provide no service flow.

The various types of goods are produced and sold in sub-period 2. The markets in which the different goods are sold differ in the means of payment that are accepted. It is assumed, as in Lucas and Stokey (1987), that "cash goods" are sold only for cash that is transferred from the buyer to the seller at that time; the cash balances used for this purpose must have been acquired in sub-period 1 by the household to which that shopper belongs. (The liquidity premium associated with cash is thus determined in the exchange of cash for other financial claims in subperiod 1.) "Credit goods" are instead sold to shoppers on credit; this means (as in Lucas and Stokey) that accounts are settled between buyers and sellers only at the end of the period, at which point the various household members have again pooled their resources, so that charges by shoppers during the period can be paid out of the income received by workers for goods sold during that same period. The only constraint on the amount of credit of this kind that a household can draw upon is assumed to be determined by a no-Ponzi condition (that is, the requirement that a household's debts be able to be paid off eventually out of future income, rather than rolled over indefinitely). "Investment goods" are sold on credit in the same way. "Special goods" are also assumed to be sold on credit, but in this case, the amount of credit that investors can draw upon is limited by the size of the line of credit arranged for them in subperiod 1. In particular, it is assumed that a given credit limit must be negotiated by the household before it is learned whether a crisis will occur in subperiod 2, and thus whether investors will have an opportunity to bid on "fire sale" assets. The existence of the non-state-contingent credit limit for purchases by investors (both their purchases of special goods and their purchases of risky durables liquidated by the bankers in a fire sale) is important in order to capture the idea that only a limited quantity of funds can be mobilized (by potential buyers with the expertise required to evaluate the assets) to bid on the assets sold in a fire sale.[9]

The nature of the "cash" that can be used to purchase cash goods requires further comment. Unlike Lucas and Stokey, we do not assume that only monetary liabilities of the government constitute "cash" that is acceptable as a means of payment in this market. We instead identify "cash" with the class of short-term safe instruments (STSIs) discussed by Carlson *et al.* (2014) in the case of the U.S., which includes U.S. Treasury bills (and not simply monetary liabilities of the Federal Reserve), and certain types of collateralized short-term debt of private financial institutions. The assumption that only these assets can be used to purchase cash goods is intended to stand in for the convenience provided by these special instruments, that accounts for their lower equilibrium yields relative to the short-period holding returns on other assets.[10] The fact that all assets of this type, whether issued by the government (or

[9]In the model of Stein (2012), this limit is ensured by assuming that the "patient investors" have a budget that is fixed as a parameter of the model. Here we endogenize this budget, by allowing it to be chosen optimally by the household in subperiod 1; but it is important that we still assume that it cannot be changed in subperiod 2.

[10]One interpretation of the "cash-in-advance" constraint is that it actually represents a constraint on the type of assets that can be held by money-market mutual funds (MMMFs). But such a

central bank) or by bankers, are assumed equally to satisfy the constraint is intended to capture the way in which the demand for privately-issued STSIs is observed to vary with the supply of publicly-issued STSIs, as shown by Carlson *et al.* (2014).

We do not, of course, deny that there are *also* special uses for base money (currency and reserve balances held at the Fed) as a means of payment, of the kind that Lucas and Stokey sought to model. In particular, when the supply of reserves by the Fed is sufficiently restricted, as was chronically the case prior to the financial crisis of 2008, the special convenience of reserve balances in facilitating payments between financial intermediaries results in a spread between the yield on reserves and that on STSIs such as Treasury bills; and the control of this spread by varying the supply of reserves was the focus of monetary policy prior to the crisis. Nonetheless, the spread between the yield on reserves and the T-bill rate (or federal funds rate) is not the one of interest to us here. Under the circumstances in which the Fed has conducted its experiments with "quantitative easing," the supply of reserves has been consistently well beyond the level needed to drive the T-bill yield down to (or even below) the yield on reserves. Hence while certain kinds of payments by banks are constrained by their reserve balances, we may assume that this has not been a binding constraint in the period in which we wish to consider the effects of further changes in the central-bank balance sheet. And granting that reserves have special uses that can result in a liquidity premium specific to them (under circumstances no longer relevant at present) does not in any way imply that STSIs cannot *also* have special uses for which other assets will not serve, giving rise to another sort of money premium — one that need not be zero simply because the premium associated with reserve balances has been eliminated.

The acceptability of a financial claim as "cash" that can be used to purchase cash goods is assumed to depend on its having a value at maturity that is completely certain, rather than being state-contingent. This requires not only that it be a claim to a fixed nominal quantity at a future date, but that it be viewed as completely safe, for one of two possible reasons: either it is a liability of the government (or

constraint gives rise to a "money premium" only to the extent that there are special advantages to investors of holding wealth in MMMFs; the ability to move funds quickly from them to make purchases is one such advantage. Rather than explicitly introducing a demand for cash on the part of MMMFs and assuming that households use their MMMF balances to make certain types of purchases, we obtain the same equilibrium money premium more simply by supposing that the STSIs can directly be used as a means of payment in certain transactions.

central bank),[11] or it is collateralized in a way that allows a holder of the claim to be certain of realizing a definite nominal value from it. We suppose that bankers can issue liabilities that will be accepted as cash, but that these liabilities will have to be backed by specific risky durables as collateral, and that the holder of the debt has the right to demand payment of the debt at any time, if they cease to remain confident that the collateral will continue to guarantee the fixed value for it.

When bankers purchase risky durables in the first subperiod, they can finance some portion of the purchase price by issuing safe debt (that can be used by the holder during the second sub-period to purchase cash goods), collateralized by the durables that are acquired. If in the second subperiod, the "no-crisis" state is reached, the durables can continue to serve as collateral for safe debt, as the value of the asset in the third subperiod can in this case be anticipated with certainty. In this case, bankers are able to roll over their short-term collateralized debt, and continue to hold the durables. If instead the "crisis" state is reached, the durables can no longer collateralize safe debt, as there is now a positive probability that in the third subperiod the durables will be worthless. In this case, holders of the safe debt demand repayment in the second sub-period, and the bankers must sell durables in a fire sale, in the amount required to pay off the short-term debt. It is the right to force this liquidation that makes the debt issued by bankers in the first sub-period safe.

To be more specific, we suppose that the sale of goods (and in particular, cash goods) occurs at the beginning of the second subperiod: after it has been revealed whether the crisis state will occur, but before the decision whether to demand immediate repayment of the short-term debt is made. Thus at the time that shoppers seek to purchase cash goods, they may hold liabilities issued by bankers that grant the holder the right to demand repayment at any time; it is the fact that the short-term debt has this feature that allows it to be accepted as cash in the market for cash goods. After the market for cash goods has taken place, the holders of the bankers' short-term debt (who may now include the sellers of cash goods) decide whether to demand immediate repayment of the debt. At this point, these holders (whether shoppers or workers) only care about the contribution that the asset will make to

[11]Of course, a claim on a government need not be completely safe. If, however, a government borrows in its own fiat currency, and if it is committed to ensure that its nominal liabilities are paid with certainty (by monetizing them if necessary), then it is possible for it to issue debt that is correctly viewed as completely safe (in nominal terms).

the household's pooled end-of-period budget. In the crisis state, they will choose to demand repayment, since this ensures them the face value of the debt, whereas if they do not demand repayment, they will receive the face value of the debt with probability $q < 1$, but will receive nothing if the "asset collapse" state occurs. If they demand repayment, they receive a claim on the investors who purchase the collateral in the fire sale; such a claim is assumed to guarantee payment in the end-of-period settlement, if within the bound of the line of credit arranged for the investor in the first subperiod.

The other source of assets that count as cash is the government. Some very short-term government liabilities (Treasury bills) count as cash. In addition, we shall suppose that the central bank can issue liabilities that also count as cash. If the central bank increases its supply of SFSIs by purchasing Treasury bills (that are themselves SFSIs), the overall supply of cash will be unchanged. (This is again a demonstration that our concept of "cash" differs importantly from that of Lucas and Stokey.) But if the central bank purchases non-cash assets (either longer-term Treasury bonds, that are less able to facilitate transactions than are shorter-term bills, or assets subject to other kinds of risk) and finances these purchases by creating new short-term safe liabilities, it can increase the net supply of SFSIs. We are interested in the effects of this latter kind of policy.

1.2 Budget Constraints and Definition of Equilibrium

Each household begins period t with I_{t-1} units of the investment good (purchased in the previous period) and financial wealth A_t, which may represent either claims on the government or on other households, and is measured in terms of the quantity of cash that would have the same market value in subperiod 1 trading (even though the assets aggregated in A_t need not all count as cash). In the first subperiod, the investment good is used to produce $F(I_{t-1})$ units of the durable good, which can sold on a competitive market at price Q_t per unit.[12] The banker in each household purchases a quantity s_t of these durables, financed partly from funds provided by the household for this purpose, and partly by issuing short-term collateralized debt in

[12]We may alternatively suppose that the investment goods are purchased by construction firms that produce the durables and sell them to bankers, and that households simply begin the period owning shares in these construction firms. The explicit introduction of such firms would not change the equilibrium conditions presented below.

quantity D_t. Here D_t is the face value of the debt, the nominal quantity to which the holder is entitled (with certainty) in the settlement of accounts at the end of period t. The price Q_t of the risky asset is quoted in the same (nominal, end-of-period) units; thus the quantity of funds that the household must provide to the banker is equal to $Q_t s_t - D_t$ in those units.

The household's other uses of its beginning-of-period financial wealth are to acquire cash, in quantity M_t, for use by the shopper, or to acquire (longer-term) bonds B_t, which are government liabilities that do not count as cash. The quantity M_t represents the end-of-period nominal value of these safe assets; thus if interest is earned on cash (as we allow), M_t represents the value of the household's cash balances inclusive of the interest earned on them, rather than the nominal value at the time that they are acquired.[13] The quantity of bonds B_t is measured in terms of the number of units of cash that have the same market value in subperiod 1 trading (as with the measurement of A_t). Hence the household's choices of s_t, D_t, M_t and B_t in the first subperiod are subject to an interim budget constraint

$$(Q_t s_t \;-\; D_t) \;+\; M_t \;+\; B_t \leq \; A_t \;+\; Q_t F(I_{t-1}). \tag{1.2}$$

The financing decisions of bankers are also subject to a constraint that safe debt D_t cannot be issued in a quantity beyond that for which they can provide sufficient collateral, given their holdings of the durable s_t.[14] This requires that

$$D_t \;\leq\; \Gamma_t \, s_t, \tag{1.3}$$

where Γ_t is the market price of the durable good in the fire sale, should one occur in period t. (Here Γ_t is quoted in terms of the units of nominal value to be delivered by

[13]If we think of cash as Treasury bills, M_t represents their face value at maturity, rather than the discounted value at which they are purchased.

[14]We might suppose that bankers can also issue debt that is not collateralized, or not collateralized to this extent. But such liabilities would not be treated as cash by the households that acquire them, so that allowing such debt to be issued by a banker would have no consequences any different from allowing the household itself to issue such debt in the first subperiod, in order to finance a larger equity contribution to its banker. And allowing households to trade additional kinds of non-cash financial liabilities would make no difference for the equilibrium conditions derived here; it would simply allow us to price the additional types of financial claims. The ability of bankers to issue collateralized short-term debt that counts as cash instead matters; this is not a type of claim that a household can issue other than by having its banker issue it (because it must be collateralized by risky durable goods), and issuing such claims has special value because they can relax the cash-in-advance constraint.

investors in the end-of-period settlement of accounts. Note that while it is not yet known in subperiod 1 whether a crisis will occur, the price Γ_t that will be realized in the fire sale *if* one occurs is perfectly forecastable.) Constraint (1.3) indicates the amount of collateral required to ensure that whichever state is reached in subperiod 2, the value of the collateralized debt will equal D_t, since sale of the collateral in a fire sale will yield at least that amount.

Regardless of the state reached in subperiod 2, cash goods purchases of the shopper must satisfy the cash-in-advance constraint

$$P_t\, c_{1t} \;\leq\; M_t, \tag{1.4}$$

where P_t is the price of "normal goods" in period t (that may depend on the state reached in subperiod 2), quoted in units of the nominal value to be delivered in the end-of-period settlement. It is this constraint that provides a reason for the household to choose to hold cash balances M_t. The common price for all normal goods follows from the fact that these goods are perfect substitutes from the point of view of their producers (workers), and that all payments that guarantee the same nominal value in the end-of-period settlement are of equal value to the sellers, once the problem of verifying the soundness of payments made in the cash goods market has been solved.[15]

There is no similar constraint on credit goods or investment goods purchases by the shopper, as these are sold on credit. The investor's purchases c_{3t} of special goods, and purchases s_t^{*d} of durables in the fire sale[16] must however satisfy a state-contingent budget constraint

$$\tilde{P}_t\, c_{3t} \;+\; \eta_t\, \Gamma_t\, s_t^{*d} \;\leq\; F_t, \tag{1.5}$$

where $\tilde{P}_t$ is the price of special goods (in the same units as P_t, and that similarly may depend on the state reached in subperiod 2); η_t is an indicator variable for the occurrence of a crisis in period t;[17] and F_t is the line of credit arranged for the investor in subperiod 1, quoted in units of the nominal quantity that the investor can promise

[15]Cash goods and credit goods sell for the same price in any given period for the same reason in the model of Lucas and Stokey (1987).

[16]We use the notation s_t^* for the quantity of durables liquidated in the fire sale, if one occurs in period t. An additional superscript d is used for the quantity demanded on this market, and a superscript s for the quantity supplied. Note that s_t^{*d} and s_t^{*s} are two independent choice variables for an individual household, and need not be chosen to be equal, even though in equilibrium they must be equal (given common choices by all households) in order for the market to clear.

[17]That is, $\eta_t = 1$ if a crisis occurs, while $\eta_t = 0$ if the no-crisis state is reached.

to deliver in the end-of-period settlement, and with a value that must be independent of the state that is realized in subperiod 2. (Note that (1.5), like (1.4), is actually two constraints, one for each possible state that may be reached in subperiod 2.)

If the crisis state is reached in subperiod 2, the banker offers s_t^{*s} units of the durable for sale in the fire sale, which quantity must satisfy the bounds

$$D_t \;\leq\; \Gamma_t\, s_t^{*s} \leq \Gamma_t\, s_t. \tag{1.6}$$

The first inequality indicates that the banker must liquidate assets sufficient to allow repayment of the short-term debt (given that in this state, the holders will necessarily demand immediate repayment); the second inequality follows from the fact that the banker cannot offer to sell more shares of the durable than she owns. (The range of possible quantity offers defined in (1.6) is non-empty only because (1.3) has been satisfied; thus a plan that satisfies (1.6) necessarily satisfies (1.3), making the earlier constraint technically redundant.)

Given these decisions, the durables owned by the household in subperiod 3 will equal

$$\underline{s}_t \;=\; s_t \,+\, \eta_t\,[s_t^{*d} - s_t^{*s}] \tag{1.7}$$

if the durables prove to be valuable, while $\underline{s}_t = 0$ regardless of the household decisions in the "asset collapse" state. The household's pooled financial wealth at the end of the period (in nominal units) will be given by

$$W_t \;=\; M_t + (R_t^b/R_t^m)B_t + P_tY_t - P_t[c_{1t}+c_{2t}+I_t] + \tilde{P}_t x_t + \eta_t\Gamma_t s_t^{*s} - D_t - F_t + T_t. \tag{1.8}$$

This consists of the household's cash balances at the end of subperiod 1, plus the end-of-period value of the bonds that it holds at the end of subperiod 1, plus additional funds obtained from the sale of both normal goods and special goods in subperiod 2, plus funds raised in the fire sale of assets in the event of a crisis, minus the household's expenditure on normal goods of the various types in subperiod 2, and the amounts that it must repay at the end of the period (if not sooner) to pay off the collateralized debt issued by the banker, and to pay for the line of credit arranged for the investor, plus the nominal value T_t of net transfers from the government. We assume that the household must pay F_t regardless of the extent to which the line of credit is used; we then do not need to subtract expenditure by the investor, as this has already been paid for when F_t is paid.[18] Note also that bonds that cost the same amount as one unit of cash in subperiod 1 are worth as much as R_t^b/R_t^m units of cash at the end of the period, where R_t^m is the gross nominal yield on cash (assumed to be known when the cash is acquired in subperiod 1, since these assets are riskless in nominal terms) and R_t^b is the gross nominal holding return on bonds (which may depend on the state reached by the end of the period).

We assume that each household is subject to a borrowing limit

$$W_t \;\geq\; \underline{W_t}, \tag{1.9}$$

expressed as a lower bound on its net worth after the end-of-period settlement of accounts. (We do not further specify the precise value of the borrowing limit, but note that it can be set tight enough to ensure that any end-of-period net indebtedness can eventually be repaid while at the same time being loose enough so that the constraint (1.9) never binds in any period.) Finally, the household carries into period $t+1$ the investment goods I_t purchased in subperiod 2 of period t, and financial wealth in the amount

$$A_{t+1} \;=\; R_{t+1}^m \, W_t, \tag{1.10}$$

where the multiplicative factor R_{t+1}^m converts the value of the household's financial wealth at the beginning of period $t+1$ into an equivalent quantity of cash (measured in terms of the face value of the STSIs rather than their cost in subperiod 1 trading).

A feasible plan for a household is then a specification of the quantities M_t, B_t, s_t, D_t, F_t, s_t^{*s}, s_t^{*d} for each period t, as a function of the history ξ_t of shocks up until then, and a specification of the quantities $c_{1t}, c_{2t}, c_{3t}, I_t, Y_t, x_t$ for each period t, as a function of both ξ_t and η_t (that is, whether a crisis occurs in period t), that satisfies the constraints (1.2)–(1.3) for each possible history ξ_t and the constraints (1.4)–(1.10) for

[18] The assumption that F_t must be paid whether or not the full line of credit is used is important because it prevents the household from simply asking for a large line of credit, as much as would be desired in the crisis state, and then not using all of it in the non-crisis state. If that were possible at no cost, the non-state-contingency of the credit available to the investor would have no bite. The assumption that the line of credit must be paid for whether used or not makes this costly, and results in the household's wishing *ex post* in the crisis state that it had provided more funds to the investor — though it also wishes *ex post* in the non-crisis state that it had provided *less* credit to the investor. This device implies that the credit available to the investor will be optimal *on average*, though not optimal in each state because it cannot be state-contingent.

each possible history (ξ_t, η_t), given initial financial wealth A_0 and pre-existing investment goods I_{-1}, and given the state-contingent evolution of the prices, net transfers from the government to households, and the borrowing limit. An optimal plan is a feasible plan that maximizes (1.1).

Equilibrium requires that all markets for goods and assets clear. Thus it requires that in the first subperiod of period t,

$$M_t \;=\; \tilde{M}_t \,+\, D_t, \tag{1.11}$$

$$B_t \;=\; B_t^s, \tag{1.12}$$

$$s_t \;=\; F(I_{t-1}), \tag{1.13}$$

where $\tilde{M}_t$ is the public supply of cash (short-term safe liabilities of the government or of the central bank) and B_t^s is the supply of longer-term government bonds (not held by the central bank). Note that we assume for simplicity that durables fully depreciate after supplying a service flow (in the event that there is no asset collapse) in the period in which they are produced and acquired by bankers; thus the supply of durables to be acquired by bankers in period t is given simply by the new production $F(I_{t-1})$, and is independent of the quantity $\underline{s}_{t-1}$ of valuable durables in the previous period.

Equilibrium also requires that in the second subperiod, if a crisis occurs,

$$s_t^{*d} \;=\; s_t^{*s}, \tag{1.14}$$

and that in either the crisis or in the non-crisis state,

$$c_{1t} \,+\, c_{2t} \,+\, I_t \;=\; Y_t, \tag{1.15}$$

and

$$c_{3t} \;=\; x_t. \tag{1.16}$$

We can then define a (flexible-price) equilibrium as a specification of prices Q_t, Γ_t and yield R_t^m on cash for each history ξ_t, and prices $P_t, \tilde{P}_t$ and bond yields R_t^b for each history (ξ_t, η_t), together with a plan (as described above) for the representative household, such that (i) the plan is optimal for the household, given those prices, and (ii) the market-clearing conditions (1.11)–(1.14) are satisfied for each history ξ_t and conditions (1.15)–(1.16) are satisfied for each history (ξ_t, η_t).

1.3 Fiscal Policy and Central-Bank Policy

The equilibrium conditions above involve several variables that depend on government policy: the supplies of outside financial assets $\tilde{M}_t$ and B_t^s, the net transfers T_t, and the yields R_t^m and R_t^b on the outside financial assets. Fiscal policy determines the evolution of end-of-period claims on the government,

$$L_t \equiv \tilde{M}_t + (R_t^b/R_t^m)B_t^s + T_t, \tag{1.17}$$

by varying state-contingent net transfers to households appropriately. The Treasury also has a debt management decision: at the beginning of each period t, it must decide how much of existing claims on the government will be financed through STSIs (issuance of Treasury bills), as opposed to longer-term debt that cannot be used to satisfy the cash-in-advance constraint. If we let $\tilde{M}_t^g$ be T-bill issuance by the Treasury in the first subperiod of period t, it follows that the total supply of longer-term debt by the Treasury will equal[19]

$$B_t^g = R_t^m L_{t-1} - \tilde{M}_t^g. \tag{1.18}$$

Of these longer-term securities issued by the Treasury, a quantity B_t^{cb} will be held as assets of the central bank, backing central-bank liabilities $\tilde{M}_t^{cb}$ of equal value. We shall suppose that all of these central-bank liabilities are STSIs that count as cash. The supply of outside assets to the private sector is then given by

$$\tilde{M}_t \equiv \tilde{M}_t^g + \tilde{M}_t^{cb}, \tag{1.19}$$

$$B_t^s \equiv B_t^g - B_t^{cb}. \tag{1.20}$$

In equilibrium, the net wealth W_t of the representative household at the end of period t must equal net claims L_t on the government. (A comparison of the definition of W_t in (1.8) with the definition of L_t in (1.17) shows that the market-clearing conditions imply that $W_t = L_t$.) It then follows from (1.10) and (1.18) that the beginning-of-period assets A_t of the representative household must equal

$$A_t = \tilde{M}_t^g + B_t^g.$$

[19] Note that liabilities with a market value the same as $\tilde{M}_t^g + B_t^g$ units of cash in subperiod 1 will have a market price of $(\tilde{M}_t^g + B_t^g)/R_t^m$.

