Online Appendix for "Bailout Stigma"

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A Technical Preliminaries

In this section, we derive technical results that will be used frequently in the proofs.

Definition A.1. *Define the following:*

(i) $m(a,b) := \mathrm{E}[\theta \,|\, \theta \in [a,b]\,] \quad \forall\, 0 \le a \le b \le 1$*;*

(ii) $\gamma(a) := \max\{\theta \ge a \,|\, \theta \le m(a,\theta) + S\} \quad \forall\, a \in [0,1]$*;*

(iii) $\mathring{m}(a,b,c,d) := \mathrm{E}[\theta \,|\, \theta \in [a,b] \cup [c,d]\,] \quad \forall\, 0 \le a \le b < c \le d \le 1$*;*

(iv) $\mathring{\gamma}(a,b,c) := \max\{\theta \ge a \,|\, \theta \le \mathring{m}(a,b,c,\theta) + S\} \quad \forall\, 0 \le a \le b < c \le 1.$

Given that θ is a continuous random variable, m and $\mathring{m}$ are well-defined and differentiable with respect to every argument. Next, recall our log concavity assumption:

Assumption A.1 (Log-concavity)**.** $\frac{d^2}{d\theta^2} \log f(\theta) < 0$ *for all* $\theta \in [0,1]$.

Based on the above, we obtain the following results.

Lemma A.1. *(i)* $\frac{\partial}{\partial a} m(a,b),\ \frac{\partial}{\partial b} m(a,b) \le 1 \ \ \forall\, 0 \le a \le b \le 1$*; (ii)* γ *is a well-defined function and* $\frac{\partial}{\partial a}\gamma(a) \le 1 \ \ \forall\, a \in [0,1]$*; (iii)* $\frac{\partial}{\partial d}\mathring{m}(a,b,c,d) \le 1 \ \ \forall\, 0 \le a \le b < c \le d \le 1$*; (iv)* $\mathring{\gamma}$ *is a well-defined function.*

Proof. See Bagnoli and Bergstrom (2005) for the proof of part (i).

For part (ii), note that (i) implies $\theta - m(a,\theta)$ is increasing and continuous in θ for all $\theta \ge a$. Thus $\gamma(a)$ defined in Definition A.1-(ii) exists and is uniquely determined for every

$a \in [0,1]$ as the minimum of the unique solution to $\theta - m(a,\theta) = S$ and 1. To prove the second part of (ii), first suppose $\gamma(a) = m(a,\gamma(a)) + S$. Applying the implicit function theorem, we have $\frac{d}{da}\gamma(a) = \frac{\frac{\partial}{\partial a}m(a,\gamma(a))}{1-\frac{\partial}{\partial \gamma}m(a,\gamma(a))}$. From Corollary 1 of Szalay (2012), we have $\frac{\partial}{\partial a}m(a,b) + \frac{\partial}{\partial b}m(a,b) \leq 1$ for all $0 \leq a \leq b \leq 1$ if $f(\cdot)$ is log-concave. This implies $\frac{d}{da}\gamma(a) = \frac{\frac{\partial}{\partial a}m(a,\gamma(a))}{1-\frac{\partial}{\partial \gamma}m(a,\gamma(a))} \leq 1$. If $\gamma(a) = 1$, then $\gamma(x) = 1$ for all $x > a$, implying that the right derivative of $\gamma(a)$ is 0 for all $a \in [0,1]$ such that $\gamma(a) = 1$.

To prove part (iii), define $\check{F}(\theta) := F(\theta) - [F(c) - (F(b) - F(a))]$ for every $\theta \geq c$. $\check{F}(\theta)$ is log-concave as shown below:

$$\begin{aligned}
\frac{d^2 \log \check{F}(\theta)}{d\theta^2} &= \frac{d}{d\theta}\left(\frac{f(\theta)}{F(\theta) - [F(c) - (F(b) - F(a))]}\right) \\
&= \frac{d}{d\theta}\left(\frac{f(\theta)}{F(\theta)} \times \frac{F(\theta)}{F(\theta) - [F(c) + (F(b) - F(a))]}\right) \\
&= \frac{d}{d\theta}\left(\frac{f(\theta)}{F(\theta)} \times \left(1 + \frac{F(c) + (F(b) - F(a))}{F(\theta) - [F(c) + (F(b) - F(a))]}\right)\right) \\
&\leq 0.
\end{aligned}$$

In the above, the last inequality follows since $1 + \frac{F(c)+(F(b)-F(a))}{F(\theta)-[F(c)+(F(b)-F(a))]}$ is positive and decreasing in θ, and $\frac{f(\theta)}{F(\theta)}$ is decreasing in θ by the log-concavity $f(\theta)$. Using the log-concavity of $\check{F}(\theta)$, we now show $\frac{\partial \mathring{m}}{\partial d} \leq 1$. Differentiating $\mathring{m}$ with respect to d, we have

$$\begin{aligned}
\frac{\partial \mathring{m}}{\partial d} &= -\frac{f(d)}{[(F(d) - F(c)) + (F(b) - F(a))]^2}\int_a^b \theta dF(\theta) + \frac{\partial}{\partial d}\left(\frac{1}{(F(d) - F(c)) + (F(b) - F(a))}\int_c^d \theta dF(\theta)\right) \\
&< \frac{\partial}{\partial d}\left(\frac{1}{(F(d) - F(c)) + (F(b) - F(a))}\int_c^d \theta dF(\theta)\right).
\end{aligned}$$

To prove $\frac{\partial \mathring{m}}{\partial d} \leq 1$, it suffices to show $\frac{\partial}{\partial d}\left(\frac{1}{(F(d)-F(c))+(F(b)-F(a))}\int_c^d \theta dF(\theta)\right) \leq 1$. Define a map $\check{\delta} : [c,1] \to \mathbb{R}$ as

$$\check{\delta}(\theta) := \theta - \frac{1}{(F(\theta) - F(c)) + (F(b) - F(a))}\int_c^\theta u dF(u),$$

which is a variant of $\delta(\theta)$ introduced by Bagnoli and Bergstrom (2005). Since $\mathring{m}(a,b,c,\theta) = \frac{1}{(F(\theta)-F(c))+(F(b)-F(a))}\int_c^\theta u dF(u)$, we need to show $\check{\delta}(\theta)$ is increasing in θ. Since $\check{F}(\theta) = (F(\theta) - F(c)) + (F(b) - F(a))$, we have $dF = d\check{F}$ for all $\theta \geq c$. Thus $\check{\delta}(\theta)$ can be rewritten as $\check{\delta}(\theta) = \theta - \frac{1}{\check{F}(\theta)}\int_c^\theta u d\check{F}(u)$. Integrating $\int_c^\theta u d\check{F}(u)$ by parts yields

$$\check{\delta}(\theta) = \theta - \frac{\theta\check{F}(\theta) - \int_c^\theta \check{F}(u)du}{\check{F}(\theta)} = \frac{\int_c^\theta \check{F}(u)du}{\check{F}(\theta)}.$$

From Theorem 1-(ii) in Bagnoli and Bergstrom (2005), $\int_c^\theta \check{F}(u)du$ is log-concave for all $\theta \geq c$ if $\check{F}(u)$ is log-concave for all $\theta \geq c$. Thus $\frac{\int_c^\theta \check{F}(u)du}{\check{F}(\theta)}$ is increasing in θ, which implies $\check{\delta}(\theta)$ is also increasing in θ.