Since $\tilde{M}_t^{cb} = B_t^{cb}$, we alternatively have

$$A_t = \tilde{M}_t + B_t^s, \tag{1.21}$$

in terms of the supplies of outside assets to the private sector.

At the end of period t, the assets of the central bank are worth $(R_t^b/R_t^m)B_t^{cb}$, while its liabilities are worth $\tilde{M}_t^{cb} = B_t^{cb}$. In general, these quantities will not be equal; we suppose, however, that net balance-sheet earnings must be rebated to the Treasury at the end of the period, in a transfer of magnitude

$$T_t^{cb} = (R_t^b/R_t^m)B_t^{cb} - \tilde{M}_t^{cb}.$$

A transfer from the central bank to the Treasury allows the Treasury to make a larger transfer to the private sector while achieving the same target for end-of-period claims on the government. However, this does not change formula (1.17) for the size of net transfer that is made to the private sector, because that equation was already written in terms of a consolidated budget constraint for the Treasury and central bank. If instead we write

$$T_t^g = L_t - \tilde{M}_t^g - (R_t^b/R_t^m)B_t^g$$

for the net transfer from the Treasury required to achieve the target L_t neglecting any transfers from the central bank, then

$$T_t = T_t^g + T_t^{cb}.$$

Finally, in addition to choosing the size of its balance sheet, the central bank can choose the nominal interest rate R_t^m paid on its liabilities. In our model, where central-bank liabilities (reserves, reverse repos, or central-bank bills) are treated as perfect substitutes for all other forms of cash (Treasury bills or STSIs issued by private banks), this policy decision directly determines the equilibrium yield on those other forms of cash as well.[20] There are thus two independent dimensions of central-bank

[20]In a more complex model in which reserve balances at the central bank play a special role that other STSIs cannot fulfill, and are in sufficiently scarce supply, there will be a spread between the interest rate paid on reserves and the equilibrium yield on other STSIs, though the central bank will still have relatively direct control over the equilibrium yield on STSIs, by varying either the interest rate paid on reserves or the degree of scarcity of reserves. Even before the increased size of central-bank balance sheets resulting from the financial crisis, many central banks implemented their interest-rate targets largely by varying the interest rate paid on reserve balances, as discussed in Woodford (2003, chap. 1).

policy each period, each of which can be chosen independently of fiscal policy (that is, of the evolution of both total claims on the government L_t and the supply of short-term safe government liabilities), except to the extent that perhaps B_t^{cb} must be no greater than B_t^g.[21] These can alternatively be described as implementation of the central bank's target for the interest rate paid on cash, and variation in the size of its balance sheet holding fixed its target for that interest rate.

There is a further potential dimension of central-bank policy, which is choice of the composition of its balance sheet. Above we have assumed that the central bank holds only longer-term Treasury securities, but it might also hold Treasury bills on its balance sheet (as indeed the Fed does). In our model, however, it is easy to see that central-bank acquisition of T-bills (financed by issuing central-bank liabilities that are perfect substitutes for T-bills and pay the same rate of interest) will have no effect on any other aspect of equilibrium. To simplify the algebra, we do not even allow for this possibility in the notation introduced above.

2 Determinants of Intertemporal Equilibrium

We turn now to a characterization of equilibrium in the model just described. We shall give particular attention to the determinants of the supply of and demand for safe assets, and the supply of and demand for risky durables, both when originally produced and in the event of a fire sale.

2.1 Conditions for Optimal Behavior

We begin our characterization of equilibrium by noting some necessary conditions for optimality of the representative household's behavior. An optimal plan for the

[21] In fact, within the logic of the model, there is no problem with allowing B_t^{cb} to exceed B_t^g; this would simply require negative holdings of government bonds by the private sector (issuance of "synthetic" bonds by the private sector), which can already be accommodated in the constraints specified above.

household (as defined in the previous section) is one that maximizes a Lagrangian

$$\begin{aligned}
\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \quad & \{u(c_{1t}, c_{2t}) + \tilde{u}(c_{3t}) + \gamma[(1-\eta_t)s_t + \eta_t q(s_t + s_t^{*d} - s_t^{*s})] - v(Y_t) - w(x_t) \\
& -\varphi_{1t}\left[M_t + B_t + Q_t(s_t - F(I_{t-1})) - A_t - D_t\right] - \eta_t\varphi_{2t}\left[D_t - \Gamma_t s_t^{*s}\right] \\
& -\eta_t\varphi_{3t}\left[\Gamma_t s^{*s_t} - \Gamma_t s_t\right] - \varphi_{4t}\left[P_t c_{1t} - M_t\right] - \varphi_{5t}\left[\eta_t\Gamma_t s_t^{*d} + \tilde{P}_t c_{3t} - F_t\right] \\
& -\varphi_{6t}[(A_{t+1}/R_{t+1}^m) - M_t - (R_t^b/R_t^m)B_t - P_t(Y_t - c_{1t} - c_{2t} - I_t) - \tilde{P}_t x_t \\
& \qquad -\eta_t\Gamma_t s_t^{*s} + D_t + F_t - T_t]\}, \qquad (2.1)
\end{aligned}$$

where we have substituted (1.7) for $\underline{s}_t$ in the utility function, and (1.8) for W_t in (1.10), in order to eliminate two variables and constraints from the maximization problem (and thus allow simplification of the Lagrangian). We have also included no term corresponding to the constraint (1.9), as in the equilibria discussed below we assume that the borrowing constraint is set so as not to bind in any period.[22]

Differentiating the Lagrangian with respect to the choice variables $M_t, B_t, D_t, s_t, s_t^{*s}$, $s_t^{*d}, F_t, c_{1t}, c_{2t}, c_{3t}, I_t, Y_t, x_t$, and A_{t+1} respectively, we obtain the first-order conditions

$$\varphi_{1t} = \mathrm{E}_t[\varphi_{4t} + \varphi_{6t}], \qquad (2.2)$$

$$\varphi_{1t} = \mathrm{E}_t[(R_t^b/R_t^m)\varphi_{6t}], \qquad (2.3)$$

$$\varphi_{1t} = (1-p)\varphi_{2t} + \mathrm{E}_t\varphi_{6t}, \qquad (2.4)$$

$$\varphi_{1t}Q_t = \gamma[p + (1-p)q] + (1-p)\varphi_{3t}\Gamma_t, \qquad (2.5)$$

$$\gamma q = (\varphi_{2t} - \varphi_{3t})\Gamma_t + \varphi_{6t}^c\Gamma_t, \qquad (2.6)$$

$$\gamma q = \varphi_{5t}^c\Gamma_t, \qquad (2.7)$$

$$\mathrm{E}_t\varphi_{5t} = \mathrm{E}_t\varphi_{6t}, \qquad (2.8)$$

$$u_1(c_{1t}, c_{2t}) = P_t[\varphi_{4t} + \varphi_{6t}], \qquad (2.9)$$

$$u_2(c_{1t}, c_{2t}) = P_t\varphi_{6t}, \qquad (2.10)$$

$$\tilde{u}'(c_{3t}) = \tilde{P}_t\varphi_{5t}, \qquad (2.11)$$

$$\beta\varphi_{1,t+1}Q_{t+1}F'(I_t) = P_t\varphi_{6t}, \qquad (2.12)$$

[22]We assume a borrowing limit that constrains the asymptotic behavior of the household's net wealth position far in the future, so as to preclude running a "Ponzi scheme," but that does not constrain the household's borrowing over any finite number of periods.

$$v'(Y_t) \;=\; P_t \varphi_{6t}, \tag{2.13}$$

$$w'(x_t) \;=\; \tilde{P}_t \varphi_{6t}, \tag{2.14}$$

and

$$\varphi_{6t} \;=\; \beta R^m_{t+1} \varphi_{1,t+1}, \tag{2.15}$$

for each $t \geq 0$.

In these conditions, it should further be understood that the first 7 choice variables (M_t through F_t) must be chosen only as a function of the history ξ_t (i.e., the state at the beginning of period t), while the other 7 variables (c_{1t} through A_{t+1}) may depend on η_t (i.e., whether a crisis occurs in period t) as well as ξ_t. This means that while there is only one condition corresponding to each of the equations (2.2)–(2.8) for each history ξ_t, each of the equations (2.9)–(2.15) actually corresponds to two conditions for each history ξ_t, one for each of the two possible states that may be reached in subperiod 2 (crisis or non-crisis). Similarly, the Lagrange multipliers $\varphi_{1t}, \varphi_{2t}, \varphi_{3t}$ will each have a single value for each history ξ_t, but the values of the multipliers $\varphi_{4t}, \varphi_{5t}, \varphi_{6t}$ may differ depending on the state reached in subperiod 2. The conditional expectation $\mathrm{E}[\cdot]$ that appears in conditions such as (2.2) refers to the expected value (as of the first subperiod of period t) of variables that may take different values depending which state is reached in subperiod 2.

The superscript c appearing on Lagrange multipliers in equations (2.6)–(2.7) indicates the value of the multiplier in the case that the crisis state occurs in subperiod 2. Thus (2.6) indicates the way in which the values of the multipliers $\varphi_{2t}, \varphi_{3t}$ (which relate to constraints that apply only in the event that the crisis state is reached) depend on the value of the multiplier φ_{6t} in the event of a crisis in period t; but note that this value may be different from the value of φ_{6t} if no crisis occurs.

In writing the FOCs in this form, we have assumed for simplicity that any random disturbances (other than learning whether or not an "asset collapse" occurs, after a crisis state is reached in subperiod 2) are realized in subperiod 2 of some period. Under this assumption, there is no difference between the information set in the first subperiod of period $t+1$ (which we have denoted ξ_{t+1}) and the information set in subperiod 2 of period t.[23] We also assume that while the yield R^b_{t+1} on longer-term

[23]There is of course the difference that by the beginning of period $t+1$, it will be known whether an asset collapse occurred in period t, while this is not yet known in period in subperiod 2 of period t (in the case that the crisis state is reached). However, because of the assumption of full depreciation

government debt may depend on the state reached in subperiod 2 of period $t+1$, the yield R^m_{t+1} on safe short-term liabilities of the central bank does not; hence this also must be known as of subperiod 2 of period t. Thus the central bank's decision about the policy rate R^m_{t+1} (which should actually be regarded as the *period t interest-rate decision*[24]) must be announced in subperiod 2 of period t.[25] This allows us to write conditions (2.12) and (2.15) without conditional expectations, as the variables with subscripts $t+1$ in these equations are ones with values that are already perfectly predictable in subperiod 2 of period t.

In addition to the FOCs (2.2)–(2.15), the household's decision variables must satisfy the constraints of the household problem, together with a set of complementary slackness conditions. We can see from condition (2.13), together with the assumption that $v'(Y) > 0$ for all possible values of Y, that $\varphi_{6t} > 0$ necessarily; similarly, if we assume non-satiation in special goods, (2.11) implies that $\varphi_{5t} > 0$ necessarily. Because it is associated with an inequality constraint (condition (1.4)), the multiplier φ_{4t} is necessarily non-negative; condition (2.2) then implies that $\varphi_{1t} > 0$ necessarily. The remaining multipliers, $\varphi_{2t}, \varphi_{3t}, \varphi_{4t}$, are associated with inequality constraints and so are necessarily non-negative, but may be equal to zero if the constraints in question do not bind. (We discuss further below when this will occur.) If any of these multipliers has a positive value, the corresponding inequality constraint must hold with equality.

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of existing durables at the end of each period, while the occurrence of an asset collapse affects the utility of the household, it has no consequences for the assets carried by the household into the following period, the amounts of which are already predictable in subperiod 2 as long as no other random disturbances (such as an unexpected change in the size of net transfers T_t) are allowed to occur in subperiod 3. We assume that policy in periods $t+1$ and later is also independent of whether an asset collapse has occurred in period t. Given this, the relevant information set for equilibrium determination in subperiod 1 of period $t+1$ is independent of whether an asset collapse has occurred.

[24] Note that R^m_{t+1} is the nominal yield between the settlement of accounts at the end of period t and the settlement of accounts at the end of period $t+1$ on wealth that is held in the form of cash. This would often be called the *period t* riskless rate of interest, as it must be determined before the period for which the safe return is guaranteed. We have used the notation R^m_{t+1} rather than R^m_t for consistency with the notation R^b_{t+1} for the one-period holding return on longer-term bonds over the same time period; the latter variable is generally not perfectly predictable in subperiod 2 of period t.

[25] We similarly assume that the Treasury's decision about the T-bill supply $\tilde{M}^g_{t+1}$ and the central bank's decision about the size of its balance sheet $\tilde{M}^{cb}_{t+1}$ are announced in subperiod 2 of period t. The Treasury's decision about the size of net transfers T_t, and hence the value of total claims on the government L_t at the end of period t, are also announced in subperiod 2 of period t.

2.2 Characterizing Equilibrium

In an equilibrium, all of the necessary conditions for optimality of the household's plan just listed must hold, and in addition, the market-clearing conditions (1.11)–(1.16) must hold. We now draw some further conclusions about relations that must exist among the various endogenous variables in an equilibrium, in order to understand how they are affected by central-bank policy.

To simplify the discussion, in the present paper we shall restrict attention to the case in which any exogenous factors that change over time (apart from the occurrence of crisis states and asset collapses, as depicted in Figure 1) are purely deterministic (that is, simply a function of the date t). That is, when we consider the effects of a temporary disturbance of any other type, we shall consider only the case of a shock that occurs in the initial period $t = 0$, with consequences that are perfectly predictable after that. We shall also restrict attention to the effects of alternative monetary and fiscal policies that are similarly deterministic; this means that while we can consider the effects of responding in different ways to a one-time disturbance (in section xx below), we do not consider the effects of responding to the occurrence of a crisis that results in a fire sale of bank assets (or to an asset collapse). The reason is that our concern here is with the consequences for the risks to financial stability of alternative central-bank policies *prior* to the occurrence of a crisis; the interesting (but more complex) question of what can be achieved by suitable use of these instruments to respond to a crisis *after* it occurs is left for a later study.

Under this assumption, neither the occurrence of a crisis nor an asset collapse in any period t affects equilibrium determination in subsequent periods, and we obtain an equilibrium in which the variables listed above as functions of the history ξ_t depend only on the date t, and those listed as functions of the history (ξ_t, η_t) will depend only on the date t and the value of η_t. Moreover, because the resolution of uncertainty during the period has no effect on equilibrium in later periods, the Lagrange multiplier φ_{6t} indicating the shadow value of additional funds in the end-of-period settlement of accounts will be independent of whether a crisis occurs in period t, and as a consequence of this, the price P_t of normal goods, the quantities purchased of normal goods (c_{1t}, c_{2t}, I_t), and the quantity Y_t that are produced will all be independent whether a crisis occurs. Similarly, the Lagrange multiplier φ_{4t} associated with the cash-in-advance constraint will have a value that is independent of whether a crisis occurs.

Thus an equilibrium can be fully described by sequences $\{A_t, M_t, B_t, D_t, F_t, s_t, s_t^*, c_{1t}, c_{2t}, I_t, Y_t, c_{3t}^c, c_{3t}^n\}$ describing the choices of the representative household,[26] sequences $\{Q_t, \Gamma_t, P_t, \tilde{P}_t^c, \tilde{P}_t^n\}$ of prices and sequences $\{R_t^m, R_t^{bc}, R_t^{bn}\}$ of yields on government securities, and sequences $\{\varphi_{1t}, \varphi_{2t}, \varphi_{3t}, \varphi_{4t}, \varphi_{5t}^c, \varphi_{5t}^n, \varphi_{6t}\}$ of Lagrange multipliers. Here the superscripts c and n are used to indicate the values that variables take in a given period conditional upon whether the crisis state (superscript c) or the non-crisis state (superscript n) is reached; variables without superscripts take values that depend only on the date. In order for these sequences to represent an equilibrium, they must satisfy all of the equilibrium conditions stated above for each date, and for each of the possible states in subperiod 2. Note that conditional expectations are no longer needed in equilibrium relations such as (2.2) or (2.4), and that the c superscript is no longer needed in (2.6).

2.3 Prices and Quantities Transacted in a Crisis

We turn now to a more compact description of the conditions that must hold in equilibrium. We begin with a discussion of the relations that determine the equilibrium supply of special goods, the degree to which investors are financially constrained, and the price of durable goods in the event of a fire sale.

We first note that (2.11) and (2.14), together with the requirement that $c_{3t} = x_t$ in each state, require that

$$\frac{\tilde{u}'(c_{3t}^s)}{w'(c_{3t}^s)} \;=\; \tilde{\varphi}_{5t}^s \;\equiv\; \frac{\varphi_{5t}^s}{\varphi_{6t}} \tag{2.16}$$

for each possible state s (equal to either c or n) that may be reached in subperiod 2. Since the left-hand side of (2.16) is a monotonically decreasing function, we can solve this equation uniquely for the demand for special goods in each state,

$$c_{3t}^s \;=\; c_3(\tilde{\varphi}_{5t}^s),$$

where we introduce the notation $\tilde{\varphi}_{kt} \equiv \varphi_{kt}/\varphi_{6t}$ for any $k \neq 6$, and $c_3(\cdot)$ is the monotonically decreasing function implicitly defined by (2.16).

[26]Here we have reduced the number of separate variables by using a single symbol s_t^* to refer to both s_t^{*s} and s_t^{*d}, as these are necessarily equal in any equilibrium, and similarly eliminated separate reference to x_t since it must always be equal to c_{3t} in any equilibrium.

Here $\tilde{\varphi}_{5t}^s$ measures the degree of financial constraint of investors in state s of subperiod 2. The value $\tilde{\varphi}_{5t}^s = 1$ would imply no ex post regret in state s about the size of the credit line arranged for the investor, and a demand for special goods that is the same as if there were no constraint separating the funds of the investor from those of the rest of the household; $\tilde{\varphi}_{5t}^s > 1$ indicates that ex post, the household would wish it had arranged more credit for the investor, while $\tilde{\varphi}_{5t}^s < 1$ would imply that it would wish it had arranged less. We also note that the socially efficient level of production and consumption of special goods in either state is given by the quantity c_3^* such that

$$\frac{\tilde{u}'(c_3^*)}{w'(c_3^*)} \;=\; 1.$$

Hence special goods are under-produced or over-produced in state s according to whether $\tilde{\varphi}_{5t}^s$ is greater or less than 1.

We can then use (2.14) to obtain the implied state-contingent price of special goods (in units of end-of-period marginal utility),

$$\varphi_{6t}\tilde{P}_t^s \;=\; \tilde{p}(\tilde{\varphi}_{5t}^s) \;\equiv\; w'(c_3(\tilde{\varphi}_{5t}^s)),$$

and the implied state-contingent expenditure on special goods (in the same units),

$$\varphi_{6t}\tilde{P}_t^s c_{3t}^s \;=\; e_3(\tilde{\varphi}_{5t}^s) \;\equiv\; \tilde{p}(\tilde{\varphi}_{5t}^s)c_3(\tilde{\varphi}_{5t}^s).$$

Note that $e_3(\tilde{\varphi}_5)$ will be a monotonically decreasing function.

Since $\varphi_{5t} > 0$ in each state, budget constraint (1.5) must hold with equality in each state. The fact that F_t must not be state-contingent then implies that the left-hand side of (1.5) must be the same whether a crisis occurs or not, so that in equilibrium,

$$e_3(\tilde{\varphi}_{5t}^n) \;=\; e_3(\tilde{\varphi}_{5t}^c) \;+\; \tilde{\Gamma}_t s_t^* \tag{2.17}$$

each period, where $\tilde{\Gamma}_t \equiv \varphi_{6t}\Gamma_t$. We note also that condition (2.8) implies that

$$(1-p)\,\tilde{\varphi}_{5t}^c \;+\; p\,\tilde{\varphi}_{5t}^n \;=\; 1.$$

We can solve this equation for $\tilde{\varphi}_{5t}^n = \tilde{\varphi}_5^n(\tilde{\varphi}_{5t}^c)$, a monotonically decreasing function with the property that $\tilde{\varphi}_5^n(1) = 1$. Substituting this for $\tilde{\varphi}_{5t}^n$ in (2.17), we obtain an equation

$$\tilde{D}(\tilde{\varphi}_{5t}^c) \;=\; \tilde{\Gamma}_t s_t^*, \tag{2.18}$$

where

$$\tilde{D}(\tilde{\varphi}_5^c) \;\equiv\; e_3(\tilde{\varphi}_5^n(\tilde{\varphi}_{5t}^c)) \,-\, e_3(\tilde{\varphi}_{5t}^c)$$

is a monotonically increasing function with the property that $\tilde{D}(1) = 0$.

Finally, we note that (2.7) implies that

$$\tilde{\varphi}_{5t}^c \tilde{\Gamma}_t \;=\; \gamma q. \tag{2.19}$$

This together with (2.18) implies that

$$\tilde{\varphi}_{5t}^c\, \tilde{D}(\tilde{\varphi}_{5t}^c) \;=\; \gamma q s_t^*.$$

Since the left-hand side of this equation is a monotonically increasing function of $\tilde{\varphi}_{5t}^c$, it can be uniquely solved for

$$\tilde{\varphi}_{5t}^c \;=\; \tilde{\varphi}_5^c(s_t^*), \tag{2.20}$$

where $\tilde{\varphi}_5^c(s^*)$ is a monotonically increasing function with the property that $\tilde{\varphi}_5^c(0) = 1$.

This solution for the equilibrium value of the multiplier $\tilde{\varphi}_{5t}^c$ then allows us to solve for the implied values of $\tilde{\Gamma}_t$, $\tilde{\varphi}_{5t}^n$, c_{3t}^c, c_{3t}^n, $\varphi_{6t}\tilde{P}_t^c$, and $\varphi_{6t}\tilde{P}_t^n$, each as a function of the quantity s_t^* of durable goods that are sold in the fire sale (if one occurs) in period t. We observe that $\tilde{\varphi}_{5t}^c$ and c_{3t}^n will be increasing functions of s_t^*, and $\varphi_{6t}\tilde{P}_t^n$ will be non-decreasing, while $\tilde{\Gamma}_t$, $\tilde{\varphi}_{5t}^n$, and c_{3t}^c will be decreasing functions of s_t^* and $\varphi_{6t}\tilde{P}_t^c$ will be non-increasing.