Part (iv) follows from part (iii): since $\theta - \mathring{m}(a,b,c,\theta)$ is increasing and continuous in θ, $\mathring{\gamma}(a,b,c)$ is the minimum of the unique solution to $\theta - \mathring{m}(a,b,c,\theta) = S$ and 1. *Q.E.D.*

For future reference, we reproduce our regularity conditions below.

Assumption A.2 (Assumption 2 in the main text)**.** *(i)* $\forall\ 0 < \tilde{\theta} < \theta < 1$, $\Delta(\theta;\tilde{\theta},S) := m(\tilde{\theta},\theta) + S - \theta - (m(\theta,\gamma(\theta)) - m(\tilde{\theta},\theta))$ *is decreasing in* θ*;*

(ii) If $\Delta(0;\tilde{\theta},S) \geq 0$*, then* $\Delta(0;\tilde{\theta},S') \geq 0$ *for every* $S' > S$ *and every* $\tilde{\theta} \in (0,1)$*;*

(iii) For every $0 < \tilde{\theta} < \theta < 1$*,* $2m(\tilde{\theta},\theta) - m(0,\tilde{\theta})$ *is decreasing in* $\tilde{\theta}$*;*

(iv) $\theta_0^* - m(\theta_0^*,\gamma(\theta_0^*)) + S > 0$ *for every* $S > 0$*, where* $\theta_0^* = \gamma(0)$*;*

(v) $\theta \geq 2m(0,\theta)$ *for all* $\theta \in [0,1]$.

B Proofs for Section 3

Lemma 1. *In any equilibrium without government interventions, there is a cutoff* $0 \leq \theta_1 \leq 1$ *such that all types* $\theta \leq \theta_1$ *sell in each of the two periods at price* $m(0,\theta_1)$ *and all types* $\theta > \theta_1$ *hold out in* $t = 1$ *and are offered price* $m(\theta_1,\gamma(\theta_1))$ *in* $t = 2$*, which types* $\theta \in [\theta_1,\gamma(\theta_1))$ *accept. If* $\theta_1 = 1$*, then* $S \geq 2(1 - \mathrm{E}[\theta])$.

Proof of Lemma 1. Fix a game with discount factor $\delta \in (0,1)$ and fix an equilibrium of the required form. Let $(q_1(\theta), q_2(\theta))$ be the units of the asset a type-θ firm sells in each of the two periods and $(t_1(\theta), \theta_2(\theta))$ be the corresponding transfers.

<u>Step 1.</u> There exists $0 \leq \hat{\theta} \leq \check{\theta} \leq \tilde{\theta} \leq 1$ such that $q_1(\theta) = q_2(\theta) = 1$ for $\theta < \hat{\theta}$; $q_1(\theta) = 1, q_2(\theta) = 0$ for any $\theta \in (\hat{\theta},\check{\theta})$; $q_1(\theta) = 0, q_2(\theta) = 1$ for any $\theta \in (\check{\theta},\tilde{\theta})$; and $q_1(\theta) = q_2(\theta) = 0$ for any $\theta > \tilde{\theta}$.

Proof. In pure strategy equilibrium, we have $q_i(\theta) \in \{0,1\}$ for each θ, $i = 1,2$. The expected discounted payoff for type-θ when imitating type-θ' is $u(\theta';\theta) := q_1(\theta')[S + t_1(\theta')] + [1 - q_1(\theta')]\theta + \delta[q_2(\theta')(S + t_2(\theta')) + (1 - q_2(\theta'))\theta]$. Let $Q(\cdot) := q_1(\cdot) + \delta q_2(\cdot)$ and $t(\cdot) := q_1(\cdot)t_1(\cdot) + \delta q_2(\cdot)t_2(\cdot)$. Since we must have $u(\theta;\theta) \geq u(\theta';\theta)$ for all θ, θ' in equilibrium, it follows that $(S - \theta)[Q(\theta) - Q(\theta')] \geq t(\theta') - t(\theta)$. Similarly we must have $u(\theta';\theta') \geq u(\theta;\theta')$, which leads to $(S - \theta')[Q(\theta') - Q(\theta)] \geq$

$t(\theta) - t(\theta')$. Combining these two inequalities, we have $(\theta - \theta')[Q(\theta) - Q(\theta')] \leq 0$. This implies monotonicity: $Q(\theta) \leq Q(\theta')$ for any $\theta \geq \theta'$. Since $q_i(\theta) \in \{0, 1\}$ for each θ, $i = 1, 2$, the monotonicity implies the desired property.

Step 2. If $\check{\theta} < 1$, then those that hold out in $t = 1$ must be offered $m(\check{\theta}, \gamma(\check{\theta}))$ in equilibrium, which is accepted by types $\theta \in (\check{\theta}, \gamma(\check{\theta}))$ where $\gamma(\check{\theta}) > \check{\theta}$.

Proof. The belief in $t = 2$ for the holdouts is the truncated distribution of F on $[\check{\theta}, 1]$. This is essentially a one-shot problem with a truncated support. Thus the stated result follows from the definition of γ.

Step 3. $\hat{\theta} = \check{\theta}$.

Proof. Suppose to the contrary that $\hat{\theta} < \check{\theta}$. Let p be the price offered in $t = 1$. By the zero profit condition, p must be a break-even price for the types that accept it. Since type-$\check{\theta}$ firm must weakly prefer accepting p in $t = 1$ to not selling in either periods, we have

$$p + S + \delta\check{\theta} \geq (1 + \delta)\check{\theta} \Leftrightarrow p + S \geq \check{\theta}. \tag{B.1}$$

This means that all firms accepting p in $t = 1$ will accept the same price p in $t = 2$ if that price were offered in $t = 2$, with strict incentive for all firms with $\theta < \check{\theta}$. The fact that types $\theta \in (\hat{\theta}, \check{\theta})$ choose not to sell in $t = 2$ means that the price offered in $t = 2$ to those that accept p in $t = 1$, denoted by p^-, is strictly less than p.

Suppose first $\check{\theta} < 1$. Then, by Step 2, an offer $p_2 := m(\check{\theta}, \gamma(\check{\theta}))$ must be made in equilibrium to those that hold out in $t = 1$, which is accepted by types $\theta \in (\check{\theta}, \gamma(\check{\theta}))$. Since type-$\check{\theta}$ must be indifferent between selling at p in $t = 1$ only and selling at p_2 in $t = 2$ only, we must have

$$p + S + \delta\check{\theta} = \check{\theta} + \delta(p_2 + S). \tag{B.2}$$

In particular, $p_2 + S > \check{\theta}$, so $p + S > \check{\theta}$. This means that, if a buyer deviates by offering $p - \varepsilon > p^-$ for sufficiently small $\varepsilon > 0$ in $t = 2$ to those that accepted p in $t = 1$, then all of them must accept that offer. The buyer makes a strictly positive profit with such a deviation since p is the break-even price. We thus have a contradiction.