In the case that $s_t^* = 0$ (no assets are sold in a fire sale), we will have $c_{3t}^c = c_{3t}^n = c_3^*$ (the efficient quantity of special goods are produced in both states), $\tilde{\varphi}_{5t}^c = \tilde{\varphi}_{5t}^n = 1$ (no regret about the size of the line of credit arranged for the investor, in either state), and $\tilde{\Gamma}_t = \gamma q$ (the market price of durables in the crisis state is equal to their "fundamental" value). Instead, if $s_t^* > 0$ (that is, if any assets are sold in a fire sale), $c_{3t}^c < c_3^* < c_{3t}^n$, $\tilde{\varphi}_{5t}^n < 1 < \tilde{\varphi}_{5t}^c$, and $\tilde{\Gamma}_t < \gamma q$. This means that special goods are under-produced in the crisis state and over-produced in the non-crisis state, and that ex post, the household wishes it had supplied more credit for its investor if the crisis state occurs, while it wishes that it had supplied less credit if the crisis state does not occur. It also means that if the crisis state occurs, the price at which durables are sold in the fire sale is less than their "fundamental" value, conditional on reaching that state. Moreover, the size of these distortions is greater the larger is the aggregate value of s_t^*. The fact that households do not take these equilibrium effects into account when choosing their planned value of s_t^{*s} results in a pecuniary externality.

2.4 Implications of the Demand for Safe Assets

We turn next to a discussion of the consequences of the supply of short-term safe instruments for equilibrium purchases of cash and credit goods. We consider first the implications of optimality conditions (2.9)–(2.10), together with the cash-in-advance constraint (1.4) and the associated complementary slackness condition.

Let us first define the demand functions $c_1^*(\lambda), c_2^*(\lambda)$ as the solution to the problem of choosing c_1 and c_2 to maximize

$$u(c_1, c_2) \;-\; \lambda(c_1 + c_2)$$

for an arbitrary "price" $\lambda > 0$. Under the assumption that cash and credit goods are both normal goods, both $c_1^*(\lambda)$ and $c_2^*(\lambda)$ must be monotonically decreasing functions.[27] We can then consider the constrained problem

$$\max_{c_1, c_2} \; u(c_1, c_2) \;-\; \lambda(c_1 + c_2) \qquad \text{s.t.} \quad c_1 \leq m, \tag{2.21}$$

where $m > 0$ represents real cash balances available to the household. The solution $c_1(\lambda; m), c_2(\lambda; m)$ to problem (2.21) can be characterized as follows: if $m \leq c_1^*(\lambda)$, then $c_1(\lambda; m) = m$ and $c_2(\lambda; m)$ is implicitly defined by the equation

$$u_2(m, c_2) \;=\; \lambda. \tag{2.22}$$

If instead $m \leq c_1^*(\lambda)$, then $c_1(\lambda; m) = c_1^*(\lambda)$ and $c_2(\lambda; m) = c_2^*(\lambda)$.

The Kuhn-Tucker conditions for this latter, constrained problem are easily seen to correspond precisely to conditions (2.9)–(2.10) and constraint (1.4) together with the complementary slackness condition, where the price of normal goods in units of end-of-period marginal utility is given by $\lambda_t \equiv \varphi_{6t} P_t$, and available real cash balances are given by $m_t \equiv M_t/P_t$. It follows that our model implies that c_{1t}, c_{2t} must satisfy

$$c_{jt} \;=\; c_j(\lambda_t; M_t/P_t)$$

for $j = 1, 2$, where the functions $c_j(\lambda; m)$ are defined in the previous paragraph.

Associated with this solution will be a value for the normalized Lagrange multiplier $\tilde{\varphi}_{4t}$, given by

$$\tilde{\varphi}_{4t} \;=\; \tilde{\varphi}_4(\lambda_t; M_t/P_t),$$

[27]The paths followed by the two variables as λ is reduced correspond to the "income-expansion path" as a result of increasing the budget available to spend on these two goods, for a fixed relative price (equal prices of the two goods).

where we define

$$\tilde{\varphi}_4(\lambda; m) \;\equiv\; \frac{u_1(c_1(\lambda; m), c_2(\lambda; m))}{u_2(c_1(\lambda; m), c_2(\lambda; m))} \;-\; 1.$$

Note further that the Kuhn-Tucker conditions for the problem (2.21) imply that $\tilde{\varphi}_4(\lambda; m) = 0$ for all $m \geq c_1^*(\lambda)$, while $\tilde{\varphi}_4(\lambda; m) > 0$ for all $m < c_1^*(\lambda)$. Furthermore, in the latter case (where the cash-in-advance constraint binds), the assumption that both cash goods and credit goods are normal goods implies that $\tilde{\varphi}_4(\lambda; m)$ is a decreasing function of λ for fixed m,[28] and a decreasing function of m for fixed λ.[29]

A comparison of (2.2) with (2.4) (and recalling that the conditional expectations have been eliminated from both of these conditions) implies that under any optimal plan, we must have $\tilde{\varphi}_{4t} = (1-p)\tilde{\varphi}_{2t}$. Hence in any equilibrium where the cash-in-advance constraint binds in some period, so that $\tilde{\varphi}_{4t} > 0$, we must also have $\tilde{\varphi}_{2t} > 0$, so that the first inequality in (1.6) is also a binding constraint, and $D_t = \Gamma_t s_t^*$ (as much as collateralized debt is issued by bankers as can be repaid in the event of a crisis, given the quantity of durables that bankers plan to sell in a fire sale). More generally, we can conclude that the normalized Lagrange multiplier $\tilde{\varphi}_{2t}$ will be given by

$$\tilde{\varphi}_{2t} \;=\; \tilde{\varphi}_2(\lambda_t; M_t/P_t),$$

where we define

$$\tilde{\varphi}_2(\lambda; m) \;\equiv\; \tilde{\varphi}_4(\lambda; m)/(1-p).$$

Condition (2.2) implies that the normalized multiplier $\tilde{\varphi}_{1t}$ will similarly be given by a function

$$\tilde{\varphi}_{1t} \;=\; \tilde{\varphi}_1(\lambda_t; M_t/P_t), \tag{2.23}$$

[28] Concavity of the utility function implies that increasing c_2 while c_1 remains fixed at m implies a decrease in the marginal utility of credit goods consumption, so that increasing λ with fixed m must correspond to a reduction in the quantity of c_2 that is purchased. In order for the demand m for cash goods to remain the same despite a budget contraction that requires fewer credit goods to be purchased, the relative price of cash goods must decrease (under the assumption of normal goods). This means that u_1/u_2 must decrease, and hence that $\tilde{\varphi}_4$ must decrease.

[29] In the $\lambda - m$ plane, the level curves of the function $\tilde{\varphi}_4$ correspond to income-expansion paths, as the budget for cash and credit goods changes with the relative price of the two types of goods fixed. If the two goods are both normal goods, m must increase along such a path as λ decreases, as discussed above; hence the level curves must have a negative slope at all points. It then follows that the sign of this partial derivative follows from the sign of the one discussed in the previous footnote.

where we define

$$\tilde{\varphi}_1(\lambda; m) \;\equiv\; 1 \,+\, \tilde{\varphi}_4(\lambda; m).$$

It follows that $\tilde{\varphi}_{1t} > 1$ if and only if the cash-in-advance constraint binds, while it is equal to 1 otherwise. Note also that both $\tilde{\varphi}_1(\lambda; m)$ and $\tilde{\varphi}_2(\lambda; m)$ will be decreasing in both arguments, in the region where the cash-in-advance constraint binds.

A comparison of (2.6) with (2.7) similarly implies that under any optimal plan, we must have

$$\tilde{\varphi}^c_{5t} \,-\, 1 \;=\; \tilde{\varphi}_{2t} \,-\, \tilde{\varphi}_{3t}. \tag{2.24}$$

This allows us to solve for the implied value of the normalized multiplier $\tilde{\varphi}_{3t}$ as

$$\tilde{\varphi}_{3t} \;=\; \tilde{\varphi}_3(\lambda_t; s^*_t, M_t/P_t),$$

where we define

$$\tilde{\varphi}_3(\lambda_t; s^*_t, M_t/P_t) \;\equiv\; \tilde{\varphi}_2(\lambda_t; M_t/P_t) \,+\, 1 \,-\, \tilde{\varphi}^c_5(s^*_t). \tag{2.25}$$

The supply of real cash balances M_t/P_t and the quantity of assets s^*_t sold in the event of a fire sale must be endogenously determined in such a way as to guarantee that in equilibrium, the value of this function is always non-negative. (We show below the existence of such a solution.)

Finally, (2.5) can be used to determine the equilibrium price of risky durables in the subperiod 1 market. If we let $\tilde{Q}_t \equiv \varphi_{6t} Q_t$ denote this price in marginal-utility units, then we obtain a solution of the form

$$\tilde{Q}_t \;=\; \tilde{Q}(\lambda_t; s^*_t, M_t/P_t),$$

where we define

$$\tilde{Q}(\lambda_t; s^*_t, M_t/P_t) \;\equiv\; \frac{\tilde{Q}^* \,+\, (1-p)\tilde{\varphi}_3(\lambda_t; s^*_t, M_t/P_t)\tilde{\Gamma}(s^*_t)}{\tilde{\varphi}_1(\lambda_t; M_t/P_t)}. \tag{2.26}$$

Here we use the notation

$$\tilde{Q}^* \;\equiv\; \gamma\, [p \,+\, (1-p)q]$$

for the expected marginal utility of the anticipated service flow from a durable purchased in subperiod 1, and

$$\tilde{\Gamma}(s^*) \equiv \gamma q / \tilde{\varphi}^c_5(s^*)$$

for the solution for $\tilde{\Gamma}_t$ derived in the previous section.

Note that the "fundamental value" of a durable purchased in subperiod 1, if the anticipated future service flow were to be valued using the same pricing kernel that is used to price bonds in (2.3),[30] would equal[31]

$$\tilde{Q}_t^{fund} \equiv \frac{\tilde{Q}^*}{\tilde{\varphi}_{1t}}. \tag{2.27}$$

Thus (2.26) implies that durables will be priced at their fundamental value in subperiod 1 if and only if the second inequality in (1.5) is not a binding constraint; that is, the quantity of durables held by bankers (and thus the availability of collateral) does not constrain bankers to issue less collateralized debt than they would otherwise wish. When the constraint binds, so that $\tilde{\varphi}_{3t} > 0$, durables are over-valued in subperiod 1. Our discussion above of the equilibrium value of $\tilde{\varphi}_{3t}$ implies that in order for this to happen, the cash-in-advance constraint must bind (so that $\tilde{\varphi}_{2t} > 0$), while the supply of durables (and hence the equilibrium value of s_t^*) must not be too large (so that $\tilde{\varphi}_5^c(s_t^*)$ is not too much greater than 1).

2.5 Determinants of the Supply of Safe Assets

We turn now to the endogenous determination of the cash supply M_t, as a result of the financing decisions of bankers. Since $\varphi_5^c(s_t^*) > 1$ if $s_t^* > 0$, we can conclude that if any assets will be sold by bankers in the event of a fire sale, the left-hand side, and hence also the right-hand side, of (2.24) must be positive. But the right-hand side of (2.24) can be positive only if $\tilde{\varphi}_{2t}$ is positive, which occurs only if the cash-in-advance constraint binds. This in turn would require that $D_t = \Gamma_t s_t^*$, as argued in the previous paragraph, and hence (using (1.11)) that

$$M_t = \tilde{M}_t + \Gamma_t s_t^*. \tag{2.28}$$

On the other hand, if $s_t^* = 0$, constraint (1.5) requires that $D_t = 0$ as well, so that (2.28) must hold in this case as well. We may thus conclude that in any equilibrium,

[30] Note that is a general pricing relation for non-cash assets, since we make no particular assumption about the nature of the state-contingent return on bonds, only that this asset cannot be used as a means of payment in the cash goods market.

[31] Equation (2.3) states that an asset that yields Y_t at the end of period in marginal-utility units should have a price in subperiod 1 of $P_t^Y = \mathrm{E}_t[Y_t]/\varphi_{1t}$. (For the case of longer-term bonds, $Y_t = \varphi_{6t} R_t^b$ and the price in the subperiod 1 market is $P_t^Y = R_t^m$.)

the total supply of cash will be given by (2.28).

It remains to determine the equilibrium value of s_t^*. We first note that in marginal-utility units, (2.28) can be written

$$\hat{M}_t \equiv \varphi_{6t} M_t = \lambda_t \tilde{m}_t + \tilde{\Gamma}_t s_t^*, \tag{2.29}$$

using the notation $\tilde{m}_t \equiv \tilde{M}_t / P_t$ for the real supply of safe assets by the government. Then in any equilibrium where

$$\tilde{m}_t + \tilde{\Gamma}_t s_t^* / \lambda_t > c_1^*(\lambda_t),$$

the cash-in-advance constraint will not bind; but since this implies that $\tilde{\varphi}_{2t} = 0$, (2.24) implies that $\tilde{\varphi}_{5t}^c$ cannot be greater than 1, which requires that $s_t^* = 0$.

Hence such an equilibrium occurs if and only if

$$\tilde{m}_t > \tilde{m}^*(\lambda_t) \equiv c_1^*(\lambda_t), \tag{2.30}$$

and involves $\hat{M}_t = \lambda_t \tilde{m}_t$. In this case, (2.25) implies that $\tilde{\varphi}_{3t} = 0$, so that $\tilde{Q}_t$ is equal to the fundamental value (2.27). In addition, because $s_t^* = 0$, we must have $\tilde{\Gamma}_t = \tilde{\Gamma}(0) = 1$, so that durables are also priced at their fundamental value in subperiod 2, even if the crisis state is reached.

Let us consider now the possibility of an equilibrium in which the supply of real cash balances is no greater than $c_1^*(\lambda_t)$ (the level required for satiation in cash), but the supply of durables s_t is large enough so that bankers are unconstrained in the amount of collateralized debt that they can issue (so that $\tilde{\varphi}_{3t} = 0$). Because of (2.24), this requires a value of s_t^* such that

$$\tilde{\varphi}_5^c(s_t^*) - 1 = \tilde{\varphi}_2(\lambda_t; \tilde{m}_t + \tilde{\Gamma}(s_t^*) s_t^* / \lambda_t). \tag{2.31}$$

It follows from our discussion above that the left-hand side of this equation is an increasing function of s_t^*, while the right-hand side is a non-increasing function of s_t^* (decreasing until the point at which the cash-in-advance constraint ceases to bind, and constant thereafter).[32] Moreover, the right-hand side is at least as large as the left-hand side if $s_t^* = 0$, given our assumption now that $\tilde{m}_t \leq c_1^*(\lambda_t)$. Hence there is

[32]Recall that $\tilde{\Gamma}(s^*) s^* = \tilde{D}(\tilde{\varphi}_5^c(s^*))$ is a monotonically increasing function of s^*, and that $\tilde{\varphi}_2(\lambda; m)$ is a decreasing function of m as long as the cash-in-advance constraint binds, and independent of the value of m for all higher values.

a unique value of $0 \leq s_t^* < s_t$ that satisfies (2.31) if and only if the left-hand side is greater than the right-hand side when $s_t^* = s_t$, which is to say, if and only if

$$\tilde{\varphi}_5^c(s_t) \,-\, 1 \;>\; \tilde{\varphi}_2(\lambda_t;\, \tilde{m}_t + \tilde{\Gamma}(s_t)s_t/\lambda_t). \tag{2.32}$$

Thus such an equilibrium exists in period t if and only if the outside supply of safe assets $\tilde{m}_t$ fails to satisfy (2.30) while the supply of durables s_t does satisfy (2.32); in such a case, s_t^* is implicitly defined by (2.31), and the total supply of cash is given by (2.29). In this case, again $\tilde{\varphi}_{3t} = 0$, and hence $\tilde{Q}_t = \tilde{Q}_t^{fund}$. Moreover, if $\tilde{m}_t < c_1^*(\lambda_t)$, the solution must involve $s_t^* > 0$ and hence $\tilde{\Gamma}_t < 1$, so that durables are under-priced in the fire sale in the event of a crisis.

If, instead, $\tilde{m}_t$ does not satisfy (2.30) and the supply of durables s_t fails to satisfy (2.32), then there can only be an equilibrium in which $s_t^* = s_t$. In this case, the supply of safe assets is given by

$$\hat{M}_t \;=\; \lambda_t \tilde{m}_t + \tilde{\Gamma}(s_t)s_t. \tag{2.33}$$

The value of $\tilde{\varphi}_{3t}$ is given by (2.25), which will be positive in the case of any value of s_t such that the inequality in (2.32) is reversed. In any such case, we must have $\tilde{Q}_t > \tilde{Q}_t^{fund}$, so that durables are overvalued in subperiod 1. In addition, the fact that $s_t^* > 0$ implies that $\tilde{\Gamma}_t < 1$, so that durables are *under-priced* in the event of a fire sale, even though they are *over-priced* in subperiod 1. In this case, an asset "boom" can be followed by a "crash."

Thus we are able to completely characterize the equilibrium pricing of risky durables in any period t (both in subperiod 1 and in the event of a crisis), as a function of three quantities: the real supply $\tilde{m}_t$ of safe assets by the government (determined by fiscal policy and central-bank asset purchases), the supply of durables s_t (which follows directly from the quantity I_{t-1} of investment goods produced in the previous period), and the marginal utility λ_t that the representative household assigns to additional real end-of-period wealth. The latter quantity depends on expectations about subsequent periods, as we discuss next.

In particular, we can write the subperiod 1 equilibrium price of durables, expressed in marginal-utility units, as a function

$$\tilde{\varphi}_{1t}\tilde{Q}_t \;=\; \phi(\lambda_t; s_t, \tilde{m}_t)$$

derived in the manner just explained. It is useful for the discussion below to consider how this function depends on the supply of durables s_t. In the case of an outside cash supply satisfying $\tilde{m}_t > c_1^*(\lambda_t)$, or a supply of durables satisfying (2.32), in equilibrium we must have $\tilde{\varphi}_{3t} = 0$, so that (2.26) implies that $\phi(\lambda_t; s_t, \tilde{m}_t) = \tilde{Q}^*$. Thus the value of the function is independent of the value of s_t in either of these cases. If instead we have both an outside cash supply below the satiation level and a supply of durables too small to satisfy (2.32), the equilibrium supply of safe assets is given by (2.33). The right-hand side of this equation is a monotonically increasing function of s_t, so that $M_t/P_t = \hat{M}_t/\lambda_t$ is also an increasing function of s_t.

It follows from this that the equilibrium value of $\tilde{\varphi}_{3t}$ given by (2.25) will be a monotonically decreasing function of s_t. It then follows from (2.26) that $\tilde{\varphi}_{1t}\tilde{Q}_t$ will be a monotonically decreasing function of s_t, and hence that the function $\phi(\lambda_t; s_t, \tilde{m}_t)$ is decreasing in this argument. Thus in the case that $\tilde{m}_t < c_1^*(\lambda_t)$, the function $\phi(\lambda_t; s_t, \tilde{m}_t)$ will be a decreasing function of s_t for all supplies of durables too small to satisfy (2.32), and will instead be constant at its minimum value of $\tilde{Q}^*$ for all s_t large enough to satisfy (2.32). The function is constant (and equal to $\tilde{Q}^*$) whenever $\tilde{m}_t > c_1^*(\lambda_t)$, regardless of the value of s_t.

It will also be useful for our discussion below of intertemporal equilibrium to note that the relative value of funds available in subperiod 1 as opposed to the end of the period will be given by a function of the form

$$\tilde{\varphi}_{1t} \;=\; \hat{\varphi}_1(\lambda_t; s_t, \tilde{m}_t). \tag{2.34}$$

This function depends only on the value of λ_t, in the case that $\tilde{m}_t \geq c_1^*(\lambda_t)$, so that there is satiation in cash. It depends on both λ_t and $\tilde{m}_t$ in the case that $\tilde{m}_t < c_1^*(\lambda_t)$ but s_t is large enough to satisfy (2.32), but does not depend on s_t, since in this case bankers' collateral constraint does not bind, and s_t^* is independent of the size of s_t. Finally, in the case that $\tilde{m}_t < c_1^*(\lambda_t)$ and s_t is too small to satisfy (2.32), the value of the function depends on all three of its arguments. (In this latter case, M_t/P_t will be an increasing function of s_t, for given values of the other two arguments, as just discussed; hence $\tilde{\varphi}_{1t}$ will be a decreasing function of s_t, for s_t in this range.)

2.6 Intertemporal Equilibrium

We now consider the connections between variables in successive periods required for an intertemporal equilibrium. One such connection is given by condition (2.12) for optimal investment demand. Using the solution for the subperiod 1 equilibrium price of durables just derived, condition (2.12) can be written in the alternative form

$$\lambda_t \;=\; \beta\, \phi(\lambda_{t+1}; F(I_t), \tilde{m}_{t+1})\, F'(I_t). \tag{2.35}$$

(Here we have also used the fact that the supply of durables in period $t+1$ must equal $s_{t+1} = F(I_t)$.)

Since the right-hand side of this expression must be a monotonically decreasing function of I_t,[33] condition (2.35) has a unique solution for the equilibrium value of I_t, which we can write in the form

$$I_t \;=\; I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1}). \tag{2.36}$$

Because the right-hand side of (2.35) is a decreasing function of I_t, the function $I(\lambda; \lambda', \tilde{m})$ implicitly defined by this equation will be a monotonically decreasing function of λ. Thus we obtain a "demand curve" for investment that is a decreasing function of λ_t, similar to the demands for cash and credit goods as decreasing functions of λ_t that can be derived in the way explained above. But whereas the demands for cash and credit goods depend on s_t and $\tilde{m}_t$ along with the value of λ_t, investment demand depends on expectations regarding the values of λ_{t+1} and $\tilde{m}_{t+1}$ along with the value of λ_t.