Suppose next $\check{\theta} = 1$, meaning that all types sell in $t = 1$. Here we invoke the D1 refinement to derive a contradiction. In the candidate equilibrium, the payoff for type $\theta > \hat{\theta}$ is $p + S + \delta\theta$, and the payoff for all types $\theta < \hat{\theta}$ is some constant u^*, which must equal $p + S + \delta\hat{\theta}$ since type-$\hat{\theta}$ must be indifferent. Let $U^*(\theta)$ be the equilibrium payoff type-θ enjoys when the candidate equilibrium is played: $U^*(\theta) := \max\{u^*, p + S + \delta\theta\}$. Suppose a type θ firm deviates by refusing the bailout in $t = 1$, and suppose a buyer offers p_2' to that deviating firm in $t = 2$. Type-θ's deviation payoff is then $\theta + \delta \max\{p_2' + S, \theta\}$. Thus, when type-$\theta$ deviates by choosing holdout, the set of market's offers in $t = 2$ that dominate the candidate equilibrium for type-θ is

$$D(\text{holdout}, \theta) := \{p_2' | \theta + \delta \max\{p_2' + S, \theta\} \geq \max\{u^*, p + S + \delta\theta\}\}\,.$$

Note that for fixed p_2', the payoff difference $\theta + \delta \max\{p_2' + S, \theta\} - \max\{u^*, p + S + \delta\theta\}$ is strictly increasing in θ, so the set $D(\text{holdout}, \theta)$ is nested in the sense that $D(\text{holdout}, \theta) \subset D(\text{holdout}, \theta')$ for $\theta' > \theta$. In other words, $D(\text{holdout}, 1)$ is maximal, and more importantly, $D(\text{holdout}, \theta)$ is not maximal if $\theta < 1$. Given this, the D1 refinement entails that the belief by the market must be supported on $\theta = 1$ in case of holdout. Thus following the deviation, the market's offer must be $p_2' = 1$. This means that, for the market's offer in the candidate equilibrium to satisfy D1, type $\check{\theta} = 1$ must enjoy the payoff of at least $1 + \delta(1 + S)$ in case of deviation to holdout from the candidate equilibrium. Since type $\check{\theta} = 1$ chooses to sell in $t = 1$ in the candidate equilibrium, we must have

$$p + S + \delta \geq 1 + \delta(1 + S), \tag{B.3}$$

which implies $p + S > 1$. This in turn implies that in $t = 2$, a buyer can deviate by offering a price slightly below p and induce acceptance from all types that accepted p in $t = 1$. Once again, the buyer makes a strictly positive profit with such a deviation, hence a contradiction.

Step 4. All types $\theta < \hat{\theta} = \check{\theta}$ (if is nonempty) are offered a single price in both periods equal to $m(0, \hat{\theta})$. All types $\theta > \hat{\theta}$ are offered price $m(\hat{\theta}, \gamma(\hat{\theta}))$, which is accepted by types $\theta \in (\hat{\theta}, \gamma(\hat{\theta}))$.

Proof. Suppose there are two distinct prices p, p' that are accepted by positive measures of firms. By the zero profit condition, both prices must be breaking even for the types that accept them. But then, no type will accept the lower price, hence a single price is offered to all types $\theta < \hat{\theta}$. The market's break-even condition then pins down the price to $m(0, \hat{\theta})$. The second statement follows from Step 2.

Step 5. If $\hat{\theta} = 1$, then $S \geq 2(1 - \mathrm{E}[\theta])$.

Proof. Applying the D1 argument as in Step 3, type-$\hat{\theta}$'s equilibrium payoff $(1+\delta)(p+S)$ should not be smaller than $1+\delta(1+S)$ where $p = m(0, 1) = \mathrm{E}[\theta]$ by Step 4. From $(1+\delta)(p+S) \geq 1+\delta(1+S)$ follows the stated condition as $\delta \to 1$. *Q.E.D.*

Theorem 2.

(i) There is an equilibrium in which firms with $\theta \leq \theta_1^*$ sell at price $p_1^* := m(0, \theta_1^*)$ in both periods, firms with $\theta \in (\theta_1^*, \theta_2^*)$ sell only in $t = 2$ at price $p_2^* := m(\theta_1^*, \theta_2^*)$, and firms with $\theta > \theta_2^*$ never sell, where θ_1^* and θ_2^* are defined by $\Delta(\theta_1^*; S) = 0$ and $\theta_2^* = \gamma(\theta_1^*)$, respectively. We have $\theta_1^* \leq \theta_0^* \leq \theta_2^*$, hence $p_1^* \leq p_0^* \leq p_2^*$, where the inequalities hold strictly if the cutoff in the one-period model satisfies $\theta_0^* \in (0, 1)$. Given Assumption 1-(i), there is at most one such equilibrium with an interior θ_1^*.

(ii) Given Assumption 1-(ii), the $t = 1$ market in equilibrium is fully active if $S \geq \overline{S}^*$, suffers from partial freeze if $S \in (\underline{S}^*, \overline{S}^*)$, and full freeze if $S < \underline{S}^*$, where $\underline{S}^*$ and $\overline{S}^*$ are defined by $\Delta(0; \underline{S}^*) = 0$ and $\Delta(1; \overline{S}^*) = 0$, respectively, and satisfy $\underline{S}^* > \underline{S}_0$ and $\overline{S}^* > \max\{\overline{S}_0, \underline{S}^*\}$.

(iii) In addition, there is an equilibrium with full market freeze in $t = 1$ for any S.

Proof of Theorem 2. To prove (i), note first that the existence of cutoffs θ_1 and θ_2 was established by Lemma 1. Thus it suffices to show $\theta_1 \leq \theta_0 \leq \theta_2$ with strict inequalities if $\theta_0^* \in (0, 1)$. Consider the $t = 1$ cutoff θ_1. If $\theta_1 < 1$, then it satisfies $\Delta(\theta_1, S) \leq 0$. Thus $\theta_1 \leq 2m(0, \theta_1) - m(\theta_1, \gamma(\theta_1)) + S < m(0, \theta_1) + S$ since $m(0, \theta_1) < m(\theta_1, \gamma(\theta_1))$. Since $\theta_0^* := \sup\{\theta | \theta \leq m(0, \theta) + S\}$, we have $\theta_1 \leq \theta_0^*$ with strict inequality if $\theta_0^* < 1$. If $\theta_1 = 1$, then $\Delta(\theta_1, S) \geq 0$ by the D1 refinement, which in turn implies $\theta_0^* = 1$. Next, the $t = 2$ cutoff $\theta_2 = \gamma(\theta_1)$ satisfies $\theta_2 \leq m(\theta_1, \gamma(\theta_1)) + S$. Since $\theta_1 \geq 0$, we have $\theta_2 = \gamma(\theta_1) \geq \gamma(0) = \theta_0^*$, where the inequality is strict for $\theta_0^* < 1$ and $\theta_1 > 0$.

For (ii), note that, by Assumption 1, $\theta_1 - 2m(0, \theta_1) + m(\theta_1, \gamma(\theta_1))$ is strictly increasing in θ_1 for each S. Since $\theta_1 < 1$ satisfies $\Delta(\theta_1, S) \leq 0$, a unique θ_1 can be found, which is increasing in S. From this and the first claim follows the second claim.

For (iii), consider the candidate equilibrium in which the market in $t = 1$ completely freezes. This means that, in $t = 2$, we have one period equilibrium with cutoff given by θ_0^* and price p_0. In this equilibrium, the payoff for type $\theta \leq \theta_0^*$ is $\theta + \delta(p_0 + S) = \theta + \delta\theta_0^*$ and the payoff for type $\theta > \theta_0^*$ is $(1 + \delta)\theta$. Thus the equilibrium payoff for type θ is $U^*(\theta) = \max\{\theta + \delta\theta_0^*, (1 + \delta)\theta\}$. Suppose a buyer deviates and offers p_1 in $t = 1$. Let p_2 be the market's offer in $t = 2$ to those that accept the deviation offer p_1. Then the payoff to type θ from accepting p_1 is $p_1 + S + \delta \max\{p_2 + S, \theta\}$. As in the proof of Lemma 1, define the set

$$D(\text{sell}, \theta) := \{p_2 | p_1 + S + \delta \max\{p_2 + S, \theta\} \geq \max\{\theta + \delta\theta_0^*, (1 + \delta)\theta\}\}.$$

For fixed p_2, the payoff difference is strictly decreasing in θ, hence $D(\text{sell}, 0)$ is maximal. Then by the D1 refinement, the market's belief must be supported on $\theta = 0$ when a firm accepts a deviation offer p_1 in $t = 1$. Given this belief, no firm will accept p_1 if $p_1 \leq p_0$. If $p_1 > p_0$, then all types $\theta \leq p_1 + S$ accept the deviation offer p_1. Then the buyer will lose money given $p_1 > p_0$. *Q.E.D.*

C Proofs for Section 4

In Section C.1, we establish necessary conditions for various equilibria. Section C.2 presents the main characterization of equilibria. To appreciate the main points, readers can skip Section C.1 and jump directly to Section C.2.

C.1 Necessary Conditions for Equilibria

We first characterize the various types of equilibria and derive necessary conditions for the existence of each type of equilibria.

C.1.1 Equilibrium Cutoff Structure

Lemma 2. *In any equilibrium, there are four possible cutoffs $0 \leq \theta_g \leq \theta_1 \leq \theta_{g\varnothing} \leq \theta_2 \leq 1$ such that types $\theta \in \Theta_g := [0, \theta_g)$ sell to the government in $t = 1$ and to the market in $t = 2$, types $\theta \in \Theta_1 := (\theta_g, \theta_1)$ sell to the market in both periods, types $\theta \in \Theta_{g\varnothing} := (\theta_1, \theta_{g\varnothing})$ sell only in $t = 1$ to the government, types $\theta \in \Theta_2 := (\theta_{g\varnothing}, \theta_2)$ sell only in $t = 2$ to the market, and types $\theta > \theta_2$ sell in neither period.*

Proof of Lemma 2. Similar to the proof of Lemma 1, fix a game with discount factor $\delta \in (0, 1)$ and the probability of market collapse $\varepsilon \in (0, 1)$. Also, fix any equilibrium of the required form. Let $q_g(\theta)$ be the unit of the asset a type-θ firm sells to the government, $(q_1(\theta), q_2(\theta))$ be the units of the asset the firm sells in each of the two periods, and $(t_g(\theta), t_1(\theta), t_2(\theta))$ be the corresponding transfers. The expected payoff for a type-θ firm when playing as if it is type-θ' is

$$\begin{aligned} u(\theta'; \theta) = {} & q_g(\theta')\big[[S + t_g(\theta')] + \varepsilon\delta\theta\big] + (1 - q_g(\theta'))\varepsilon(1 + \delta)\theta \\ & + (1 - \varepsilon)\{1 - q_g(\theta')\}\big[q_1(\theta')\{S + t_1(\theta)\} + \{1 - q_1(\theta')\}\theta\big] \\ & + (1 - \varepsilon)\delta\big[q_2(\theta')\{S + t_2(\theta')\} + \{1 - q_2(\theta')\}\theta\big]. \end{aligned}$$

Let $Q(\cdot) := q_g(\cdot) + (1-\varepsilon)\{1-q_g(\cdot)\}q_1(\cdot) + (1-\varepsilon)\delta q_2(\cdot)$ and $T(\cdot) := q_g(\cdot)t_g(\cdot) + (1-\varepsilon)\{1-q_g(\cdot)\}q_1(\cdot)t_1(\cdot) + (1-\varepsilon)\delta q_2(\cdot)t_2(\cdot)$. Since $u(\theta; \theta) - u(\theta'; \theta) \geq 0$ and $u(\theta'; \theta') - u(\theta; \theta') \geq 0$ for every

$\theta \neq \theta'$ in equilibrium, it follows that $(S-\theta)[Q(\theta)-Q(\theta')] \geq T(\theta')-T(\theta)$ and $(S-\theta')[Q(\theta')-Q(\theta)] \geq T(\theta)-T(\theta')$. Combining these inequalities, we have $(\theta'-\theta)[Q(\theta)-Q(\theta')] \geq 0$, which implies that $Q(\theta)$ is decreasing in θ. Since $1 > (1-\varepsilon) > (1-\varepsilon)\delta$ and $q_j(\theta) \in \{0,1\}$ for every $j = g, 1, 2$ in pure-strategy equilibrium, there exist cutoffs $0 \leq \theta_g \leq \theta_1 \leq \theta_{g\o} \leq \theta_{1\o} \leq \theta_2 \leq 1$ such that: *(i)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (1,0,1)$ if $\theta \in [0, \theta_g]$; *(ii)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (0,1,1)$ if $\theta \in (\theta_g, \theta_1]$; *(iii)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (1,0,0)$ if $\theta \in (\theta_1, \theta_{g\o}]$; *(iv)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (0,1,0)$ if $\theta \in (\theta_{g\o}, \theta_{1\o}]$; *(v)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (0,0,1)$ if $\theta \in (\theta_{1\o}, \theta_2]$; *(vi)* $(q_g(\theta), q_1(\theta), q_2(\theta)) = (0,0,0)$ if $\theta > \theta_2$. Applying the same logic used for the proof of Step 3 in the proof of Lemma 1, it can be shown that $\theta_{g\o} = \theta_{1\o}$. Thus any equilibrium must be characterized by the cutoff structure $0 \leq \theta_g \leq \theta_1 \leq \theta_{g\o} \leq \theta_2 \leq 1$. *Q.E.D.*

In what follows, we describe the incentive compatibility constraints for firms in each type of equilibria. Since the equilibria are characterized by a set of cutoff types by Lemma 2, we derive conditions characterizing these cutoffs in each type of equilibria. Lastly, we find bailout terms compatible with each type of equilibria.

C.1.2 Necessary Conditions for SBS Equilibrium

In this equilibrium, there exists $\theta_{g\o} \in (0, \theta_2)$ and $\theta_2 \leq 1$ such that: types $\theta \in [0, \theta_{g\o}]$ sell to the government at price p_g in $t=1$ but cannot sell in $t=2$ due to $m(0, \theta_{g\o}) < I$; types $\theta \in (\theta_{g\o}, \theta_2]$ sell only in $t=2$ at price $m(\theta_{g\o}, \theta_2)$; the market fully freezes in $t=1$ so no trade occurs.