If we write our solution for the sum of the demands for cash and credit goods as

$$c_{1t} + c_{2t} \;=\; y(\lambda_t; s_t, \tilde{m}_t),$$

then the aggregate demand for normal goods can be written as

$$Y_t \;=\; y(\lambda_t; s_t, \tilde{m}_t) \,+\, I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1}). \tag{2.37}$$

In a flexible-price equilibrium (the kind assumed thus far), this quantity of normal goods will also have to be voluntarily supplied, which requires that condition (2.13) be satisfied. Hence the equilibrium value of λ_t must satisfy

$$v'(y(\lambda_t; F(I_{t-1}), \tilde{m}_t) \,+\, I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})) \;=\; \lambda_t. \tag{2.38}$$

Since the left-hand side of this equation is a non-increasing function of λ_t (strictly decreasing if $v'' > 0$), there will be a unique solution for λ_t corresponding to given values of $I_{t-1}, \tilde{m}_t, \tilde{m}_{t+1}$, and λ_{t+1}.

[33] Here we rely upon the demonstration above that $\phi(\lambda; s, \tilde{m})$ is a non-increasing, positive-valued function of s, in addition to our assumption that the function $F(I)$ is strictly concave.

In the initial period of the model, the value of I_{t-1} will be given as an initial condition; but in all subsequent periods, the value will be endogenously determined by (2.36). Hence for all periods after the initial period, we obtain an equilibrium relation of the form

$$v'(y(\lambda_t; F(I(\lambda_{t-1}; \lambda_t, \tilde{m}_t)), \tilde{m}_t) \ + \ I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})) \ = \ \lambda_t. \qquad (2.39)$$

Given an initial stock of investment goods I_{-1} in period $t = 0$, and a path for $\{\tilde{m}_t\}$ for all $t \geq 0$ (determined by fiscal policy and the central bank's balance-sheet policy), an intertemporal equilibrium is then a sequence of anticipated values $\{\lambda_t\}$ for all $t \geq 0$ that satisfy equation (2.38) when $t = 0$ and the second-order nonlinear difference equation (2.39) for all $t \geq 1$.

Given a solution for the path $\{\lambda_t\}$, the associated path for the production of investment goods is given by (2.36) for all $t \geq 0$. This in turn implies a supply of durables s_t for each period $t \geq 0$, using (1.13). One then has sequences of values $\{\lambda_t, s_t, \tilde{m}_t\}$ for each of the periods $t \geq 0$. The implied values for the variables $s_t^*, M_t/P_t$, and so on, as well as for the various normalized Lagrange multipliers, can then be determined for each of these periods using the results derived in the previous sections.

This gives us a solution for the allocation of resources, all relative prices and all *real* asset prices, that involves no reference to any nominal variables, as long as the central bank's balance-sheet policy is specified in real terms (since the real supply of outside safe assets is used in the above calculations). In fact, the only element of policy that matters for the determination of real variables in the flexible-price version of the model is the path of $\{\tilde{m}_t\}$. The path of government debt as a whole does not matter for the determination of any variables in the model: "Ricardian equivalence" obtains (given our assumption of a representative household and lump-sum taxes and transfers), except for the qualification that changes in the government supply of safe assets are not neutral in this model, owing to the cash-in-advance constraint.[34]

Conventional monetary policy (the central bank's control of the interest rate on cash balances R_t^m) is also irrelevant to the determination of real variables, though it can be used to control the general level of prices (the path $\{P_t\}$, and along with it the prices of other goods and assets in monetary units). Condition (2.15) requires

[34]For the same reason, it does not matter exactly what type of liabilities the government issues other than short-term safe assets; and it similarly does not matter, in this model, what type of non-cash assets are held on the balance sheet of the central bank.

that in equilibrium

$$R^m_{t+1} \;=\; (1+r^m_{t+1})\frac{P_{t+1}}{P_t}, \qquad (2.40)$$

where

$$1+r^m_{t+1} \;\equiv\; \frac{\lambda_t}{\beta\lambda_{t+1}\hat{\varphi}_1(\lambda_{t+1}; s_{t+1}, \tilde{m}_{t+1})}$$

is the equilibrium real return on cash between the end of period t and the end of period $t+1$. Note that the path of the variable $\{r^m_{t+1}\}$ is determined for all $t \geq 0$ by the path of $\{\tilde{m}_t\}$ in the manner discussed above, as with all other real variables. Equation (2.40) then describes the Fisher relation that must hold between the nominal interest rate on cash and the rate of inflation.

The equilibrium paths of the price level $\{P_t\}$ for $t \geq 0$ and of the nominal interest rate $\{R^m_{t+1}\}$ for $t \geq 0$ are jointly determined by the equilibrium relation (2.40) and the reaction function (that may for example be of the form $R^m_{t+1} = \psi(P_t/P_{t-1})$) that specifies how the central bank's interest-rate target responds to variation in the price level. The discussion of how this occurs follows exactly the lines of the discussion of price-level determination in a flexible-price "cashless economy" in Woodford (2003, chap. 2). Note that while the present model includes a number of financial frictions and other complications not present in the simple model used in that discussion, what matters is that the variable r^m_{t+1} in equation (2.40) evolves in a way that is completely exogenous with respect to the evolution of the price level and independent of the specification of (conventional) monetary policy.

It will simplify the discussion that follows if we let conventional monetary policy be specified not by a central-bank reaction function, but rather by a target path for the price level $\{P_t\}$ for all $t \geq 0$. Since this target path can be achieved by a suitable rule for setting the interest rate R^m_{t+1} (assuming that equation (2.40) does not imply a negative nominal rate at any time,[35] given the target path of prices), we will simply assume that the path of the price level conforms to the target path chosen by the central bank, and use equation (2.40) to determine the implied equilibrium evolution of the nominal interest rate on cash.

[35] The model as described above would in fact not preclude a negative nominal interest rate in equilibrium, i.e., a value $R^m_{t+1} < 1$. It is more realistic, however, to add an assumption that households can demand currency from the central bank at any time in exchange for interest-earning cash, which would for institutional reasons earn a zero nominal interest rate, and that such currency would be acceptable as payment for cash goods. The possibility of holding currency would then preclude equilibria with $R^m_{t+1} < 1$ in any period.

Finally, condition (2.3) requires that the equilibrium expected return on bonds satisfy

$$\mathrm{E}_t[R_t^b] \;=\; \hat{\varphi}_1(\lambda_t; s_t, \tilde{m}_t)$$

in all periods $t \geq 0$. Given a specification of the character of this alternative form of government debt to determine the relative value of bonds in states c and n, this relation then completely determines the state-contingent returns on bonds. Note that solution for equilibrium bond yields is not necessary in order to solve for any of the other variables discussed earlier; hence we need not further discuss the character of bonds or their equilibrium prices.

3 The Size of the Central-Bank Balance Sheet and Stationary Equilibrium

We wish to compare the effects of the two dimensions of central-bank policy: variation in its target for the interest rate R_t^m paid on cash, and variation in the size of its balance sheet holding fixed its target for that interest rate. We first compare alternative possible long-run stationary equilibria, in which the inflation rate, the various interest rates, and relative prices are all constant over time, and the real size of the central-bank balance sheet and the real supply of T-bills by the Treasury are constant over time as well. We can show that there exists a two-dimensional family of such stationary equilibria. Moreover, fixing the real supply of Treasury bills, it is still possible to move in both directions within this two-dimensional family of stationary equilibria by varying the two independent dimensions of central-bank policy. Thus even a simple consideration of stationary equilibria allows us to observe the separate effects of the two dimensions of policy.

3.1 Alternative Stationary Equilibria

In a stationary equilibrium, we assume that the government pursues a constant inflation target

$$\frac{P_t}{P_{t-1}} \;=\; \Pi \;>\; 0$$

for all $t \geq 0$, starting from some given initial price level P_{-1}, and chooses to supply a constant quantity of real outside cash balances $\tilde{m}_t = \tilde{m}$ in all periods $t \geq 0$ as well.[36] We further assume that there are no transitory disturbances to preferences, technological possibilities or financial constraints (so that the equations derived above apply in all periods, with no modifications), and that the economy starts from an initial stock of investment goods I_{-1} that takes the particular value I with the property that starting with this level of investment goods results in an equilibrium in which $I_t = I$ for all $t \geq 0$ as well. In such a case (and for choices of the targets Π and $\tilde{m}$ within suitable ranges), we can show the existence of an intertemporal equilibrium with the special property that the variables $c_{1t}, c_{2t}, c^c_{3t}, c^n_{3t}, s_t, s^*_t, \lambda_t, \tilde{Q}_t, \tilde{\Lambda}_t, \tilde{p}^c_t, \tilde{p}^n_t, \hat{M}_t, R^m_t$, and the various normalized Lagrange multipliers all have the same constant values for all $t \geq 0$, which constant values we shall simply denote c_1, c_2, and so on.

From equation (2.39) it is evident that such a stationary equilibrium must correspond to a constant value λ for the marginal-utility value of end-of-period real income that satisfies

$$v'(y(\lambda; F(I(\lambda; \lambda, \tilde{m})), \tilde{m}) \; + \; I(\lambda; \lambda, \tilde{m})) \; = \; \lambda. \tag{3.1}$$

This gives us a single equation to solve for the stationary equilibrium value of λ corresponding to a given stationary target $\tilde{m}$. Given the solution for λ from this equation, the implied stationary value of I is then given by $I = I(\lambda; \lambda, \tilde{m})$, which is the value of I_{-1} that we must assume for existence of such an equilibrium. Such an equilibrium will obviously involve a constant supply of durables, equal to $s = F(I)$. These constant values for λ_t, s_t, and $\tilde{m}_t$ in all periods then allow us to solve for constant values of all of the other variables listed above, using the methods explained in the previous section.

The constant value of the nominal interest rate on cash will be given by $R^m = (1 + r^m(\tilde{m}))\Pi$, where

$$1 + r^m \; \equiv \; \frac{1}{\beta \tilde{\varphi}_1(\tilde{m})}$$

and $\tilde{\varphi}_1(\tilde{m})$ is the stationary value of $\tilde{\varphi}_{1t}$, that depends on the value chosen for $\tilde{m}$, as discussed above, but is independent of the choice of Π. Thus for any choice of $\tilde{m}$, it

[36] Note that given our assumption of a constantly growing target path for the price level and our assumption that this target is precisely achieved each period, there is no difference between specifying the target path for the supply of outside cash balances as a constant real level or as a nominal target with a constant growth rate equal to the target inflation rate.

is possible to choose any value of Π such that

$$\Pi \geq \beta\tilde{\varphi}_1(\tilde{m}),$$

so that the required stationary nominal interest rate satisfies $R^m \geq 1$.

There is a stationary equilibrium corresponding to any value $\tilde{m} > 0$, but for all $\tilde{m}$ greater than a critical value m^*, the stationary equilibrium is the same. Here m^* is the level of outside real cash balances required for satiation in cash balances, which we can determine as follows. In a stationary equilibrium with satiation in cash balances, we must have $c_1 = c_1^*(\lambda)$ and $c_1 = c_1^*(\lambda)$. In addition, $\tilde{\varphi}_1 = 1$ $\tilde{Q} = \tilde{Q}^*$, so that $\phi(\lambda; s, \tilde{m}) = \tilde{Q}^*$, regardless of the values of λ and s. It follows that the stationary level of investment goods production I must equal $I^*(\lambda)$, the quantity implicitly defined by the equation

$$F'(I) = \frac{\lambda}{\beta\tilde{Q}^*}.$$

From this it follows that the stationary value of λ must satisfy

$$v'(c_1^*(\lambda) + c_2^*(\lambda) + I^*(\lambda)) \;=\; \lambda. \tag{3.2}$$

Since $c_1^*(\lambda), c_2^*(\lambda)$, and $I^*(\lambda)$ are all monotonically decreasing functions, it follows that the left-hand side of (3.2) is a non-increasing function of λ, and the equation must have a unique solution for λ. The associated stationary level of cash balances can be any level greater than or equal to $m^* \equiv c_1^*(\lambda)$. Hence such a stationary equilibrium exists in the case of any value of $\tilde{m}$ that is greater than or equal to m^*.

Finally, in any stationary equilibrium, the equilibrium real return on longer-term bonds (and indeed, any asset that can neither be used as cash nor used as collateral in order to issue liabilities that can be used as cash) will equal

$$E[R^b]/\Pi \;=\; R^m\tilde{\varphi}_1/\Pi \;=\; \beta^{-1}.$$

Note that this is independent of both $\tilde{m}$ and Π. Thus a higher value of $R^m/\Pi = 1 + r^m(\tilde{m})$ corresponds to a reduced spread between the returns on longer-term bonds and those on holding cash. We also note that the value of $\tilde{\varphi}_1$ (or more precisely, the log of $\tilde{\varphi}_1$) measures this spread.

We thus find that there is a two-dimensional family of possible stationary equilibria, which can be indexed by the choice of the two policy variables Π and $\tilde{m}$, which can be independently varied using the two dimensions of central-bank policy: conventional monetary policy (interest-rate policy) and balance-sheet policy (quantitative easing). These two dimensions of monetary policy have quite different effects. In our flexible-price model, interest-rate policy has no effect on any real variables, but can be used (within the limit imposed by the zero lower bound) to control inflation. Balance-sheet policy (changing the total supply of outside safe assets by increasing or reducing the quantity of longer-term bonds held by the central bank) can instead affect the steady-state values of all of the real variables in our model, except that further increases in the real supply of outside safe assets beyond the level $\tilde{m} = m^*$ have no further effects.

The possible stationary values of the various real variables that can be achieved by alternative monetary policies can thus be fully characterized by considering the one-parameter family of stationary equilibria corresponding to different values of $\tilde{m}$. These equilibria can be classified as of three possible types, according to which of the financial constraints bind. (The three possible cases correspond to the three cases discussed in our treatment in the previous section of the endogenous determination of the safe asset supply.)

First, there are equilibria in which the real outside supply of safe assets equals or exceeds the level m^* required for satiation; in these equilibria, the cash-in-advance constraint is slack, bankers finance none of their purchases of durables by issuing collateralized short-term debt (so that the collateral constraint on such issuance is also slack), and as no assets are sold in a fire sale even if the crisis state occurs, there is no ex post regret of the size of investors' credit limit (so that the constraint that this must be fixed in advance also does not bind).

Second, there are equilibria in which the real outside supply of safe assets is insufficient, and there is some private issuance of safe debt, but the quantity of safe debt issued by bankers is still small enough for the collateral constraint not to bind. And third, there are equilibria in which the incentive for issuance of safe debt by bankers is so strong that their issuance of such liabilities is limited by the availability of suitable collateral. The three cases correspond to different ranges of real outside supply of safe assets: high values of $\tilde{m}$, an intermediate range of values of $\tilde{m}$, and low values of $\tilde{m}$ respectively.

This one-parameter family of stationary equilibria can alternatively be parameterized by the associated value of $R^m/\Pi = 1 + r^m(\tilde{m})$, the stationary gross real

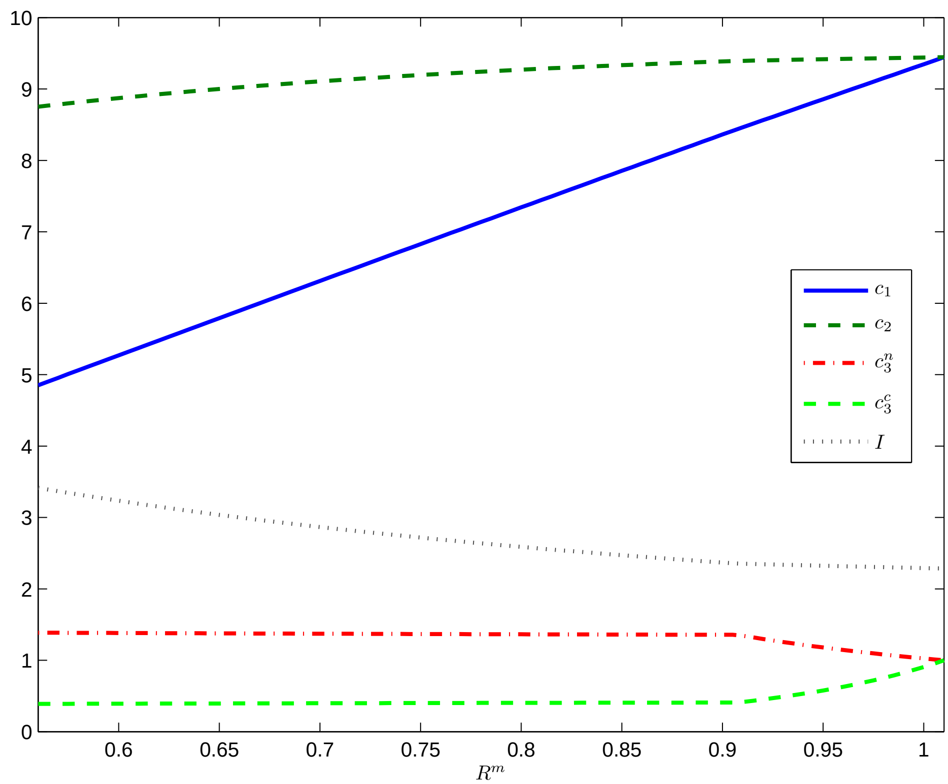

Figure 2: The allocation of resources in alternative stationary equilibria corresponding to different constant values of R^m/Π.

rate of return on cash. Values of $\tilde{m}$ increasing from 0 up to m^* correspond to values of R^m/Π increasing from some minimum value $1 + r^m(0)$ (which may well be positive, though it will generally correspond to a negative real rate of return) up to $1+r^m(m^*) = \beta^{-1} > 1$ (the point at which the spread between the return on bonds and that on cash is completely eliminated). A numerical example may usefully illustrate how systematic variation in this parameter changes the character of the stationary equilibrium.

Figure 2 shows how the stationary equilibrium values of c_1, c_2, c_3^c, c_3^n, and I vary with alternative stationary values for R^m/Π. (The figure thus completely displays the allocation of real resources in each possible equilibrium, and supplies all of the information needed to evaluate the level of expected utility of the representative household in each case, and draw conclusions about the welfare effects of alternative possible long-run policy targets.) The values of R^m/Π considered vary from $1+r^m(0)$

at the left boundary of the figure to $1 + r^m(m^*) = \beta^{-1} > 1$ at the right boundary.

In this example, cash and credit goods enter the household's utility function symmetrically, so that in an efficient allocation equal quantities of the two goods are produced and consumed; thus a comparison of the magnitudes of c_1 and c_2 can be used to see the size of the distortion created by the cash-in-advance constraint. There is no distortion ($c_1 = c_2$) at the extreme right of the figure, i.e., when $R^m/\Pi = \beta^{-1}$, so that there is no spread between the return on longer-term bonds and cash. Moving left in the figure, as the real return on cash is reduced (meaning that the spread is made progressively larger), the extent to which c_1 is less than c_2 grows progressively greater.

The efficiency of the level of production and consumption of special goods can also be seen directly from the figure. Because both the utility from consuming special goods and the disutility of supplying them are independent of which state occurs in subperiod 2, an efficient allocation requires that c_3^n equal c_3^c; and for the parameterization used in this example, the common efficient level of special goods production is equal to 1 (regardless of the level of production and consumption of other goods). Thus the degree to which c_3^n is greater than c_3^c (and to which the former quantity is greater than 1, while the latter quantity is smaller) indicates the degree to which the production and consumption of special goods is distorted by the fact that investors spend some of their resources on acquiring risky durables in the fire sale that occurs in the crisis state. As one moves from right to left in the figure, the incentive of bankers to issue collateralized short-term debt increases, but the consequence is an increasing quantity of durables that must be sold to redeem such debt in the even of a fire sale, increasing the wedge between c_3^n and c_3^c.

The three different possible types of equilibrium correspond to different regions of the horizontal axis in the figure. The possibility of an equilibrium in which the cash-in-advance constraint is slack is represented by the right boundary ($R^m/\Pi = \beta^{-1}$); while this corresponds to an entire range of possible values of $\tilde{m}$ (any $\tilde{m} \geq m^*$), they all correspond to the same real return on cash and the same allocation of resources. The case in which the cash-in-advance constraint binds but bankers' collateral constraint is slack corresponds to values of R^m/Π from around 0.91 to 1.01, while the case in which both constraints bind corresponds to all values of R^m/Π from the left boundary to about 0.91.

In the relatively high-cash-return region, because bankers' collateral constraint

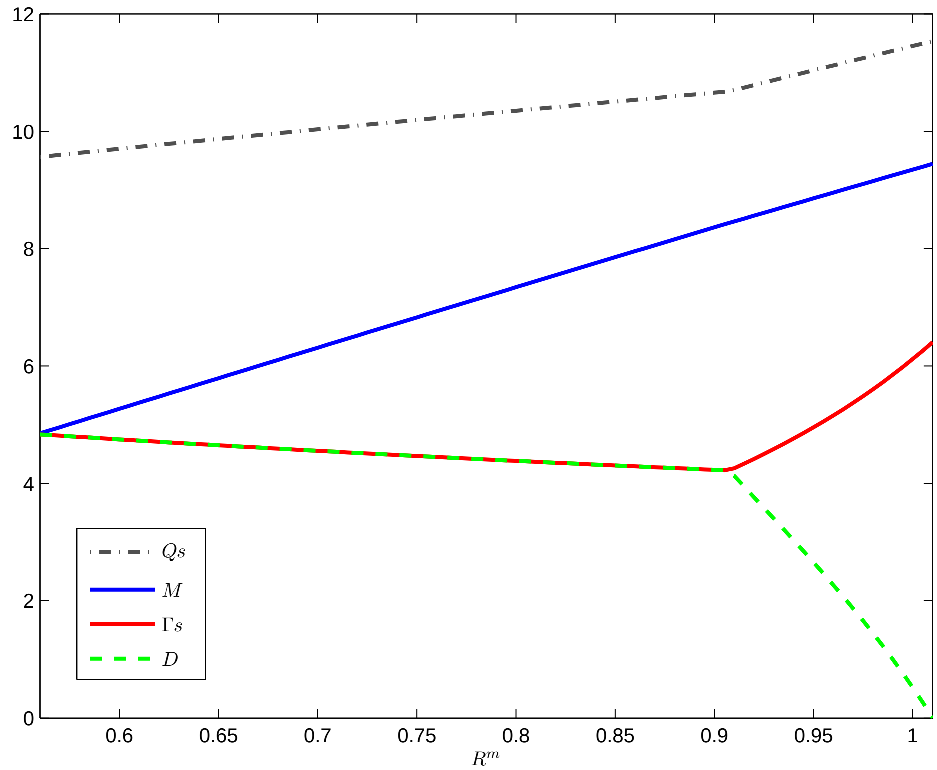

Figure 3: The endogenous supply of safe assets in alternative stationary equilibria corresponding to different constant values of R^m/Π, with its implications for both the capital structure of banks and the total supply of safe assets.

does not bind, the quantity of short-term debt issuance by bankers increases relatively rapidly as R^m/Π is decreased, as a consequence of which the wedge between c_3^n and c_3^c increases relatively sharply; but because durables are still valued at their fundamental value in subperiod 1, the production of durables does not increase greatly. In the lower-cash-return region, instead, further reductions in R^m/Π do not increase debt issuance as rapidly (because now the quantity of debt issued can increase only to the extent that the quantity of durables purchased by bankers also increases enough to provide the required additional collateral), so that the wedge between c_3^n and c_3^c no longer increases so rapidly; but because the ability of durables to allow additional short-term debt issuance increases the price of durables above their fundamental value, the equilibrium production of durables now increases more rapidly with further reductions in R^m/Π.

Figure 3 shows the stationary values of another set of variables, across the same one-parameter family of stationary equilibria: the supply of short-term collateralized debt $\tilde{D}$ (the stationary value of the variable $\tilde{D}_t \equiv \tilde{\varphi}_{6t} D_t$), the resulting total supply of cash $\hat{M}$, the upper bound $\tilde{\Gamma}s$ on issuance of short-term debt by bankers given by the expected market value of their assets in the event of a crisis, and for purposes of comparison, the market value $\tilde{Q}s$ of those same assets in subperiod 1.[37] One sees that as the equilibrium return on cash falls, and the money premium correspondingly increases, as one moves from the right boundary of the figure to the left, that the issuance of short-term debt by banks increases from an initial value of zero (when the money premium is zero) to progressively higher values. The rate of increase is sharpest in the high-cash-return region, because the upper bound on debt issuance does not bind; after that constraint begins to bind (around $R^m/\Pi = 0.91$), $\tilde{D}$ increases less sharply with further declines in R^m/Π, as it can only increase to the extent that $\tilde{\Gamma}s$ also increases. In fact, in the high-cash-return region, $\tilde{\Gamma}s$ decreases as the money premium increases; the reason is that, as short-term debt issuance increases the quantity of assets that must be sold in a fire sale in the event of a crisis increases, depressing the fire-sale value of bankers' assets. Once R^m/Π falls to around 0.91, the constraint comes to bind, both because of the increase in desired debt issuance *and* the reduction in the value of the collateral available to back such debt. Beyond this point, further increases in the size of the money premium cause $\tilde{\Gamma}s$ to increase, rather than continuing to decrease; this is because the value of relaxing the constraint on short-term debt issuance now contributes to a larger market value of durables in subperiod 1,[38] which induces a larger market supply of durables (as can be seen from the I curve in Figure 2), so that $\tilde{\Gamma}s$ increases slightly, even though the fire-sale price $\tilde{\Gamma}$ continues to fall.

[37]Each of these variables is measured in marginal-utility units, as they have a constant value in marginal-utility units in a stationary equilibrium, regardless of the rate of inflation. Also, we have shown above that the equilibrium relations determining the values of these variables are in many cases simpler when written in terms of the variables expressed in marginal-utility units.

[38]Specifically, the value of $\tilde{\varphi}_1 \tilde{Q}/\lambda$ increases, which is the ratio of the marginal-utility value of the sale price of a unit of the durable good in subperiod 1, given that payment received in subperiod 1 can be used to acquire cash for use by the shopper, to the marginal-utility value of the sale price of a unit of normal goods in subperiod 2. This relative price determines the incentive to produce additional investment goods, as shown by condition (2.12), and hence the supply of durables. Note that the stationary value of $\tilde{Q}$ does *not* increase, as can be seen from the $\tilde{Q}s$ curve in this figure.

We can also see, from the size of the gap between the solid line indicating the value of $\hat{M}$ and the dashed line indicating the value of $\tilde{D}$, how the part of the cash supply that comes from outside safe assets (the value of $\lambda\tilde{m}$, in marginal-utility units) varies across the alternative stationary equilibria. This value decreases monotonically as one proceeds from right to left in the figure, both because $\tilde{D}$ increases and because $\hat{M}$ decreases; the latter effect represents the reduction in the demand for cash balances as the opportunity cost of holding them (i.e., the money premium) increases. The fact that the equilibrium relationship between the size of the money premium and the quantity of outside safe assets is monotonic indicates how the choice of a stationary level for the supply of outside safe assets (through the combination of the Treasury's debt-management policy and the central bank's balance-sheet policy) can be used to determine the stationary value of R^m/Π, and thus to select which of the stationary equilibria depicted in these figures should occur.

Note that there is a limit to how far R^m/Π can be reduced by shrinking the supply of outside safe assets; at the left edge of the figure, $\tilde{m}$ falls to zero, while R^m/Π is still positive. (This is because this lower bound does not correspond to an opportunity cost high enough to reduce the demand for cash balances to zero; it is only necessary that the demand for cash balances fall to a low enough level that it is no greater than the quantity of safe liabilities that bankers wish to supply, which grows the larger the money premium gets.) However, this lower bound for R^m/Π can easily be well below 1 (as shown in the figure), corresponding to a negative long-run equilibrium short-term real rate. Thus our model is one in which it is perfectly possible to have an equilibrium short-term real rate that remains negative forever, as a result of a shortage of safe assets; this results in a "safety trap" in the sense of Caballero and Farhi (2013), in the case that the inflation target Π is too low. An advantage of working with a fully developed monetary equilibrium model, however, is that we see that the existence of a safety trap depends not simply on too low a supply of safe assets (or too great a demand for them), but *also* on choosing too low an inflation target, just as in the "liquidity trap" model of Krugman (1998) and Eggertsson and Woodford (2003).

Figure 4 shows how the degree to which durables are both over-valued in subperiod 1 (and at the time that the decision to divert resources into the production of durables is made) and under-valued in the event of a fire sale varies across the alternative stationary equilibria. The dashed plots the stationary value of $\tilde{\varphi}_1\tilde{Q}/\tilde{Q}^*$, which is

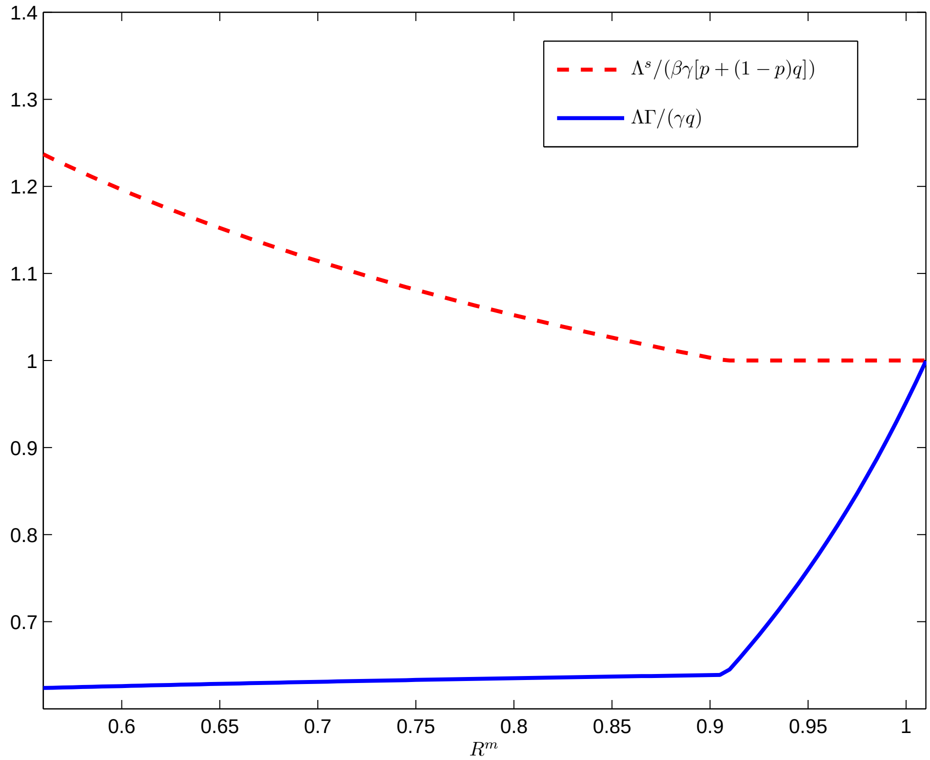

Figure 4: The degree of initial over-valuation of durables (dashed line) and the degree of their subsequent under-valuation in the event of a crisis (solid line), in alternative stationary equilibria corresponding to different constant values of R^m/Π.

to say the ratio of the subperiod 1 market price of durables to their "fundamental" value.[39] Thus durables are over-valued in subperiod 1 to the extent that this quantity exceeds 1. We see that it equals 1 (there is no over-valuation) in the high-cash-return region, given that banks do not wish to acquire additional durables for the sake of being able to issue more collateralized short-term debt. However, for all values of R^m/Π below 0.91, durables are over-valued, and the degree of over-valuation gets progressively higher the larger is the money premium.

The solid line in the same figure plots the stationary value of $\tilde{\Gamma}/\gamma q$, which is the ratio of the fire-sale price of durables to their fundamental value under this contingency

[39] Alternatively, the quantity plotted is the ratio of Λ^s to its fundamental value $\beta\tilde{Q}^*$, where Λ^s is the marginal-utility valuation assigned to an additional quantity of investment goods sufficient to allow production of an additional unit of durables, so that the demand curve for investment goods can be written as $F'(I) = \lambda/\Lambda^s$.

(which is smaller than their fundamental value in subperiod 1, since if a crisis occurs the probability that the durables are worthless is higher than previously realized). Thus durables are under-valued in the fire sale to the extent that this quantity is less than 1. As shown in the figure, durables are under-valued in the fire sale in the case of any $R^m/\Pi < \beta^{-1}$ (corresponding to any $\tilde{m} < m^*$), and the degree of under-valuation increases steadily the larger the money premium. The degree of under-valuation increases especially sharply with increases in the money premium in the high-cash-return region, since in this region s^* (the quantity of assets sold in the fire sale if one occurs) increases relatively sharply with increases in the money premium. Once the constraint that s^* can be no larger than the total quantity s of assets held by bankers becomes a binding constraint, s^* increases much less rapidly with further increases in the money premium, and the degree of equilibrium under-valuation correspondingly ceases to increase so rapidly, though it grows somewhat.

We can alternatively measure the extent to which distortions are created by financial constraints in the alternative stationary equilibria by looking not at how market valuations differ from fundamental values, but at the extent to which the constraints affect households' decisions, as indicated by the size of the Lagrange multipliers associated with the various constraints. Figure 5 plots the values of the three key (normalized) Lagrange multipliers in our model: $\tilde{\varphi}_1$ (which indicates a binding cash-in-advance constraint to the extent that it is greater than 1[40]), $\tilde{\varphi}_3$ (which indicates a binding constraint on the quantity of collateralized short-term debt that bankers can issue to the extent that it is positive), and $\tilde{\varphi}_5^c$ (which indicates a binding constraint on the ability of investors to spend as much in the crisis state as the household would wish *ex post,* to the extent that it is greater than 1).

The value of $\tilde{\varphi}_1$ is equal to u_1/u_2, the marginal rate of substitution between cash and credit goods, and the more this exceeds 1, the greater the inefficiency of the allocation of expenditure between these two types of goods (which have equal disutility of supply). We see from the figure that the magnitude of this distortion increases steadily as R^m/Π is reduced (which is to say, as the money premium increases), starting from zero distortion when $R^m/\Pi = \beta^{-1}$, so that there is no money premium. Moreover, the magnitude of the distortion is a convex function of the size of the money premium, so that the rate at which the distortion increases becomes greater

[40]Note that the quantity $\tilde{\varphi}_1 - 1$ plotted in the figure is also the value of $\tilde{\varphi}_4$, as well as $1 - p$ times the value of $\tilde{\varphi}_2$.

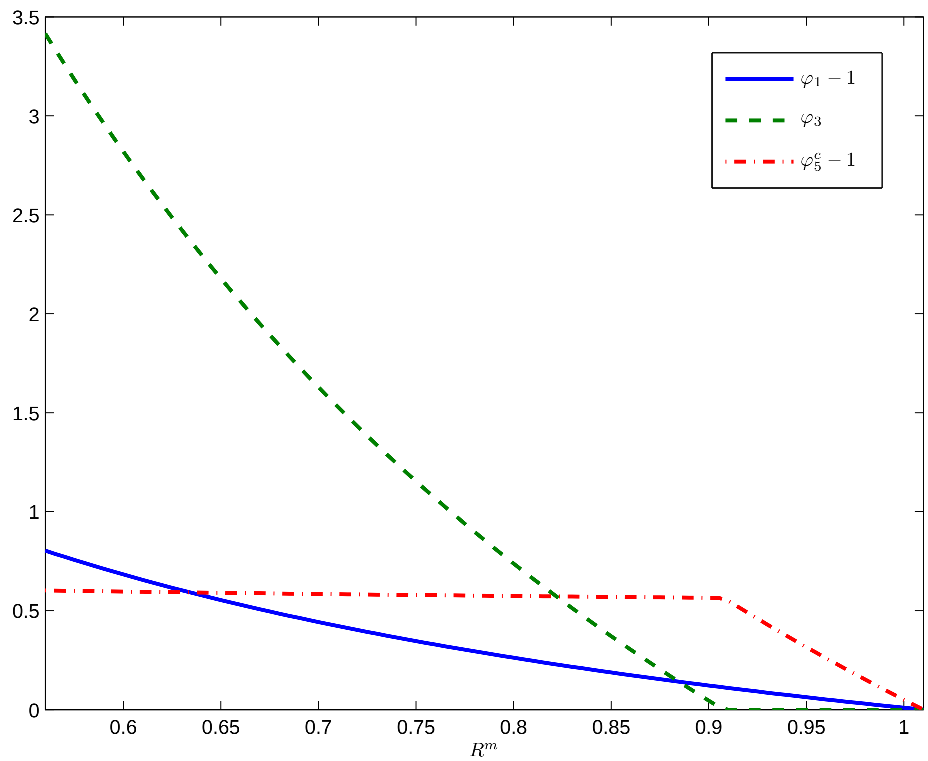

Figure 5: Lagrange multipliers indicating the degree to which the various financial constraints bind, in alternative stationary equilibria corresponding to different constant values of R^m/Π.

for larger values of the money premium.

As explained in section 2.3, the stationary equilibrium values of c_3^c and c_3^n are both monotonic functions of $\tilde{\varphi}_5^c$ (the first an increasing function, the latter a decreasing function), with $\tilde{\varphi}_5^c = 1$ corresponding to the efficient level of production c_3^* of special goods in both states. Hence the extent to which $\tilde{\varphi}_5^c$ is greater than 1 indicates the degree of inefficiency in the level of production and consumption of special goods (in both states) owing to the possibility of a fire sale of assets by banks. The figure shows that the magnitude of this distortion also increases as R^m/Π is reduced, starting from zero distortion when $R^m/\Pi = \beta^{-1}$. However, the magnitude of this distortion increases sharply with increases in the size of the money premium only in the high-cash-return region; once the availability of collateral becomes a binding constraint on issuance of short-term debt by bankers, the degree of inefficiency in the level of production of special goods increases only gradually with further increases in the size of the money premium.

Finally, the figure indicates that $\tilde{\varphi}_3 > 0$, indicating that the constraint that short-term debt issuance cannot exceed the amount that can be backed by the collateral value of bankers' assets binds, only for values of R^m/Π less than 0.91. Below this point, however, the value of the multiplier rises sharply with further increases in the money premium; this accounts for the increase in the subperiod 1 market price of durables, shown in Figure 4, over this same region.

3.2 Consequences of a Larger Central-Bank Balance Sheet

We can now consider how a quantitative easing policy that permanently increases the size of the central bank's balance sheet (in real terms, or relative to the size of the economy) — and more specifically, a policy of purchasing longer-term assets and financing these purchases by issuing short-term safe liabilities — affects the economy's long-run equilibrium. To the extent that the effects of the policy are not undercut by an offsetting shift in the maturity composition of the debt issued by the Treasury,[41] such a policy can increase the steady-state level of $\tilde{m}$. If $\tilde{m} < m^*$, so that there is not already satiation of the demand for safe assets even without any creation of safe assets by the private sector, then increasing $\tilde{m}$ will mean moving to a stationary equilibrium with a higher value of R^m/Π, corresponding to a movement further to the right in each of the figures just presented.

This has real effects, and in particular has consequences for financial stability. However, a larger supply of outside safe assets as a result of a policy of quantitative easing should *improve* financial stability. Specifically, whether the economy begins in the low-cash-return or high-cash-return region, a higher value of R^m/Π (and hence a smaller money premium) reduces private issuance of short-term debt $\tilde{D}$. As a consequence, it reduces the quantity s^* of durables that will have to be sold in a fire sale in the event of a crisis, and so reduces the severity of the distortions associated with a crisis.[42] Both the degree to which durables are under-valued in the crisis (as

[41]Note that such a shift in Treasury policy did offset a significant part of the effect of the Fed's asset purchases in recent years, as shown by Greenwood *et al.* (2014).

[42]Note that in the simple model presented here, the *probability* of a crisis is exogenous, and so cannot be affected by policy. But policy can affect the severity of a crisis, conditional on the crisis state being reached.

shown in Figure 4) and the degree of inefficiency in the level of production of special goods (as shown in Figure 2) are smaller, the larger the value of R^m/Π.

Thus from the standpoint of financial stability, a larger central-bank balance sheet is clearly to be preferred (at least as far as long-run steady states are concerned). In fact, the other real effects of a quantitative easing policy on the long-run steady state are also beneficial. A higher value of R^m/Π implies that the cash-in-advance constraint binds less tightly (as shown by the value of $\tilde{\varphi}_1$ in Figure 5), and this results in a more efficient allocation of household expenditure on cash and credit goods between the two types of goods (a ratio of c_1/c_2 closer to 1, in Figure 2). And in the low-cash-return region (where $\tilde{\varphi}_3 > 0$), a higher value of R^m/Π also results in less over-valuation of durables in subperiod 1, so that there is less inefficient over-production of durables (as is also seen in Figure 2). Thus each of these considerations points in the same direction: the equilibrium allocation of resources is more efficient (and the welfare of the representative household is increased) if the real supply of outside safe assets is increased.

The conclusion that expansion of the central bank's balance sheet is associated with a more efficient allocation of resources between cash and credit goods might seem surprising in light of the analysis of Lucas and Stokey (1987), who conclude, in the context of a similar model (but without durable goods production or fire sales), that efficiency in this respect is greater the *lower the rate of growth* of the monetary base — with the highest levels of efficiency (and hence of welfare for the representative household) being achieved only in the case of steady contraction of the size of the central bank's balance sheet. The difference in conclusions results from their assumption that the safe liabilities that count as cash must earn a nominal interest rate of zero (so that $R^m = 1$ is assumed). In that case, steady states with different values of R^m/Π must correspond to different rates of inflation Π — whereas here the choice of the inflation target Π is independent of the aspects of policy that determine R^m/Π, within the bound required by the lower bound on nominal interest rates.

Lucas and Stokey conclude, as we do, that relaxation of the cash-in-advance constraint, and a more efficient allocation of expenditure between cash and credit goods, requires a higher value of R^m/Π, but in their analysis this requires a lower rate of inflation, and hence a lower rate of growth of the *nominal* value of outside safe assets $\tilde{M}_t$. In our model, it is also true that in a long-run stationary equilibrium, the rate of growth of $\tilde{M}_t$ must equal the inflation rate. But it is possible for the central bank to control the value of the currency unit other than through its control of the path of $\tilde{M}_t$ (by appropriate variation in R_t^m), so that there is a decision to make about how large $\tilde{M}_t$ should be relative to the level of P_t aimed at through interest-rate policy, that is separate from the question of the long-run rate of growth of both variables. Thus it is not correct, more generally, to identify a decision to increase the size of the central bank's balance sheet with a decision to pursue a more *inflationary* policy; in the long run, these are two distinct issues. The short-run consequences of balance-sheet expansion are instead considered in section 5.

4 Quantitative Easing Compared with Macroprudential Policy

Another implication of increasing the supply of central-bank reserves through a quantitative easing policy, not discussed in the analysis above, is relaxation of the constraint on private banks' ability to issue money-like liabilities that may result from a requirement that they hold reserves in proportion to their issuance of such liabilities. Such reserve requirements apply (at least in some countries, like the US) to at least some kinds of short-term safe instruments issued by commercial banks — though not, even in the US, to the kind of privately issued STSIs that were most responsible for the financial fragility exposed by the recent crisis.[43] And under many traditional textbook accounts of the way that monetary policy affects the economy, the key effect of a central-bank open-market operation is precisely to relax this constraint on private bank behavior by increasing the quantity of reserves that are available to satisfy the reserve requirement. This might seem to have important implications for financial stability that would cut in the opposite direction to the analysis above; that is, it might seem that expansion of the central bank's balance sheet should have as an effect, or even as its primary effect, an increase in the extent to which private banks acquire risky assets and finance those assets by issuing money-like liabilities.

[43] The kinds of liabilities, such as retail deposits at commercial banks, to which such requirements apply were not the ones the demand for which proved to highly volatile. While these funds could in principle be withdrawn on short notice, they were not, probably owing to the existence of deposit insurance; and so they were not responsible for any appreciable funding risk.

This is in fact a key theme of the analysis by Stein (2012), and the basis for the proposal there that monetary policy decisions be considered from the standpoint of "financial-stability regulation."