We derive necessary conditions for each type to play the above equilibrium strategies. The payoffs from playing the equilibrium strategies across the two periods are $p_g + S + \theta$ for all $\theta \in [0, \theta_{g\o}]$, and $\theta + m(\theta_{g\o}, \theta_2) + S$ for all $\theta \in (\theta_{g\o}, \theta_2]$. Since type $\theta_{g\o}$ is indifferent between these choices, we must have $m(\theta_{g\o}, \theta_2) = p_g$. Meanwhile, type θ_2 must be the highest type that will be induced to sell in $t=2$. It must follow that $\theta_2 = \gamma(\theta_{g\o})$, where γ is defined in Definition A.1-(ii). Thus we have the following conditions that determine the marginal types $\theta_{g\o}$ and θ_2:

$$\theta_2 = \gamma(\theta_{g\o}); \tag{C.1}$$

$$p_g = m(\theta_{g\o}, \gamma(\theta_{g\o})). \tag{C.2}$$

By Lemma A.1, $\theta_{g\o}$ and θ_2 are uniquely determined by p_g. Moreover, since both $\gamma(\theta)$ and $m(\theta, \gamma(\theta))$ are differentiable and increasing in θ, we have $\frac{d}{dp_g}\theta_{g\o} > 0$. The case in which all firms sell to the government in $t=1$ (i.e., $\theta_{g\o} = 1$) can be supported if and only if $p_g \geq m(1, \gamma(1)) = 1$. The reason is the following. Applying the same argument used to prove Lemma 1, one can easily see that the worst off-the-path belief consistent with D1 for a deviator (one who holds out) is

$\theta_{g\o} = 1$. Given the belief, no firm deviates if and only if $p_g \geq m(1, \gamma(1)) = 1$. Since we restrict $p_g \leq 1$, such a case ($\theta_{g\o} = 1$) can be observed only if $p_g = 1$.

In addition, the SBS equilibrium requires that types $\theta \in [0, \theta_{g\o}]$ cannot sell in $t = 2$, hence

$$m(0, \theta_{g\o}) < I. \tag{C.3}$$

Furthermore, either of the following constraints on $\theta_{g\o}$ must hold:

$$\theta_{g\o} < I, \tag{C.4}$$

$$\theta_{g\o} - m(\theta_{g\o}, \gamma(\theta_{g\o})) + S \leq 0. \tag{C.5}$$

To see why, suppose the $t = 1$ market opens after the bailout and a buyer deviates and offers $p'_1 \geq I$. Since the belief in $t = 1$ is that only the types $\theta > \theta_{g\o}$ are available for asset sales, the firms accepting p'_1 are assigned the off-the-path belief as being the worst of the available types (i.e., $\theta = \theta_{g\o}$).

First, suppose $\theta_{g\o} < I$. Given the off-the-path belief, firms that deviate and sell at price p'_1 cannot sell in $t = 2$, hence the total payoff from the deviation equals $p'_1 + S + \theta$. Clearly $p'_1 > m(\theta_{g\o}, \gamma(\theta_{g\o}))$ for, otherwise, the deviating firms will not sell at p'_1. But if $p'_1 > m(\theta_{g\o}, \gamma(\theta_{g\o}))$, then all types $\theta \in (\theta_{g\o}, (p'_1 + S) \wedge 1]$ will sell at p'_1, and $m(\theta_{g\o}, (p'_1 + S) \wedge 1) - p'_1 > 0$ from the definition of $\gamma(\cdot)$. Thus no buyers in $t = 1$ deviate if $\theta_{g\o} < I$.

Second, suppose $\theta_{g\o} \geq I$. Given the off-the-path belief above, firms selling at p'_1 can sell at price $\theta_{g\o}$ in $t = 2$. Thus the total payoff to the deviating firm is $p'_1 + S + \max\{\theta, \theta_{g\o} + S\}$. In the above, we showed that $p'_1 > m(\theta_{g\o}, \gamma(\theta_{g\o}))$ is not possible. If $p'_1 \leq m(\theta_{g\o}, \gamma(\theta_{g\o}))$, then types $\theta > \theta_{g\o} + S$ do not sell at p'_1 since $p'_1 + S + \max\{\theta, \theta_{g\o} + S\} = p'_1 + S + \theta < \theta + \max\{m(\theta_{g\o}, \gamma(\theta_{g\o})) + S, \theta\}$. On the contrary, types $\theta \in (\theta_{g\o}, \theta_{g\o} + S]$ sell at p'_1 if and only if $p'_1 + \theta_{g\o} + 2S \geq \theta + m(\theta_{g\o}, \gamma(\theta_{g\o})) + S$, or equivalently $p'_1 \geq \theta + m(\theta_{g\o}, \gamma(\theta_{g\o})) - \theta_{g\o} - S$. Let $\theta'_1 := p'_1 + (\theta_{g\o} - m(\theta_{g\o}, \gamma(\theta_{g\o})) + S)$. Then types $\theta \in (\theta_{g\o}, \theta'_1]$ sell to the deviating buyer, so the deviating buyer gets the expected payoff $m(\theta_{g\o}, \theta'_1) - p'_1$. Since $\frac{d\theta'_1}{dp'_1} = 1$ and hence $\frac{d}{dp'_1}(m(\theta_{g\o}, \theta'_1) - p'_1) < 0$, we have $m(\theta_{g\o}, \theta'_1) - p'_1 < 0$ for any $p'_1 \leq m(\theta_{g\o}, \gamma(\theta_{g\o}))$ if and only if $m(\theta_{g\o}, \theta'_1) - p'_1 \leq 0$ at $p'_1 = m(\theta_{g\o}, \gamma(\theta_{g\o})) - S$. Since $m(\theta_{g\o}, \theta'_1) - p'_1 = \theta_{g\o} - m(\theta_{g\o}, \gamma(\theta_{g\o})) + S$, we must have $\theta_{g\o} - m(\theta_{g\o}, \gamma(\theta_{g\o})) + S \leq 0$.

Let P^{SBS} denote the range of p_gs for which SBS equilibrium exists.

Lemma C.1. *If P^{SBS} in non-empty, it is a convex set such that $p_g > p^*_0$ for all $p_g \in P^{SBS}$.*

Proof. Since $m(\theta, \gamma(\theta))$ is increasing in θ, $\theta^{SBS}_{g\o}(p_g)$ is increasing in p_g if $\theta^{SBS}_{g\o}$ is well-defined.

Hence there exists $\tilde{p}_g^{SBS}$ such that $\theta_{g\o}^{SBS}(p_g)$ satisfies (C.3) if and only if $p_g < \tilde{p}_g^{SBS}$. Furthermore, since $p_0^* = m(0, \theta_0^*) = m(0, \gamma(0))$, we have $p_g > p_0^*$ for $\theta_{g\o}^{SBS}(p_g) > 0$. Lastly, since $\theta - m(\theta, \gamma(\theta)) + S$ is increasing in θ, there exists $\hat{p}_g^{SBS}$ such that: *(i)* if (C.4) binds but (C.5) does not, then $\theta_{g\o}^{SBS}(p_g)$ satisfies (C.4) if and only if $p_g < \hat{p}_g^{SBS}$; *(ii)* if (C.5) binds but (C.4) does not, then $\theta_{g\o}^{SBS}(p_g)$ satisfies (C.5) if and only if $p_g \leq \hat{p}_g^{SBS}$. Therefore, $P^{SBS} = (p_0^*, \tilde{p}_g^{SBS})$ if (C.3) binds, $P^{SBS} = (p_0^*, \hat{p}_g^{SBS}]$ if (C.4) binds, and $P^{SBS} = (p_0^*, \hat{p}_g^{SBS})$ if (C.5) binds. *Q.E.D.*

For every $p_g \in P^{SBS}$, we let $\theta_{g\o}^{SBS}(p_g)$ denote the marginal type determined by (C.2). For expositional convenience, we may occasionally abbreviate $\theta_{g\o}^{SBS}(p_g)$ to $\theta_{g\o}^{SBS}$.

C.1.3 Necessary Conditions for MBS Equilibrium

In this equilibrium, there exist $\theta_g \in (0, \theta_2)$ and $\theta_2 \leq 1$ such that: types $\theta \in [0, \theta_g]$ sell to the government at price p_g in $t = 1$ and to the market at price $m(0, \theta_g)$ in $t = 2$; types $\theta \in (\theta_g, \theta_2]$ sell only in $t = 2$ at price $m(\theta_g, \theta_2)$; no asset trade occurs in the $t = 1$ market.