In the analysis here, we have abstracted from reserve requirements, since even in the US, these have not been binding constraints on banks' behavior during the period when the Fed's experiments with quantitative easing have occurred.[44] We can, however, use our framework to discuss the consequences for financial stability of increasing or decreasing the cost to financial institutions of issuing collateralized short-term debt as a source of financing, even when they hold sufficient assets to provide the collateral for such issuance. But we view this as a separate dimension of policy — *macroprudential* policy — that should be distinguished, conceptually, from both conventional monetary policy (interest-rate policy) and central-bank balance-sheet policy.[45] One might well use instruments of macroprudential policy that affect the ability and/or incentives of banks to issue money-like liabilities that are unrelated to the central bank's balance sheet (and that do not depend on the existence of reserve requirements). And even when the tool that is used is a reserve requirement, one can loosen or tighten this constraint independently of the way one that changes the size of the central bank's balance sheet; first, because one can vary the required reserve ratio as well as the supply of reserves, and second, because the central bank can vary the supply of STSIs without varying the supply of reserves, if it issues central-bank bills or engages in reverse repo transactions,[46] or by varying the quantity of Treasury bills on its own balance sheet.

The effects of varying macroprudential policy are in fact quite different from the effects (considered above) of varying the central bank's supply of outside safe assets, when the latter policy is implemented in a way that has no direct effects on financial institutions' cost of short-term debt issuance. We can introduce macroprudential

[44] They were not relevant, even earlier, for most of the kinds of financing decisions modeled in this paper. As noted earlier, the privately supplied "cash" in this model should be identified primarily with repos or asset-backed commercial paper.

[45] Macroprudential policy, modeled in a way similar to that used here, is also compared with conventional monetary policy in Sergeyev (2016), which also discusses Ramsey policy when the two distinct types of policy instruments exist. The discussion of optimal policy by Sergeyev does not treat the use of balance-sheet policies of the kind that are the central focus here.

[46] See in particular Carlson *et al.* (2014) on the usefulness of reverse repo transactions, such as the Fed's proposed ON RRP facility, for this purpose.

policy into the model set about above in the following way. Suppose that a banker who issues short-term debt with face value D_t obtains only $\xi_t D_t$ in additional funds with which to acquire assets in subperiod 1, where $0 \leq \xi_t \leq 1$; the quantity $(1-\xi_t)D_t$ represents a proportional tax on issuance of safe debt, collected by the government. The variable ξ_t (or alternatively the tax rate) then represents an instrument of macroprudential policy. Note that ξ_t may be varied from period to period, if the degree to which it is desirable to provide a disincentive to safe debt issuance varies over time; and that the choice of the path of $\{\xi_t\}$ is independent of the choice of the path of $\{\tilde{m}_t\}$, the real outside supply of safe assets.

One possible way of implementing such a tax on safe debt issuance is through a reserve requirement. Suppose that a bank that issues safe debt with face value $\hat{D}_t$ is required to hold reserves $H_t \geq k_t \hat{D}_t$, where H_t is the value of the reserves in the end-of-period settlement. Suppose furthermore that reserves pay a gross nominal interest rate of $R_t^{cb} \leq R_t^m$, which means that $\theta_t \equiv R_t^m / R_t^{cb} \geq 1$ units of cash must be paid in subperiod 1 to acquire a unit of reserves. Finally, suppose that a bank's reserve balance can be used to pay off its safe debt in subperiod 2, if the holders of the bank's short-term debt are not willing to roll it over, with one unit of reserves serving to retire one unit of short-term debt. Then the bank's collateral constraint again takes the form (1.3), and the assets sold in a fire sale must satisfy (1.6), where now $D_t \equiv \hat{D}_t - H_t$ is short-term debt issuance not covered by the bank's reserve balance. The funds obtained by the bank with which to purchase additional assets in subperiod 1 are only $\hat{D}_t - \theta_t H_t$, owing to the need to acquire reserves with some of the proceeds of the debt issuance. This quantity can alternatively be expressed as $\xi_t D_t$, where

$$\xi_t \equiv \frac{1 - k_t \theta_t}{1 - k_t} \leq 1.$$

If we assume that $k_t \theta_t \leq 1$, so that it is possible for the bank to acquire the required reserves out of the proceeds of its short-term debt issuance,[47] then $\xi_t \geq 0$ as assumed above. Thus reserve requirements are an example of the kind of macroprudential policy that can be modeled in the way proposed above (in the case that the interest rate paid on reserves is less than the rate paid on cash). Note that in this case, ξ_t can be reduced either by reducing the interest rate R_t^{cb} paid on reserves (relative to the

[47]Note that tighter reserve requirements than this would have no effect, since when $k_t \theta_t = 1$, banks are already completely precluded from raising any funds by issuing short-term debt.

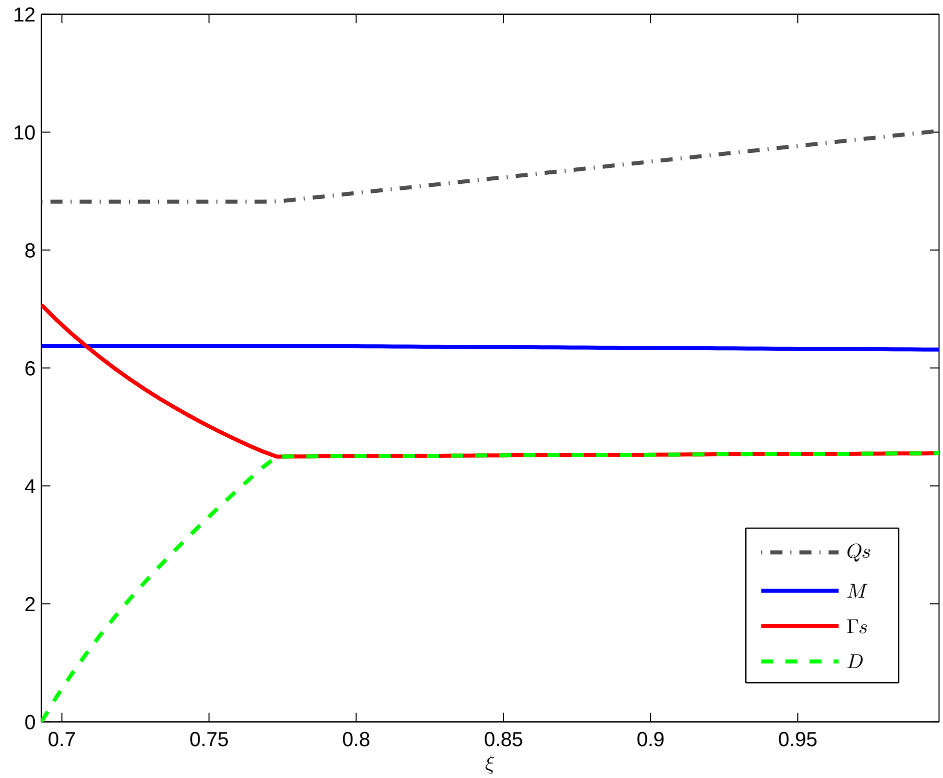

Figure 6: Short-term debt issuance by banks (and the other variables shown in Figure 3) in alternative stationary equilibria corresponding to different constant values of ξ, for a fixed value of R^m/Π.

central bank's target for the interest rate paid on cash), or by increasing the required reserve ratio k_t.

The FOCs that characterize optimal household behavior are not changed by the introduction of macroprudential policy, except that (2.4) now takes the more general form

$$\xi_t \varphi_{1t} \;=\; (1-p)\varphi_{2t} + \mathrm{E}_t \varphi_{6t}. \tag{4.1}$$

With this change, the derivation of the conditions for an intertemporal equilibrium proceeds as in section 2. The equilibrium paths of the endogenous variables now depend on the specification of the series $\{P_t, \tilde{m}_t, \xi_t\}$, representing three distinct dimensions of policy: conventional monetary policy, the determination of the outside supply of safe assets by debt management policy and quantitative easing, and macroprudential policy.

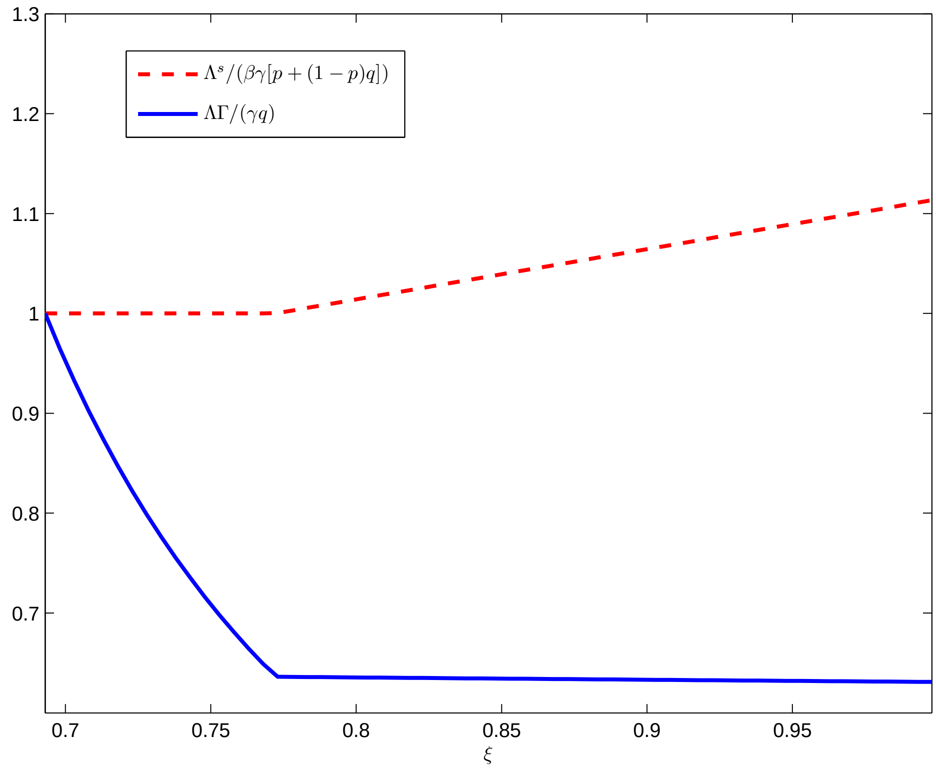

Figure 7: The degree of initial over-valuation of durables (dashed line) and the degree of their subsequent under-valuation in the event of a crisis (solid line), in alternative stationary equilibria corresponding to different constant values of ξ, for the same fixed value of R^m/Π as in Figure 6.

In this more general version of the model, we obtain a three-parameter family of stationary equilibrium, indexed by stationary values $\Pi, \tilde{m}$, and ξ. The stationary real allocation of resources depends only the stationary values of $\tilde{m}$ and ξ. In the previous section, we have shown how variation in $\tilde{m}$ (or alternatively, in R^m/Π) affects the stationary equilibrium values of real variables and relative prices, for a fixed value of ξ. (In that section, we assumed $\xi = 1$; but similar qualitative conclusions would obtain in the case of any fixed value of ξ.) Here we consider instead the consequences of varying the stationary value of ξ, and in particular, the extent to which the effects of varying the strength of macroprudential policy (perhaps by relaxing or tightening reserve requirements) are equivalent to the effects of variations in the supply of reserves, discussed in the previous section.

Figure 6 shows again the stationary values of the variables plotted in Figure 3 (that compare short-term debt issuance by banks with the total supply of cash and with the available collateral to back such issuance), for alternative constant values of $\xi \leq 1$, holding fixed the target that determines the central bank's balance-sheet policy (here assumed to be a fixed target for the term premium associated with longer-term bonds, or equivalently a fixed value of R^m/Π). In the case shown in the figure, the target for R^m/Π is low enough that, in the absence of any reserve requirement or other regulation of short-term debt issuance by banks (i.e., the case $\xi = 1$), the stationary equilibrium is of the "low-cash-return" type discussed in the previous section; that is, the incentive for short-term debt issuance by banks is great enough for the collateral constraint to bind, resulting in over-valuation of durables in subperiod 1, so that durables are over-supplied. We consider this case for our numerical illustration because it is the case in which there is the most reason to be interested in whether macroprudential policy can reduce the distortions resulting from banks' excessive incentive to issue short-term debt. The corresponding stationary values for the market valuation of durables are shown in Figure 7.

Figure 6 shows that as we increase the tax rate on short-term debt issuance (or increase the effective tax rate, by increasing the required reserve ratio or reducing the rate of interest paid on reserves), and thus lower ξ, the stationary value of $\tilde{D}$ falls. And for a sufficiently large tax rate (the case of ξ less than 0.77, in our numerical example), the collateral constraint ceases to bind; this implies that durables are no longer over-valued in subperiod 1, as shown in Figure 7. In the case of an even larger tax rate (though still less than 100 percent taxation of the proceeds from issuing short-term debt), short-term debt financing of banks is completely driven out ($\tilde{D} = 0$), because the macroprudential tax fully offsets the value of the money premium to issuers of financial claims that can be used as cash. (In our numerical example, this occurs when $\xi = 0.69$, the left boundary of the figures.) When this occurs, bankers no longer have to sell assets in a fire sale, even if the crisis state occurs, and the under-valuation of durables in the crisis state is eliminated, as is also shown in Figure 7. Further reductions in ξ below this value are irrelevant, as banks' issuance of short-term debt cannot be further reduced.

The implications of these alternative equilibria for the allocation of resources are shown in Figure 8. Because balance-sheet policy is used to fix the value of R^m/Π, the stationary value of $\tilde{\varphi}_1$, and hence the stationary value of $\tilde{\varphi}_4$, are unaffected by

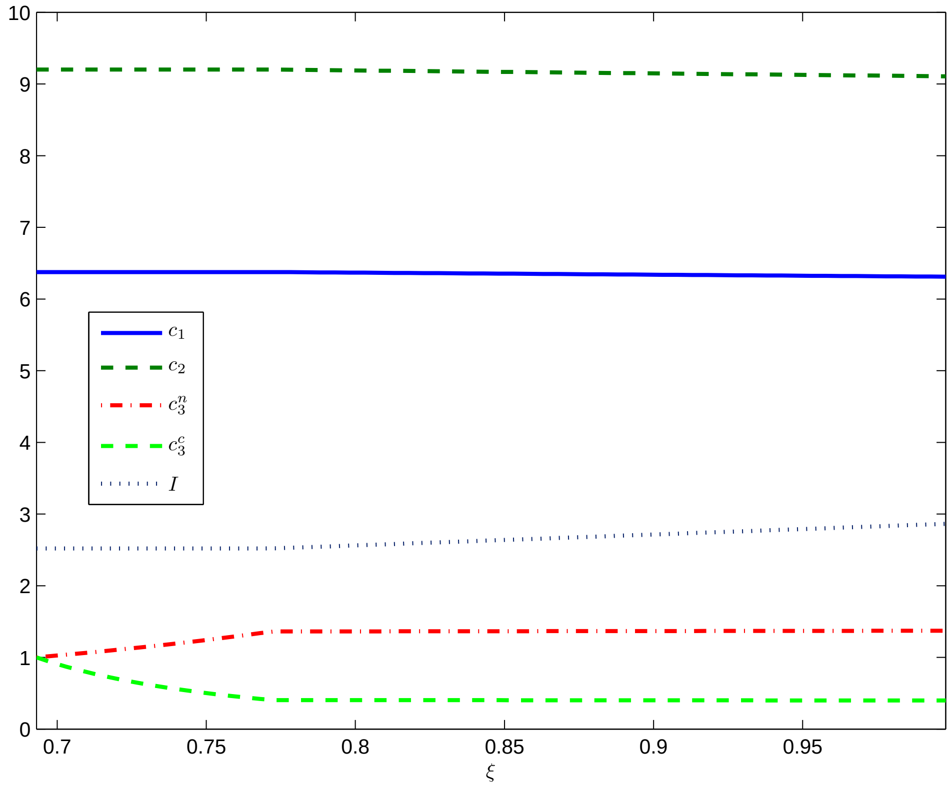

Figure 8: The allocation of resources in the same set of alternative stationary equilibria shown in Figures 6 and 7.

changing ξ. This means that the degree of inefficiency in the allocation of expenditure between cash and credit goods (as measured by the degree to which the marginal rate of substitution u_1/u_2 is greater than 1, the relative cost of producing them) is unaffected, and hence the equilibrium levels of production of cash and credit goods are little affected. However, as ξ is decreased from 1 (while still greater than 0.77), the degree of inefficient over-production of investment goods is reduced, owing to the decrease in the degree to which banks are willing to pay to relax their collateral constraints. (Once ξ is less than 0.77, the collateral constraint no longer binds, as shown in Figure 6; hence further reductions in ξ produce no further reductions in this distortion.) Moreover, because reductions in ξ reduce short-term debt issuance (as long as ξ remains greater than 0.69), and hence the value of s^*, they reduce the degree of inefficiency in the production and consumption of special goods: both c_3^c and c_3^n move closer to the efficient level of 1, which they reach exactly if ξ is reduced to 0.69.

We can now ask to what extent the effects of expanding the supply of central-bank liabilities through quantitative easing are equivalent, or even similar, to the effects of relaxing a reserve requirement that limits the ability of banks to issue money-like liabilities. In the context of our model, the former sort of policy corresponds to an increase in $\tilde{m}$ (resulting in an increase in R^m/Π, if there is not already satiation in cash balances), which can be implemented while keeping ξ fixed; the latter sort of policy corresponds to an increase in ξ (assuming a reserve requirement tight enough to bind), which can be implemented while keeping $\tilde{m}$ fixed, or with an appropriate adjustment of the central bank's balance sheet, while keeping R^m/Π fixed.

A comparison of Figures 6-8 with Figures 2-4 shows that not only are these two policies not equivalent, their effects are in many respects exactly the opposite. An expansion of the central bank's balance sheet while fixing ξ corresponds to a movement from left to right in Figures 2-4: short-term debt issuance by private banks falls, both the over-valuation of durables in subperiod 1 and their under-valuation in the event of a crisis are reduced, the over-production of durables is reduced, and level of production of special goods in both the c and n states becomes more nearly efficient. A relaxation of a binding reserve requirement while fixing R^m/Π corresponds instead to a movement from left to right in Figures 6-8, which essentially reverses the effects seen in the earlier figures: short-term debt issuance by private banks increases, both the over-valuation of durables in subperiod 1 and their under-valuation in the crisis state increase, the over-production of durables in increased, and the level of production of special goods is progressively more severely distorted.

In fact, both an expansion of the outside supply of safe assets and a tightening of reserve requirements (or other forms of macroprudential policy) have similar consequences for financial stability, insofar as both reduce the extent to which banks finance themselves by issuing short-term safe debt. Either of these policies, pursued far enough, will completely eliminate private issuance of money-like claims (the right boundary of Figures 2-4, or the left boundary of Figures 6-8), and consequently eliminate the distortions resulting from the risk of a fire sale of assets and from the desire of bankers to obtain assets that can be used to collateralize short-term debt issuance. Thus each of these policies, either of which is welfare-enhancing (when not irrelevant), can serve to some extent as a substitute for the other. It is worth noting, however, that while a sufficient increase in the outside supply of safe assets would make macroprudential policy unnecessary in our model (since private issuance of money-like claims can be completely eliminated, even if $\xi = 1$), the reverse is not true: even a macroprudential policy of the maximum possible stringency (one that completely prevents private issuance of STSIs) will not eliminate the welfare gains from further expansion of the outside supply of safe assets, since even when $\tilde{D} = 0$ (as in the case with $\xi = 0.69$ in Figures 6-8), there will still be inefficient under-consumption of cash goods, owing to the binding cash-in-advance constraint, as long as $\tilde{m} < m^*$.

5 Conventional and Unconventional Monetary Policy in the Presence of Nominal Rigidities

In the analysis thus far, all prices have been assumed to be perfectly flexible, and to clear markets each period. In such a model, conventional monetary policy has no real effects, and affects only the general level of prices in terms of the monetary unit. It follows that conventional monetary policy has no consequences for financial stability. This establishes a sharp distinction between the effects of conventional monetary policy (interest-rate policy) and balance-sheet policy, since as shown above, the central bank's balance sheet (specifically, the *real* supply of safe assets by the central bank) does have consequences for financial stability.

Such an analysis is adequate for consideration of the possible long-run stationary equilibria achievable under alternative policies, as in the previous two sections. But it does not suffice for an analysis of the considerations at play when alternative dimensions of monetary policy are used to address short-run macroeconomic stabilization objectives, and this is the context in which central banks' recent experiments with quantitative easing have been conducted. To address the issues raised by recent policies, we need to consider the consequences for financial stability of using quantitative easing as a substitute for an interest-rate cut that is prevented by the effective lower bound on short-term nominal interest rates, in a situation where such an interest-rate cut would otherwise be desired in order to achieve a higher level of output.

The notion that an interest-rate cut would be desired in order to increase real activity only makes sense in the presence of nominal rigidities of some kind. Here we discuss a simple extension of the model presented above, that shows how sticky prices allow conventional monetary policy to have real effects in the short run, while it affects only the general level of prices in the long run. We can then compare the effects of quantitative easing to those of an interest-rate cut, both with respect to the effects of these policies on aggregate demand, and their consequences for financial stability.

5.1 Equilibrium with a Sticky Price for Normal Goods

We suppose that it is only the price P_t of normal goods that must be set in advance, while the prices of special goods, durable goods, and all financial assets are assumed to be perfectly flexible, as above. (Because all three types of normal goods are perfect substitutes from the standpoint of their suppliers, we suppose that a single price P_t is posted, at which goods of any of these types can be purchased; we suppose that the buyer determines which type of good will be obtained.) For simplicity, we also consider here the case of a single unexpected aggregate shock (apart from the kind of uncertainty represented in Figure 1) at some date t, in response to which monetary policy (both interest-rate policy and balance-sheet policy) may be adjusted; we suppose that there is no further uncertainty (except for the kind depicted in Figure 1) about how the economy will evolve after this shock occurs, and that the shock is completely unanticipated prior to its occurrence.

The fact that the shock is completely unexpected means that before it occurs, people expect an equilibrium in which there will never be any random developments except the kind depicted in Figure 1. We may suppose that this equilibrium is a stationary equilibrium of the kind described in section 3. Note that in such an equilibrium, the price P_τ of normal goods in any period τ is a deterministic function of time; it does not depend on which state is reached in subperiod 2 of period τ, nor does it depend on the history ξ_τ of states revealed in previous periods. Hence we may suppose that the same price P_τ is set for normal goods in all periods $\tau \leq t$ as would clear markets in the flexible-price stationary equilibrium analyzed above, even if the price P_τ must be set before subperiod 2 of period τ is reached. For purposes of the present discussion, we need not discuss how exactly the predetermined price of normal goods is determined, beyond the assumption that in an environment where the future is perfectly predictable (except for the uncertainty each period depicted in Figure 1), the price that is set each period is the one that would clear the market for normal goods.

Let us suppose that period t is one in which no crisis occurs in subperiod 2 (though it is not known up until this time that this would be the case). But let us also suppose that in subperiod 2 of period t, an unexpected shock occurs, as a result of which the utility of cash and credit goods consumption is equal to $\chi u(c_{1t}, c_{2t})$, and the disutility of supplying normal goods is equal to $\chi v(Y_t)$, for some factor $\chi > 0$ that need not equal 1; the other components of the utility function are unaffected by the shock. The factor χ is assumed to take a value different from 1 only in period t (and prior to period t, it is assumed to equal 1 with probability 1 in period t as well). The point of assuming a shock of this particular type is that for a given level of production of investment goods, the efficient level of production and consumption of cash and credit goods would not be changed by the shock χ; however, the real rate of interest required to sustain that level of demand will change (will be lower if χ is lower). Hence the shock χ represents a "demand disturbance" to which it would be desirable to respond by lowering interest rates, if this is not precluded by the interest-rate lower bound.

Both conventional monetary policy and balance-sheet policy are allowed to respond to the occurrence of the shock, though their paths are assumed to be perfectly predictable from then on, as with all other exogenous variables. We suppose that R^m_τ and $\tilde{m}_\tau$ are both determined in subperiod 2 of period $\tau - 1$. Hence neither R^m_t nor $\tilde{m}_t$ can be affected by the value of χ; these variables are both equal to their values in the stationary equilibrium. But R^m_τ and $\tilde{m}_\tau$ can both differ from their stationary equilibrium values in periods $\tau \geq t + 1$.

For simplicity, we shall here consider only policy responses to the shock of a special sort. We continue to suppose that from period $t + 1$ onward, conventional monetary policy (that is, the choice of $R^m_\tau + 1$ for all $\tau \geq t + 1$) is used to ensure that the path of normal goods prices $\{P_\tau\}$ grows at the constant rate π^* in all periods $\tau \geq t + 1$.[48] We similarly continue to suppose that balance-sheet policy is used to

[48]Note that P_{t+1} is set in advance on the basis of expectations about the demand for normal goods in period $t+1$, which will depend on the interest rate R^m_{t+2} because of condition (2.15). Thus the rule for setting $R^m_{\tau+1}$ in periods $\tau \geq t + 1$ can be used to ensure that the market-clearing price for normal goods in all periods $\tau \geq t + 1$ are consistent with the inflation target. This desideratum leaves the value of R^m_{t+1} undetermined. Note that the value of P_t reflects expectations about how R^m_{t+1} would be set; but these are expectations about monetary policy in period t that were held prior to the unexpected shock, that may not be confirmed, as a result of the shock.

achieve a real outside supply of cash $m_{\tau+1}$ equal to the stationary equilibrium value $\tilde{m}$ for all $\tau \geq t+1$. We can then reduce the set of alternative monetary policies that we consider to a two-parameter family, corresponding to different possible choices of R^m_{t+1} and $\tilde{m}_{t+1}$ (both of which must be chosen in subperiod 2 of period t, but which may depend on the value of χ).[49]

Because the price P_t has been fixed in advance, it is assumed to be independent of the value of χ, and equal to the price associated with the stationary equilibrium that had previously been expected to continue. Once the shock χ occurs, there is no further uncertainty about how the economy will evolve from then on (except the uncertainty depicted in Figure 1). Hence we may suppose that the price P_τ of normal goods in each period $\tau \geq t+1$ is set so as to clear the market for normal goods in that period. (While we assume that P_τ must be set prior to subperiod 2 of period τ, we suppose that it is not set prior to subperiod 2 of period $\tau - 1$.[50]) Thus in the equilibrium considered in this section, the only period in which the market for normal goods need not clear is period t (the period in which the shock χ occurs); in that period, P_t is set at the level that would clear the market in the event that $\chi = 1$.

More generally, we shall suppose that all variables that are determined in subperiod 1 of period t, or earlier, are determined in the way that they are in the equilibrium in which $\chi = 1$ is expected (that is, as in the stationary equilibrium with flexible prices implied by the initial policy). Thus the values of $A_t, M_t, B_t, D_t, F_t, s_t$, and Q_t are unaffected by the shock, in addition to P_t and all variables dated $t-1$ or earlier. Instead the variables $c_{1t}, c_{2t}, c_{3t}, I_t, Y_t, x_t$, and $\tilde{P}_t$, as well as all variables dated $t+1$ or later, are determined in a way that takes account of the occurrence of the shock χ.[51] The Lagrange multipliers $\varphi_{4t}, \varphi_{5t}$, and φ_{6t} are jointly determined with this latter set of variables (as well as the Lagrange multipliers for later periods).

The variables that are affected by the shock χ are determined by a system of intertemporal equilibrium conditions of the form stated earlier, with the following

[49] For simplicity, in this section we abstract from the possible use of macroprudential policy, as in sections 1-3; that is, we consider only equilibria in which $\xi_t = 1$ at all times.

[50] This means that the length of time for which prices are sticky is limited in the proposed model. A quantitatively realistic model would doubtless need to allow some prices to remain fixed for a longer period, but the simple case considered here suffices to illustrate the qualitative effects of temporary stickiness of prices.

[51] Note that the variables s^*_t and Γ_t are undefined, as we have assumed that the crisis state does not occur in period t.

exceptions. First, the fact that the suppliers of normal goods must supply whatever quantity of such goods is demanded at the predetermined price P_t means that the FOC (2.13) need not be satisfied in period t *ex post* (that is, after the shock χ occurs). However, the other FOCs for optimal household behavior stated above continue to apply, and (2.13) also must hold in periods $t+1$ and later (since normal goods prices in those periods are set in a way that clears the market). Thus we drop one (but only one) of the conditions that would determine a flexible-price intertemporal equilibrium from subperiod 2 of period t onward, replacing it by the requirement that P_t equal a predetermined value, whether this clears the market for normal goods or not. Second, the partial derivatives $u_i(c_1, c_2)$ in FOCs (2.9)–(2.10) are replaced by $\chi u_i(c_1, c_2)$ (for $i = 1, 2$) in period t only. All other FOCs and market-clearing conditions continue to take the forms stated above.

The demand for cash and credit goods in period t will then be given by

$$c_{1t} \;=\; c_1(\lambda_t/\chi; M_t/P_t), \qquad c_{2t} \;=\; c_2(\lambda_t/\chi; M_t/P_t),$$

where M_t/P_t is unaffected by the shock. Aggregate demand for normal goods in period t is accordingly

$$Y_t \;=\; c_1(\lambda_t/\chi; M_t/P_t) \,+\, c_2(\lambda_t/\chi; M_t/P_t) \,+\, I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1}). \tag{5.1}$$

Since $c_1(\lambda; m)$ and $c_2(\lambda; m)$ are both non-increasing functions of λ, and at least c_2 must be decreasing, it follows that aggregate demand is a monotonically increasing function of χ, for given values of λ_t and λ_{t+1}.

Condition (2.40) continues to be a requirement for equilibrium, as a result of which we must have

$$\lambda_t \;=\; \beta\, \lambda_{t+1}\, \hat{\varphi}_1(\lambda_{t+1}; F(I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})), \tilde{m}_{t+1})\, R^m_{t+1}\, \frac{P_t}{P_{t+1}}. \tag{5.2}$$

This equation indicates the way in which the choice of R^m_{t+1} in period t affects the value of λ_t, and through it aggregate demand Y_t, for given expectations about conditions in period $t+1$. Note that if the price P_t were required to clear the market for normal goods, substitution of (5.1) into (2.13) would yield a condition to determine the required value of λ_t in equilibrium; (5.2) would then indicate the interest rate R^m_{t+1} required to achieve the price-level target P_t. Under the assumption that P_t is predetermined and need not clear the market, it is possible for R^m_{t+1} to change in response to the shock, resulting in a value of Y_t that need not satisfy the voluntary supply condition (2.13).

In each period from $t+1$ onward, we effectively have flexible prices, so that condition (2.39) is again required for equilibrium. Thus for any specification of R^m_{t+1} and of the path $\{\tilde{m}_\tau\}$ for all $\tau \geq t+1$, the equilibrium sequence $\{\lambda_\tau\}$ for $\tau \geq t$ is determined by condition (5.2) and the sequence of conditions of the form (2.39) for each period from $t+1$ onward. Given a solution for the sequence $\{\lambda_\tau\}$, aggregate demand for normal goods is determined by (5.1) in period t, and by (2.37) in each period from $t+1$ onward. The implied equilibrium values of other variables are then determined in the way discussed in section 2.

5.2 Real Effects of Conventional and Unconventional Monetary Policy

We can now compare the effects of quantitative easing with those of conventional interest-rate policy, as possible responses to a shock χ. Note that if both R^m_{t+1} and the path $\{\tilde{m}_\tau\}$ for $t \geq t+1$ remain fixed at the values associated with the stationary equilibrium in which there is no shock, then the values $\lambda_\tau = \bar{\lambda}$ for all $\tau \geq t$ will satisfy condition (2.39) in period t and condition (2.39) for each of the periods $t+1$ and later, where $\bar{\lambda}$ is the constant value of λ_τ in the stationary equilibrium. Aggregate demand for normal goods in period t is then given by (5.1). If $\chi = 0$, this implies $Y_t = \bar{Y}$, the constant level of output in the stationary equilibrium. If instead $\chi < 0$, we will have $Y_t < \bar{Y}$. This reduction in the production of normal goods will be inefficient, since it will imply that

$$u_2(c_{1t}, c_{2t}) \;=\; \chi\bar{\lambda} \;<\; \bar{\lambda} \;=\; v'(\bar{Y}) \;\leq\; v'(Y_t),$$

so that the marginal utility of additional consumption of normal goods would exceed the marginal disutility of supplying them.

We consider now the extent to which monetary policy can be used to respond to such a shock. In addition to the effects of policy on production and consumption, we shall be interested in how each of our possible dimensions of central-bank policy influence financial conditions. Two measures of financial conditions are especially useful. One is the size of the money premium earned by cash, which we can measure by the extent to which the ratio

$$\frac{E_t[R^b_{t+1}]}{R^m_{t+1}} \;=\; \tilde{\varphi}_{1,t+1}$$

is greater than one. This is a measure of financial conditions that determines the incentives for short-term debt issuance by banks. Another important measure is the expected one-period real return on longer-term bonds,

$$1 + \bar{r}^b_{t+1} \;\equiv\; E_{t+1}\left[\frac{R^b_{t+1}P_t}{P_{t+1}}\right] \;=\; \frac{\lambda_t}{\beta\bar{\lambda}}.$$

(Note that this aspect of financial conditions can alternatively be measured by the value of λ_t.) This is the measure of financial conditions that is relevant for determining the aggregate demand for non-durable normal goods, as a result of (5.1). Below we analyze the effects of each of the dimensions of policy on both of these measures of financial conditions.

5.2.1 Conventional Monetary Policy

Conventional monetary policy can be used to mitigate the effects of a χ shock by lowering R^m_t (if this is not prevented by the lower bound on the nominal interest rate). The effects of such policy are most easily seen in the special case that $v(Y)$ is linear, so that $v'(Y) = \bar{\lambda}$ regardless of the value of Y. Then (2.39) requires that $\lambda_{t+1} = \bar{\lambda}$, and (5.2) reduces to

$$\lambda_t \;=\; \beta\,\bar{\lambda}\,\hat{\varphi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})\,R^m_{t+1}/\Pi. \tag{5.3}$$

Here we have also substituted the target gross inflation rate Π for P_{t+1}/P_t, on the assumption that interest-rate policy in period $t+1$ is used to ensure that the target inflation rate is realized, regardless of other conditions.

For simplicity, we discuss here only the case of an equilibrium in which bankers' collateral constraint binds in period $t+1$, so that durables are over-valued in subperiod 1. (Note that this is the case in which risks to financial stability are of the greatest concern.) In this case, we can use

$$M_t/P_t \;=\; \tilde{m}_t \;+\; \lambda_t^{-1}\tilde{\Gamma}(s_t)s_t$$

(which follows from (2.29)), together with the fact that $\tilde{\varphi}_1(\lambda; m)$ is decreasing in both arguments (as shown in section 2), to conclude that $\hat{\varphi}_1(\lambda; s, \tilde{m})$ will be a decreasing function of both s and $\tilde{m}$, for any fixed value of λ.

In addition, (2.35) requires that

$$\lambda_t \;=\; \beta\, \phi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})\, F'(I_t). \tag{5.4}$$

This establishes an equilibrium relationship between investment demand I_t and financial conditions as measured by λ_t (though it is important to note that the value of $\tilde{m}_{t+1}$ remains of independent relevance). Equating the right-hand sides of (5.3) and (5.4), we see that equilibrium investment I_t must satisfy

$$\hat{\varphi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}) \;=\; \frac{\Pi}{\bar{\lambda}\, R^m_{t+1}}\, \phi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})\, F'(I_t). \tag{5.5}$$

Again restricting attention to the case where bankers' collateral constraint binds in period $t+1$, we can alternatively express the function $\phi(\lambda; s, \tilde{m})$ in terms of the function $\hat{\varphi}_1$. Note that

$$\begin{aligned} \tilde{\varphi}_{1t}\tilde{Q}_t &= \tilde{Q}^* \,+\, (1-p)\tilde{\varphi}_{3t}\,\tilde{\Gamma}_t \\ &= \tilde{Q}^* \,+\, (1-p)[(\tilde{\varphi}_{2t} \,+\, 1)\,\tilde{\Gamma}_t \,-\, \gamma q] \\ &= \gamma p) \,+\, [\tilde{\varphi}_{1t} \,-\, p]\,\tilde{\Gamma}_t, \end{aligned}$$

using (2.5), (2.6), and (2.4) in succession. It follows that we can express the function ϕ as

$$\phi(\lambda; s, \tilde{m}) \;\equiv\; \hat{\phi}(\hat{\varphi}_1(\lambda; s, \tilde{m}); s), \tag{5.6}$$

where

$$\hat{\phi}(\tilde{\varphi}_1; s) \;\equiv\; \gamma p \,+\, [\tilde{\varphi}_1 \,-\, p]\,\tilde{\Gamma}(s). \tag{5.7}$$

Condition (5.5) can then be written alternatively in the form

$$\tilde{\varphi}_{1,t+1} \;=\; \frac{\Pi}{\bar{\lambda}\, R^m_{t+1}}\, \hat{\phi}(\tilde{\varphi}_{1,t+1}; F(I_t))\, F'(I_t) \tag{5.8}$$

using (5.6). This describes a relationship that must exist between investment demand I_t and the money premium $\tilde{\varphi}_{1,t+1}$ in the case of any given specification of conventional monetary policy. It is worth noting (for the sake of our discussion below of quantitative easing) that this relationship is unaffected by the value of $\tilde{m}_{t+1}$.

Because $\tilde{\Gamma}(s) \leq \gamma q < \gamma$, (5.7) implies that

$$0 \ < \ \frac{\partial \hat{\phi}}{\partial \tilde{\varphi}_1} \ < \ \frac{\hat{\phi}}{\tilde{\varphi}_1}, \qquad \frac{\partial \hat{\phi}}{\partial s} \ < \ 0.$$

It follows from these that (5.8) implicitly defines a function

$$\tilde{\varphi}_{1,t+1} \ = \ \bar{\varphi}_1(I_t, R^m_{t+1}) \tag{5.9}$$

which is decreasing in both arguments.

Using this together with (2.34), we see that the equilibrium level of investment I_t must satisfy

$$\hat{\varphi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}) \ = \ \bar{\varphi}_1(I_t, R^m_{t+1}). \tag{5.10}$$

This can be solved for the effects of monetary policy on investment. Given that $\hat{\varphi}_1(\lambda; s, m)$ is a decreasing function of s (when the collateral constraint binds), we see that both sides of (5.10) are decreasing functions of I_t.

The comparative statics of I_t in response to a change in either R^m_{t+1} or $\tilde{m}_{t+1}$ then depend on the relative slopes of these two schedules. We shall assume that in the initial equilibrium, relative to which we wish to consider the effects of a change in monetary policy,

$$\frac{\partial \bar{\varphi}_1}{\partial I} \ < \ \frac{\partial \hat{\varphi}_1}{\partial I} \ < \ 0, \tag{5.11}$$

as shown in the upper part of Figure 9. In this case we obtain the conventional signs for the short-run effects of interest-rate policy.[52] In particular, because $\bar{\varphi}_1(I, R^m)$ is a decreasing function of R^m, a reduction of R^m_{t+1} will increase I_t, as shown in the figure.

It also follows from (5.6) and the fact that $\hat{\varphi}_1(\lambda; s, m)$ is a decreasing function of s that $\phi(\lambda; s, \tilde{m})$ is also a decreasing function of s. Thus the right-hand side of (5.4) is a decreasing function of I_t, for any fixed value of $\tilde{m}_{t+1}$. Hence (5.4) establishes an inverse relationship between λ_t and I_t that must hold regardless of the value chosen

[52]Note that if the inequality (5.11) is reversed, our model would imply that a reduction in the interest rate on cash is associated with a decrease, rather than an increase, in aggregate demand. Condition (5.4) establishes an inverse relationship between λ_t and I_t, regardless of whether (5.11) holds; so if a reduction of R^m_{t+1} were associated with a reduction of I_t, this would have to mean an increase in λ_t, and hence a decrease in all three terms on the right-hand side of (5.1). Such an effect would be contrary to familiar evidence regarding the effects of interest-rate policy, and would also preclude the possibility of a "liquidity trap" in which the lower bound on nominal interest rates prevents rates from being cut enough to maintain a desired level of real activity.

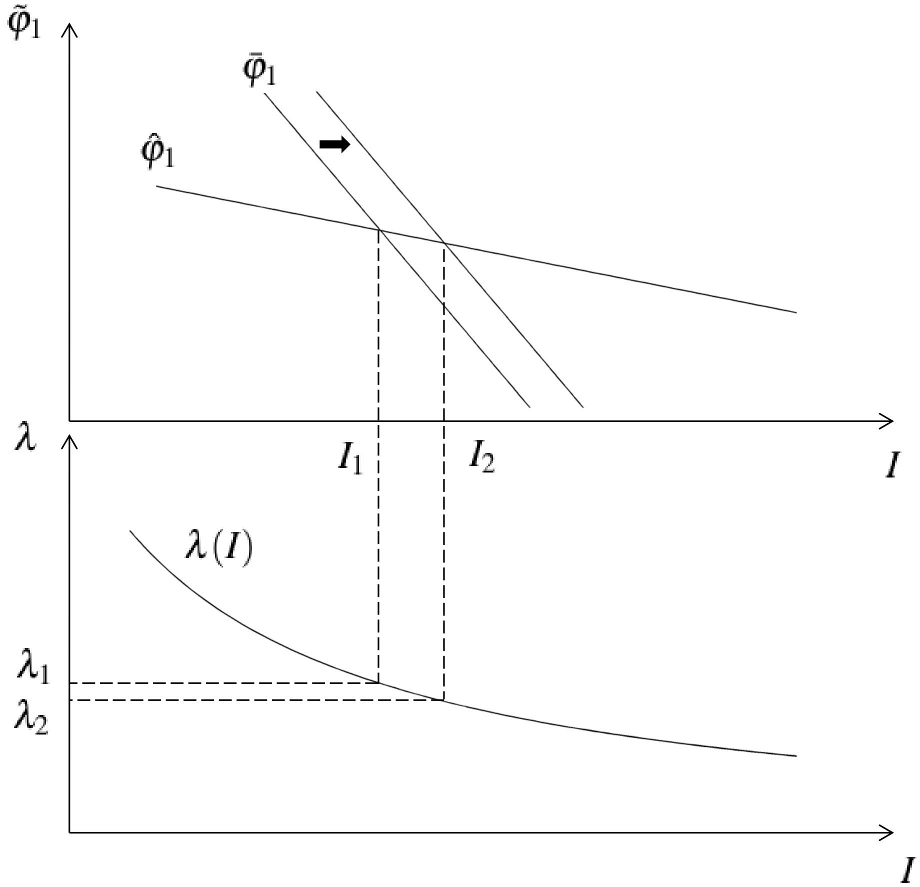

Figure 9: The effects of a reduction in R^m_{t+1}, the interest rate paid on safe assets, on the money premium and equilibrium investment in risky real assets (upper graph) and on financial conditions as measured by λ_t (lower graph).

for R^m_{t+1}. This relationship is graphed in the lower part of Figure 9. Since a reduction in R^m_{t+1} increases I_t, it must also reduce λ_t, as shown in the figure. This in turn will imply an increase in Y_t, because of (5.1).[53]

While interest-rate policy can be used to stimulate aggregate demand in this way, and so to reduce some of the distortions created by the demand shock χ, it has the side effect of increasing risks to financial stability. Part of the increase in aggregate demand associated with a reduction of λ_t will be an increase in the production of

[53]Note that the expression $I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})$ in this equation is just the quantity I_t, that we have shown must increase.

investment goods, as a result of which s_{t+1} will be higher. In the case of an equilibrium in which the collateral constraint binds, this will mean a correspondingly higher value of s^*_{t+1}, as a consequence of which the degree of under-valuation of durables in the event of a crisis and fire sale will be more severe. Thus in the sticky-price version of the model, it is indeed the case that reducing short-term nominal interest rates increases risk-taking by banks in a way that makes the distortions associated with a crisis more severe, should one occur.

5.2.2 Effects of Unconventional Policies

In the event that the lower bound on interest rates prevents R^m_{t+1} from being reduced to the extent that would be necessary to maintain aggregate demand at the desired level, quantitative easing provides an alternative channel through which aggregate demand may be increased. Like conventional interest-rate policy, an expansion of the supply of short-term safe assets by the central bank affects aggregate demand by easing financial conditions, as indicated by a reduction in λ_t (which can be thought of as the price of a particular very-long-duration indexed bond).