In equilibrium, the total payoffs for the firms are $p_g + m(0, \theta_g) + 2S$ for all $\theta \in [0, \theta_g]$, $\theta + m(\theta_g, \theta_2) + S$ for all $\theta \in (\theta_g, \theta_2]$, and 2θ for all $\theta > \theta_2$. For expositional ease, we treat the case in which θ_2 is interior (so it is characterized by an indifference condition). As one can see clearly, our characterization also works for the boundary case $\theta_2 = 1$. From type θ_g's indifference condition, we have $p_g + m(0, \theta_g) + S - \theta_g - m(\theta_g, \theta_2) = 0$. Similarly, type θ_2's indifference condition leads to $\theta_2 = m(\theta_g, \theta_2) + S = \gamma(\theta_g)$. Thus we have the following conditions that determine θ_g and θ_2:

$$\theta_2 = \gamma(\theta_g), \tag{C.6}$$

$$p_g = \theta_g - m(0, \theta_g) - S + m(\theta_g, \gamma(\theta_g)). \tag{C.7}$$

Since $\theta - m(0, \theta) + m(\theta, \gamma(\theta))$ is continuous and increasing in θ, the marginal type θ_g is uniquely determined by and increasing in p_g. If p_g is very large in that $p_g \geq 1 - m(0, 1) - S + m(1, \gamma(1)) = 2 - \mathrm{E}[\theta] - S$, then we assign $\theta_g = 1$. Such an assignment is supported by the off-the-path belief at the $t = 2$ market that the holdouts in $t = 1$ are perceived as the highest type $\theta = 1$. Once again, one can check this is the only belief that satisfies the D1 refinement.

In addition, the bailout recipients also sell in $t = 2$. Thus, we have

$$m(0, \theta_g) \geq I. \tag{C.8}$$

Furthermore, the $t = 1$ market freezes completely. From the analysis of SBS equilibrium (recall

(C.4) and (C.5)), we showed that the $t = 1$ market freezes fully if and only if either $\theta_g < I$ or $\theta_g - m(\theta_g, \gamma(\theta_g)) + S \leq 0$. Since $\theta_g > m(0, \theta_g) \geq I$ from (C.8), the latter condition must hold:

$$\theta_g - m(\theta_g, \gamma(\theta_g)) + S \leq 0. \tag{C.9}$$

Lastly, all types $\theta \in [0, \theta_g]$ sell assets at price $m(0, \theta_g)$ in $t = 2$. Thus we must have $\theta_g \leq m(0, \theta_g) + S$, or equivalently

$$\theta_g \leq \theta_0^*. \tag{C.10}$$

Lemma C.2. *There exist $\underline{p}_g^{MBS} \leq \overline{p}_g^{MBS}$ such that (C.7) admits a unique θ_g that satisfies (C.8) – (C.10) if and only if $p_g \in [\underline{p}_g^{MBS}, \overline{p}_g^{MBS}]$.*

Proof. Since $\theta - m(0, \theta) + m(\theta, \gamma(\theta)) - S$ is increasing in θ, $\theta_g^{MBS}(p_g)$ is increasing in p_g if well-defined. Moreover, since $m(0, \theta)$ is increasing in θ, there exists $\underline{p}_g^{MBS}$ such that $\theta_g^{MBS}(p_g)$ is well-defined and satisfies (C.8) if and only if $p_g \geq \underline{p}_g^{MBS}$. Furthermore, since $\theta - m(0, \theta) + S$ is increasing in θ, there exists $\overline{p}_g^{MBS}$ such that $\theta_g^{MBS}(p_g)$ (if well-defined) satisfies (C.9) and (C.10) if and only if $p_g \leq \overline{p}_g^{MBS}$. Putting all these results together, we have $P^{MBS} = [\underline{p}_g^{MBS}, \overline{p}_g^{MBS}]$. *Q.E.D.*

For every $p_g \in [\underline{p}_g^{MBS}, \overline{p}_g^{MBS}]$, we let $\theta_g^{MBS}(p_g)$ denote the marginal type θ_g determined by (C.7). For expositional convenience, we may abbreviate $\theta_g^{MBS}(p_g)$ to θ_g^{MBS}. We also let $P^{MBS} := [\underline{p}_g^{MBS}, \overline{p}_g^{MBS}]$.

C.1.4 Necessary Conditions for MR Equilibrium

In this equilibrium, a positive measure of firms sell to the market in $t = 1$ (in addition to a positive measure of firms selling to the government). Bailout rejuvenates the $t = 1$ market. There are two possible types of MR equilibria: **MR1** in which $\Theta_g, \Theta_1, \Theta_2 \neq \emptyset$, but $\Theta_{g\emptyset} = \emptyset$, and **MR2** in which $\Theta_g, \Theta_1, \Theta_{g\emptyset}, \Theta_2 \neq \emptyset$.

MR1 equilibrium: In this equilibrium, there exist $0 < \theta_g < \theta_1 \leq \theta_2 \leq 1$ such that: types $\theta \in [0, \theta_g]$ sell to the government in $t = 1$, and to the market in $t = 2$ at price $m(0, \theta_g)$; types $\theta \in (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods; types $\theta \in (\theta_1, \theta_2]$ sell to the market at price $m(\theta_1, \theta_2)$ only in $t = 2$; types $\theta > \theta_2$ do not sell in either period.

In equilibrium, firms' total payoffs are $p_g + m(0, \theta_g) + 2S$ for all $\theta \in [0, \theta_g]$, $2m(\theta_g, \theta_1) + 2S$ for all $\theta \in (\theta_g, \theta_1]$, $\theta + m(\theta_1, \theta_2) + S$ for all $\theta \in (\theta_1, \theta_2]$, and 2θ for all $\theta > \theta_2$. From these payoffs, it is straightforward to see the three marginal types must satisfy relevant incentive constraints (i.e., indifference in the case of an interior solution):

$$p_g = 2m(\theta_g, \theta_1) - m(0, \theta_g); \tag{C.11}$$

$$\theta_1 = \max\{\theta \in [\theta_g, 1] \,|\, m(\theta_g, \theta) + S - \theta - (m(\theta, \gamma(\theta)) - m(\theta_g, \theta)) \geq 0\}; \tag{C.12}$$

$$\theta_2 = \gamma(\theta_1). \tag{C.13}$$

(C.11) is the indifference condition for type-θ_g firm, (C.12) that for type-θ_1 firm, and (C.13) that that for type-θ_2 firm. Assumption A.2-(i) ensures that θ_1 is uniquely determined by θ_g. As before (C.12) allows for the possibility that $\theta_1 = 1$. The D1 refinement suggests that if $\theta_1 = 1$, the worst off-the-path belief for a deviating holdout firm is $\theta_1 = 1$, so (C.12) ensures that given that belief, no firm wishes to deviate.