Consider the effects of an increase in $\tilde{m}_{t+1}$, holding R^m_{t+1} fixed. We can again determine the effect on equilibrium investment demand using (5.10). The schedule corresponding to the right-hand side of this equation does not shift as a result of an increase in $\tilde{m}_{t+1}$, but the fact that $\hat{\varphi}_1(\lambda; s, \tilde{m})$ is a decreasing function of $\tilde{m}$ means that the schedule corresponding to the left-hand side of the equation shifts down for each possible value of I_t, as shown now in the upper part of Figure 10. Then again assuming that the relative slopes of the two schedules are given by (5.11), we can again conclude that I_t must increase while $\tilde{\varphi}_{1,t+1}$ must decrease.

We can also again use (5.4) to determine the change in λ_t required by a given size increase in I_t. As argued above, an increase in I_t reduces the right-hand side of this equation, for any given value of $\tilde{m}_{t+1}$. In addition, (5.6) together with the result above that $\hat{\varphi}_1(\lambda; s, \tilde{m})$ is a decreasing function of $\tilde{m}$ implies that the function $\phi(\lambda; s, \tilde{m})$ is also a decreasing function of $\tilde{m}$. This means that the curve (the graph of equation (5.4)) graphed in the lower part of Figure 10 shifts down as a result of an increase in $\tilde{m}_{t+1}$, as shown in the figure. It then follows that λ_t is reduced by an increase in $\tilde{m}_{t+1}$, both because of the decrease in I_t (the shift along the curve) and because of the direct effect of an increase in $\tilde{m}_{t+1}$ (the downward shift of the curve).

It follows that an increase in $\tilde{m}_{t+1}$ must loosen financial conditions, in the sense

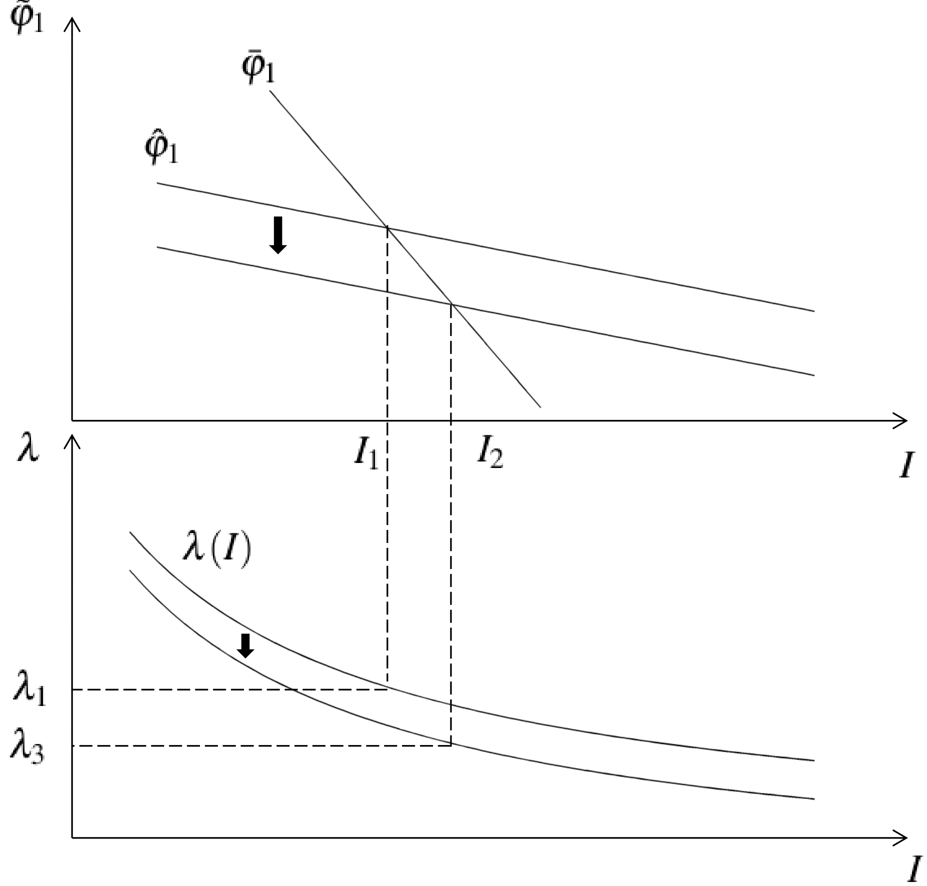

Figure 10: The effects of an increase in the central-bank supply of safe liabilities $\tilde{m}_{t+1}$ on the money premium and equilibrium investment in risky real assets (upper graph) and on financial conditions as measured by λ_t (lower graph).

that λ_t is reduced. This in turn means that, as in the case of an interest-rate cut, Y_t must increase because of (5.1). Thus the effects of quantitative easing are qualitatively similar to those of an interest-rate cut: financial conditions are eased, the aggregate demand for normal goods increases (both because of an increase in the demand for credit goods and an increase in the demand for investment goods), but at the same time risks to financial stability increase, because of an increase in short-term debt issuance by banks, leading to larger expected distortions in the case that a crisis state occurs in period $t+1$.

Nonetheless, the two policies do not have quantitatively equivalent effects. If we compare an interest-rate cut (reduction in R^m_{t+1}) and an increase in the net supply of safe assets by the central bank (increase in $\tilde{m}_{t+1}$) that increase the equilibrium demand for investment goods I_t by the same amount, the increase in $\tilde{m}_{t+1}$ reduces λ_t by a greater amount.[54] This can be seen from the fact that (5.4) must apply in either case. If, by hypothesis, I_t increases by the same amount in both cases, then the only difference in the implied value for λ_t is that $\tilde{m}_{t+1}$ increases in the second case, but remains constant in the first; and this implies a lower value of λ_t in the second case. This means that in the case of quantitative easing, a greater share of the total increase in aggregate demand comes from increased demand for credit goods, as opposed to increased demand for investment goods. Thus a given degree of aggregate demand stimulus can be achieved with less risk to financial stability if it is brought about through an expansion of the central bank's balance sheet, rather than by cutting the interest rate paid on cash.

We can also consider the effects of aggregate demand stimulus through relaxation of macroprudential constraints (that is, an increase in ξ_{t+1}). Let us generalize the analysis presented in the earlier part of this section to allow for a macroprudential tax (or reserve requirement), so that ξ_t need not equal 1 (as assumed thus far in this section). Conditions (5.1), (5.3) and (5.4) continue to be required for an equilibrium, and the definition of the function $\hat{\varphi}_1(\lambda; s, \tilde{m})$ is unchanged; but in (5.4), the expression $\phi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})$ must be replaced by $\phi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}, \xi_{t+1})$, where we define

$$\phi(\lambda; s, \tilde{m}, \xi) \;\equiv\; \hat{\phi}(\xi\hat{\varphi}_1(\lambda; s, \tilde{m}); s),$$

generalizing (5.6).

Condition (5.8) then takes the more general form

$$\tilde{\varphi}_{1,t+1} \;=\; \frac{\Pi}{\bar{\lambda}\, R^m_{t+1}}\, \hat{\phi}(\xi_{t+1}\tilde{\varphi}_{1,t+1}; F(I_t))\, F'(I_t).$$

As above, this implicitly defines a function

$$\tilde{\varphi}_{1,t+1} \;=\; \bar{\varphi}_1(I_t, R^m_{t+1}, \xi_{t+1}), \tag{5.12}$$

where now $\bar{\varphi}_1(I, R^m, \xi)$ is decreasing in I and R^m, and increasing in ξ. The equilibrium level of investment is again determined by (5.10), but now the schedule corre-

[54] Compare Figures 9 and 10. In Figure 10, the amount of quantitative easing is chosen so as to achieve the same increase in investment (from I_1 to I_2) as the interest-rate cut in Figure 9. The reduction in λ_t is instead larger (from λ_1 to $\lambda_3 < \lambda_2$).

sponding to the left-hand side is shifted only by $\tilde{m}_{t+1}$, while the schedule corresponding to the right-hand side is shifted by changes in either R^m_{t+1} or ξ_{t+1}. To a linear approximation (which is to say, in the case of small enough policy changes), then, an increase in ξ_{t+1} (a relaxation of macroprudential policy, as by reducing the required reserve ratio) has the same effects on I_t and $\tilde{\varphi}_{1,t+1}$ as a certain size of cut in R^m_{t+1}.

We can also use (5.12) to rewrite (5.3) in the form

$$\lambda_t \;=\; \beta\,\bar{\lambda}\,\bar{\varphi}_1(I_t, R^m_{t+1}, \xi_{t+1})\,R^m_{t+1}/\Pi. \tag{5.13}$$

From this it follows that since a relaxation of macroprudential policy reduces the value of $\tilde{\varphi}_{1,t+1}$, it must reduce the value of λ_t. Hence it increases demand for credit goods, and so must increase Y_t, like the other two policies just considered. But it also follows from (5.13) that in the case of two policy changes (a cut in R^m_{t+1} or an increase in ξ_{t+1}) that reduce $\tilde{\varphi}_{1,t+1}$ to the same extent, and that therefore reduce $\bar{\varphi}_1(I_t, R^m_{t+1}, \xi_{t+1})$ to the same extent, the interest-rate cut must reduce λ_t by more, and so must stimulate demand for credit goods to a greater extent. Thus an even greater share of the increase in aggregate demand achieved by relaxing macroprudential policy comes from an increase in investment demand, as opposed to an increase in the demand for credit goods, than in the case of an increase in aggregate demand achieved by cutting the interest rate on cash.

We can therefore order the three types of expansionary policy: for a given degree of increase in aggregate demand, achieving it by increasing $\tilde{m}_{t+1}$ increases I_t the least, achieving it by reducing R^m_{t+1} increases I_t to an intermediate extent, and achieving it by increasing ξ_{t+1} increases I_t (and hence short-term debt issuance by banks, and risks to financial stability) the most. One consequence of this is that increasing aggregate demand through monetary policy *need not* involve any increased risks to financial stability at all. For example, one might combine an increase in $\tilde{m}_{t+1}$ with a tightening of macroprudential policy (reduction of ξ_{t+1}) that exactly offsets the effects of the quantitative easing on desired investment demand, so that there is no net change in I_t. Since the former policy change will reduce λ_t more than the latter policy change increases it, the net effect will be a loosening of financial conditions, and an increase in the demand for credit goods. Since there is (by hypothesis) no change in investment demand, aggregate demand Y_t will increase; but there will be no associated increase in s_{t+1}, and hence no increase in the severity of the distortions associated with a crisis state in period $t+1$.

We conclude that while quantitative easing *may* increase risks to financial stability in the case that nominal rigidities allow short-run effects of monetary policy on aggregate demand, it *need not* have any such effect. If the increase in the central bank's balance sheet is combined with an increase in the interest rate paid on cash, or a tightening of macroprudential policy to a sufficient extent, it can increase aggregate demand without any adverse consequences for financial stability. It is particularly easy to achieve this outcome by combining the quantitative easing with macroprudential policy, if a suitable macroprudential instrument exists; for in our model, reduction of ξ_{t+1} provides an even greater disincentive to issuance of short-term debt by banks than does raising R^m_{t+1}, for a given degree of reduction in aggregate demand.

These results imply that quantitative easing may be a useful addition to a central bank's monetary policy toolkit, even when interest-rate policy is not yet constrained by the effective lower bound on short-term nominal interest rates. In the case of a contractionary shock χ, the effects on aggregate demand can be offset purely through a reduction in R^m_{t+1}, if the lower bound does not prevent the size of rate cut that is needed; but such a response increases risks to financial stability more than is necessary. One could alternatively counter the effects of the χ shock by increasing $\tilde{m}_{t+1}$, while leaving R^m_{t+1} unchanged; and this would have the advantage of posing less of a threat to financial stability. Even better, one could combine a somewhat larger increase in $\tilde{m}_{t+1}$ with a tightening of macroprudential policy, allowing the effects of the χ shock on aggregate demand to be offset, with even less of an increased risk to financial stability, possibly none at all.

6 Conclusions

We can now assess the validity of the concerns about the consequences of quantitative easing for financial stability sketched in the introduction, in the light of the model just presented. Our model is one in which monetary policy does indeed influence risks to financial stability; in particular, policies that loosen financial conditions, either by lowering the central bank's operating target for its policy rate (conventional monetary policy), or by relaxing reserve requirements (or other macro-prudential constraints), should each increase the attractiveness of private issuance of "money-like" liabilities, resulting in increased leverage and as a consequence an increased risk of serious resource misallocation in the event of a funding crisis. This means that there can at times be a tension between the monetary policy that would be preferable strictly from the standpoint of aggregate demand management and inflation stabilization, on the one hand, and the policy that would minimize risks to financial stability.

The question is whether it is correct to think of quantitative easing as a policy analogous to these, and hence that poses similar risks to financial stability. Our model implies that such an analogy is imperfect. A quantitative easing policy (increasing the public supply of safe assets, by issuing additional safe liabilities of the central bank, used to purchase assets that do not earn a similar safety premium) similarly increases aggregate demand by lowering the equilibrium rate of return on non-safe assets. But unlike conventional monetary policy, it does this by lowering the equilibrium safety premium (by making safe assets less scarce), rather than by lowering the equilibrium return on safe assets; and this does not have the same consequences for financial stability. Lowering the equilibrium return on risky investments (such as the "durable goods" modeled here, which one may think of as housing) by lowering the return on safe assets works only insofar as the *increased* spread between the two returns that would result if the return on risky investments did not also fall increases the incentive to finance additional risky investment by issuing safe liabilities, thus increasing the leverage of the banks and the degree to which they engage in liquidity transformation; this results in a reduced equilibrium return on risky investment, but not by enough to fully eliminate the increased spread that induces banks to issue additional safe asset-backed liabilities. This mechanism necessarily increases the risk to financial stability at the same time as it increases aggregate demand. Quantitative easing instead *decreases* the spread between these two returns, at least in the absence of any change in the private supply of safe liabilities. This reduction in the spread reduces the incentive for private issuance of such liabilities, and reduced issuance of safe asset-backed liabilities by banks offsets some of the reduction in the spread, but does not completely eliminate it as otherwise banks would not have a reason to reduce their issuance. Hence in this case the reduction in the equilibrium return on risky investments is associated with a *reduction* of the incentive for liquidity transformation by banks, rather than an increase.

Similarly, quantitative easing increases the total supply of safe assets and so reduces the safety premium; but unlike a reduction in reserve requirements (or relaxation of macroprudential policy), it achieves this by increasing the public supply of safe assets (and actually reducing the incentive for private issuance), rather than by increasing the incentive that banks have to finance risky investment by issuing safe asset-backed liabilities. Again the consequences for the degree of liquidity transformation by the banking sector, and hence the risk to financial stability, are entirely different.

Likewise, it is true that quantitative easing eases financial conditions by reducing the spread between the required return on risky investments and the return on safe assets; but this does not mean that risk premia are "artificially" reduced in a way that distorts incentives for prudent behavior, leading to excessive risk-taking. In the model presented here, quantitative easing reduces the safety premium, but because the public supply of safe assets for private investors to hold is increased, and not because anyone is misled into under-estimating the degree of risk involved in undertaking risky investments. Moreover, the reduced spread reduces the incentive for private issuance of safe liabilities, and instead favors financing of investment through issuance of non-safe liabilities, which is desirable on financial stability grounds. Rather than threatening financial stability by encouraging more risk-taking, it favors it by encouraging forms of financing that reduce the magnitude of the distortions associated with a funding crisis.

In our model, we can compare the effects of three alternative policies that can increase aggregate demand by easing financial conditions: reducing the central bank's operating target for the nominal interest rate on safe assets (conventional monetary policy, on the assumption that the zero lower bound does not yet preclude such easing), relaxing reserve requirements (or other macro-prudential constraints), and quantitative easing. We find that among these, quantitative easing is the policy that increases risks to financial stability the least, for any given degree of increase in aggregate demand. Not only does quantitative easing make it possible for a central bank to increase aggregate demand even when conventional monetary policy is constrained by the zero lower bound on nominal interest rates, but it is possible, at least in principle, to increase aggregate demand without the collateral effect of an increased risk to financial stability, if the increased supply of safe liabilities by the central bank is combined with a sufficient tightening of macroprudential measures. (The latter measures would by themselves reduce aggregate demand; but when combined with quantitative easing, the net effect is an increase in aggregate demand, even when the degree of macroprudential tightening is enough to fully offset any increase in risks to financial stability as a result of the balance-sheet policy.)

This indicates that a concern for the effects of monetary policy on financial stability need not preclude using quantitative easing to stimulate aggregate demand in circumstances where (as in the US in the aftermath of the recent crisis) conventional monetary policy is constrained by the zero lower bound. In fact, the fact that demand stimulus through quantitative easing poses smaller risks to financial stability than demand stimulus through lowering short-term nominal interest rates suggests that balance-sheet policy may be a useful tool of monetary stabilization policy even when a central bank is far from the zero lower bound. In the model presented here, aggregate demand stimulus achieved by lowering nominal interest rates increases the risk to financial stability more than would a quantitative easing policy that is equally effective in increasing aggregate demand. This implies that (even if macroprudential policy is unavailable or ineffective) it should be possible to increase aggregate demand without increasing the risk to financial stability by combining expansionary balance-sheet policy with an appropriate size of *increase* in the policy rate. In such a case, conventional monetary policy would essentially be used for macroprudential purposes (to control the risk to financial stability) while balance-sheet policy is used for demand stabilization.

Further study of the effects of quantitative easing policies would therefore seem to be warranted, not simply for the sake of having a more effective policy toolkit for use the next time that conventional policy is again constrained by the zero lower bound, but also, arguably, in order to improve the conduct of stabilization policy under more normal circumstances as well. The availability of this additional dimension of monetary policy is particularly likely to be of use under circumstances where additional monetary stimulus through interest-rate reduction is unattractive owing to concerns about financial stability. Such a situation could easily arise even when interest rates are well above their effective lower bound.

References

[1] Adrian, Tobias, and Hyun Song Shin, "Financial Intermediaries and Monetary Economics," *Handbook of Monetary Economics,* vol. 3A, Amsterdam: Elsevier Press, 2010.

[2] Bernanke, Ben S., "Opening Remarks: Monetary Policy Since the Onset of the Crisis," in *The Changing Policy Landscape,* Kansas City: Federal Reserve Bank of Kansas City, 2012.

[3] Brunnermeier, Markus K., "Deciphering the Liquidity and Credit Crunch 2007-2008," *Journal of Economic Perspectives,* 23(1): 77-100 (Winter 2009).

[4] Brunnermeier, Markus K., and Lasse Heje Pederson, "Market Liquidity and Funding Liquidity," *Review of Financial Studies* 22: 2201-2238 (2009).

[5] Caballero, Ricardo J., and Emmanuel Farhi, "A Model of the Safe Asset Mechanism (SAM): Safety Traps and Economic Policy," NBER Working Paper no. 18737, January 2013.

[6] Carlson, Mark, Burcu Duygan-Bump, Fabio Natalucci, William R. Nelson, Marcelo Ochoa, Jeremy Stein, and Skander Van den Heuvel, "The Demand for Short-Term Safe Assets and Financial Stability: Some Evidence and Implications for Central Bank Policies," FEDS paper no. 2014-102, Federal Reserve Board, November 25, 2014.

[7] Eggertsson, Gauti B., and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity* 2003-1, pp. 139-211.

[8] Giavazzi, Francesco, and Alberto Giovannini, "Central Banks and the Financial System," in S. Eijffinger and D. Masciandaro, eds., *Central Banking, Financial Regulation and Supervision After the Financial Crisis,* Cheltenham, UK: Edward Elgar, 2012.

[9] Gorton, Gary, *Slapped by the Invisible Hand: The Panic of 2007,* Oxford: Oxford University Press, 2010.

[10] Gorton, Gary, and George Pennacchi, “Financial Intermediaries and Liquidity Creation,” *Journal of Finance* 45: 49-71 (1990).

[11] Gorton, Gary, Stefan Lewellen, and Andrew Metrick, “The Safe-Asset Share,” *American Economic Review* 102(3): 101-106 (2012).

[12] Greenwood, Robin, Samuel G. Hanson, Joshua S. Rudolph, and Lawrence H. Summers, “Government Debt Management at the Zero Lower Bound,” Hutchins Center Working Paper no. 5, September 30, 2014.

[13] Greenwood, Robin, Samuel G. Hanson, and Jeremy C. Stein, “A Gap-Filling Theory of Corporate Debt Maturity Choice,” *Journal of Finance* 65: 993-1028 (2010).

[14] Jeanne, Olivier, and Anton Korinek, “Managing Credit Booms and Busts: A Pigouvian Taxation Approach,” NBER Working Paper no. 16377, September 2010.

[15] Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, “The Aggregate Demand for Treasury Debt,” *Journal of Political Economy* 120: 233-267 (2012).

[16] Krugman, Paul R., “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap,” *Brookings Papers on Economic Activity* 1998-2, pp. 137-206.

[17] Lorenzoni, Guido, “Inefficient Credit Booms,” *Review of Economic Studies* 75: 809-833 (2008).

[18] Lucas, Robert E., Jr., and Nancy L. Stokey, “Money and Interest in a Cash-in-Advance Economy,” *Econometrica* 55: 491-513 (1987).

[19] Sergeyev, Dmitriy, “Optimal Macroprudential and Monetary Policy in a Currency Union,” working paper, Università Bocconi, March 2016.

[20] Shleifer, Andrei, and Robert W. Vishny, “Liquidation Values and Debt Capacity: A Market Equilibrium Approach,” *Journal of Finance* 47: 1343-1366 (1992).

[21] Shleifer, Andrei, and Robert W. Vishny, “Fire Sales in Finance and Macroeconomics,” *Journal of Economic Perspectives*, 25(1): 29-48 (Winter 2011).

[22] Stein, Jeremy C., "Monetary Policy as Financial-Stability Regulation," *Quarterly Journal of Economics* 127: 57-95 (2012).

[23] Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press, 2003.