There are additional necessary conditions for the MR1 equilibrium. First, types $\theta \in [0, \theta_g]$ should be able to finance their projects in $t = 2$ (an implication of Lemma 2):

$$m(0, \theta_g) \geq I. \tag{C.14}$$

Second, types $\theta \in (\theta_1, \gamma(\theta_1)]$ must prefer selling only in $t = 2$ to either selling to the market or selling to the government in $t = 1$, which requires $m(\theta_1, \gamma(\theta_1)) \geq m(\theta_g, \theta_1)$ and $m(\theta_1, \gamma(\theta_1)) \geq p_g$. Since the first inequality holds trivially, we only state the second condition:

$$m(\theta_1, \gamma(\theta_1)) \geq p_g. \tag{C.15}$$

The conditions (C.11) – (C.15) will be used later when characterizing the set of bailout terms that support the MR1 equilibrium. One can also see that the same conditions are also sufficient for MR1 to arise.

Lemma C.3.

(i) There exists $\check{p}_g^{MR1}$ such that (C.11) and (C.12) admit a unique (θ_g, θ_1) that satisfies $0 < \theta_g < \theta_1 \leq 1$ if and only if $p_g > \check{p}_g^{MR1}$.

(ii) There exists $\underline{p}_g^{MR1} \geq \check{p}_g^{MR1}$ such that θ_g determined by (C.11) and (C.12) satisfies (C.14) if and only if $p_g \geq \underline{p}_g^{MR1}$.

Proof. We first prove part (i). Given Assumption A.2-(iii), (C.11) defines θ_1 as a decreasing function of θ_g, labelled $\tilde{\theta}_1(\theta_g)$, whenever well-defined. Furthermore, given Assumption A.2-(i),

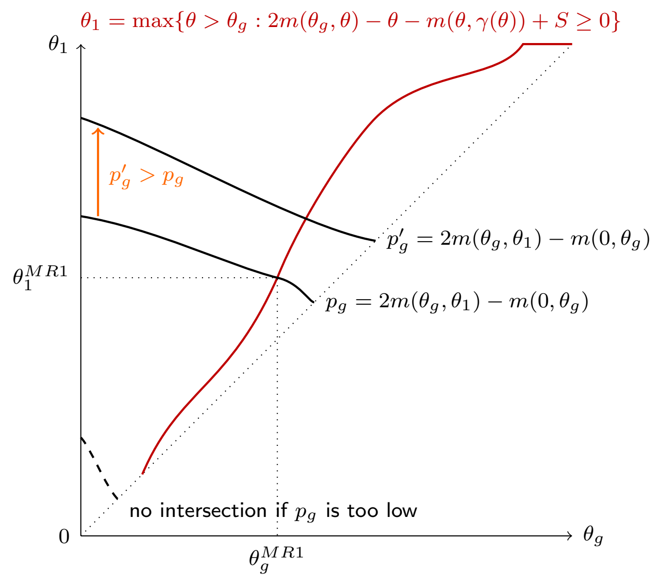


Figure 1 – Characterization of θ_g^{MR1} and θ_1^{MR1}

(C.12) defines θ_1 as an increasing function of θ_g, labelled $\theta_1^I(\theta_g)$, whenever well-defined. In fact, there exists $\underline{\theta}_g < 1$ such that $\theta_1^I(\theta_g)$ is well-defined if and only if $\theta_g > \underline{\theta}_g$.[1] Therefore, (θ_g, θ_1) satisfying (C.11) and (C.12), if well-defined, is characterized by a unique point of intersection between $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$. Moreover, it can be shown that $\tilde{\theta}_1(\theta_g)$ shifts up as p_g increases, which is also illustrated in Figure 1. Lastly, one can find that there exists a $p_g \leq 1$ such that $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$ intersect.[2] Putting all results together, there exists $\check{p}_g^{MR1}$ such that two curves $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$ intersect at a unique point if and only if $p_g > \check{p}_g^{MR1}$.

We next prove part (ii). Since $\tilde{\theta}_1(\theta_g)$ shifts up as p_g increases, θ_g determined by (C.11) and (C.12), is increasing in p_g for all $p_g > \underline{p}_g^{MR1}$. Therefore, there exists $\underline{p}_g^{MR1} \geq \check{p}_g^{MR1}$ such that θ_g determined by (C.11) and (C.12) satisfies $m(0, \theta_g) \geq I$ if and only if $p_g \geq \underline{p}_g^{MR1}$. *Q.E.D.*

For any $p_g > \check{p}_g^{MR1}$, let $\theta_g^{MR1}(p_g)$ and $\theta_1^{MR1}(p_g)$ denote the marginal types θ_g and θ_1 determined by (C.11) and (C.12). For expositional convenience, we may abbreviate $\theta_g^{MR1}(p_g)$ and $\theta_1^{MR1}(p_g)$ to θ_g^{MR1} and θ_1^{MR1}, respectively.

[1] From Assumption A.2-(i), $\theta_1^I(\theta_g)$ is well-defined if and only if $\theta_g - m(\theta_g, \gamma(\theta_g)) + S > 0$. Since $\theta - m(\theta, \gamma(\theta))$ is increasing in θ and $1 - m(1, \gamma(1)) + S = S > 0$, there exists $\underline{\theta}_g < 1$ such that $\theta_g - m(\theta_g, \gamma(\theta_g)) + S > 0$ if and only if $\theta_g > \underline{\theta}_g$.

[2] To show this, we prove that there exists p_g at which $\tilde{\theta}_1(\underline{\theta}_g) = \theta_1^I(\underline{\theta}_g)$. From the definitions of $\tilde{\theta}_1(\cdot)$ and $\theta_1^I(\cdot)$, $\tilde{\theta}_1(\underline{\theta}_g) = \theta_1^I(\underline{\theta}_g)$ is equivalent to

$$p_g = (\underline{\theta}_g - m(\underline{\theta}_g, \gamma(\underline{\theta}_g)) - S) + m(\underline{\theta}_g, \gamma(\underline{\theta}_g)). \tag{C.16}$$

Since $\underline{\theta}_g < \theta_0^*$ from Assumption A.2-(iv) and $\underline{\theta}_g < 1$, there exists $p_g < 1$ that solves (C.16).

MR2 equilibrium: In this equilibrium, there exist $0 < \theta_g < \theta_1 < \theta_{g\o} \leq \theta_2 \leq 1$ such that: types $\theta \in [0, \theta_g]$ sell to the government in $t = 1$ and to the market in $t = 2$ at price $m(0, \theta_g)$; types $\theta \in (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods; types $\theta \in (\theta_1, \theta_{g\o}]$ sell to the government in $t = 1$ but do not sell in $t = 2$; types $\theta \in (\theta_{g\o}, \theta_2]$ sell at price $m(\theta_{g\o}, \theta_2)$ only in $t = 2$; types $\theta > \theta_2$ do not sell in either period.

As before, $(\theta_g, \theta_1, \theta_{g\o}, \theta_2)$ must satisfy the following conditions:

$$p_g = 2m(\theta_g, \theta_1) - m(0, \theta_g); \tag{C.11}$$

$$\theta_1 = m(0, \theta_g) + S; \tag{C.17}$$

$$p_g = m(\theta_{g\o}, \gamma(\theta_{g\o})); \tag{C.18}$$

$$\theta_2 = \gamma(\theta_{g\o}). \tag{C.19}$$

Similar to the boundary case of the SBS equilibrium, the case in which all firms sell in $t = 1$ (i.e., $\theta_{g\o} = 1$) can be supported if and only if $p_g \geq m(1, \gamma(1)) = 1$. Indeed, the worst off-the-path belief consistent with D1 for a deviator (one who holds out) is $\theta_{g\o} = 1$. Given this belief, no firm deviates if and only if $p_g \geq 1$. Since we restrict $p_g \leq 1$, such a case ($\theta_{g\o} = 1$) can be observed only for $p_g = 1$.

As before, the marginal type θ_g must also satisfy (C.14). Note that (C.14) implies

$$m(\theta_{g\o}, \gamma(\theta_{g\o})) > m(\theta_g, \theta_1) > I.$$

Second, since $\Theta_{g\o} \neq \emptyset$, we must have

$$\theta_1 < \theta_{g\o}. \tag{C.20}$$

Lemma C.4. *There exist* $\underline{p}_g^{MR2}$ *and* $\overline{p}_g^{MR2}$ *such that* (C.11) *and* (C.17) *admit a unique* (θ_g, θ_1) *that satisfies* $0 < \theta_g < \theta_1$ *and* (C.14) *if and only if* $p_g \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$.

Proof. Similar to the proof of Lemma C.3, (C.11) defines θ_1 as a decreasing function of θ_g, labelled $\tilde{\theta}_1(\theta_g)$, whenever well-defined. Moreover, (C.17) defines θ_1 as an increasing function of θ_g, labelled $\theta_1^{II}(\theta_g)$, whenever well-defined. Therefore, (θ_g, θ_1) satisfying (C.11) and (C.17), if well-defined, is characterized by a unique point of intersection between $\tilde{\theta}_1(\theta_g)$ and $\theta_1^{II}(\theta_g)$. Moreover, one can

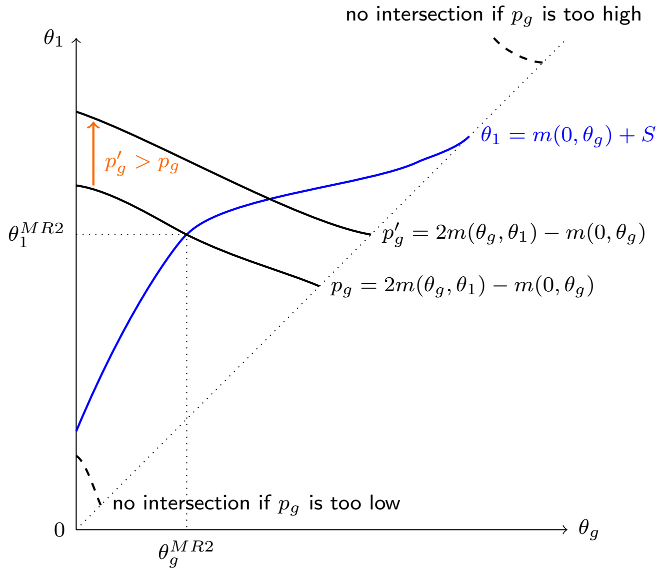


Figure 2 – Characterization of θ_g^{MR2} and θ_1^{MR2}

find that there exists $p_g \leq 1$ at which $\tilde{\theta}_1(\theta_g)$ and $\theta_1^{II}(\theta_g)$ intersect.[3] Since $\tilde{\theta}_1(\theta_g)$ shifts up as p_g increases, there exists $\check{p}_g^{MR2}$ such that (C.11) and (C.17) admit a unique (θ_g, θ_1) satisfying $\theta_g > 0$ if and only if $p_g > \check{p}_g^{MR2}$. Furthermore, as depicted in Figure 2, θ_g determined by (C.11) and (C.17) is increasing in p_g for all $p_g > \check{p}_g^{MR2}$. Since $\theta_1^{II}(\theta_g) \leq \theta_g$ for all $\theta_g \geq \theta_0^*$, there exists $\overline{p}_g^{MR2} > \check{p}_g^{MR2}$ such that (θ_g, θ_1) determined by (C.11) and (C.17) satisfies $0 < \theta_g < \theta_1$ if and only if $p_g \in (\check{p}_g^{MR2}, \overline{p}_g^{MR2})$. Lastly, there exists $\underline{p}_g^{MR2} > \check{p}_g^{MR2}$ such that θ_g determined by (C.11) and (C.17) satisfies (C.14) if and only if $p_g \geq \underline{p}_g^{MR2}$. *Q.E.D.*

For any $p_g > \check{p}_g^{MR2}$, let $\theta_g^{MR2}(p_g)$, $\theta_1^{MR2}(p_g)$, and $\theta_{g\o}^{MR2}(p_g)$ denote the marginal types θ_g, θ_1, and $\theta_{g\o}$ determined by (C.11), (C.17), and (C.18). For expositional convenience, we may abbreviate $\theta_g^{MR1}(p_g)$, $\theta_1^{MR1}(p_g)$, and $\theta_{g\o}^{MR2}(p_g)$ to θ_g^{MR1}, θ_1^{MR1}, and $\theta_{g\o}^{MR2}$, respectively.

Lemma C.5. *Fix p_g. Suppose both $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ and $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ are well-defined at that p_g. If the former violates (C.15), then the latter satisfies (C.20). Conversely, if the former satisfies (C.20), then the latter violates (C.15).*

Proof. We first establish some technical results used in the proof.

[3] We first show that if MR2 equilibrium exists, then it requires $S < 1$. Suppose to the contrary MR2 equilibrium exists for some $S \geq 1$. Then $\theta_1^{II}(\theta_g) = 1$ for all $\theta_g \geq 0$, so any (θ_g, θ_1) satisfying (C.17) violates (C.20), a contradiction. We next show that there exists p_g at which $\tilde{\theta}_1(0) = \theta_1^{II}(0)$. From the definitions of $\tilde{\theta}_1(\cdot)$ and $\theta_1^{II}(\cdot)$, $\tilde{\theta}_1(0) = \theta_1^{II}(0)$ is equivalent to $p_g = 2m(0, S)$. Since $1 > S > 2m(0, S)$ from Assumption A.2-(v), $\tilde{\theta}_1(0) = \theta_1^{II}(0)$ at $p_g = 2m(0, S) < 1$. Thus, there is a unique point of intersection between $\tilde{\theta}_1(\theta_g)$ and $\theta_1^{II}(\theta_g)$ for any $p_g \in (2m(0, S), 1)$, as is also seen in Figure 2.

Claim 1. *Suppose θ_g and θ_1 satisfy (C.11) and (C.12). Then θ_g and θ_1 satisfy (C.15) if and only if $\theta_1 \leq m(0, \theta_g) + S$.*

Proof. Suppose there exist θ_g and θ_1 determined by (C.11) and (C.12) that satisfy (C.15). Then we have

$$\begin{aligned}\theta_1 &\leq 2m(\theta_g, \theta_1) - m(\theta_1, \gamma(\theta_1)) + S \\ &= p_g + m(0, \theta_g) - m(\theta_1, \gamma(\theta_1)) + S \\ &= m(0, \theta_g) + S + (p_g - m(\theta_1, \gamma(\theta_1))) \\ &\leq m(0, \theta_g) + S,\end{aligned}$$

where the first inequality follows from (C.12), the second equality follows from (C.11), and the last inequality follows from (C.15). Conversely, one can show that if θ_g and θ_1 satisfy (C.11) and (C.12), then $\theta_1 \leq m(0, \theta_g) + S$ is sufficient for them to satisfy (C.15). *Q.E.D.*

Claim 2. *Suppose θ_g, θ_1, and $\theta_{g\o}$ satisfy (C.11), (C.17), and (C.18). Then θ_1 and $\theta_{g\o}$ satisfy (C.20) if and only if $2m(\theta_g, \theta_1) - \theta_1 - m(\theta_1, \gamma(\theta_1)) + S > 0$.*

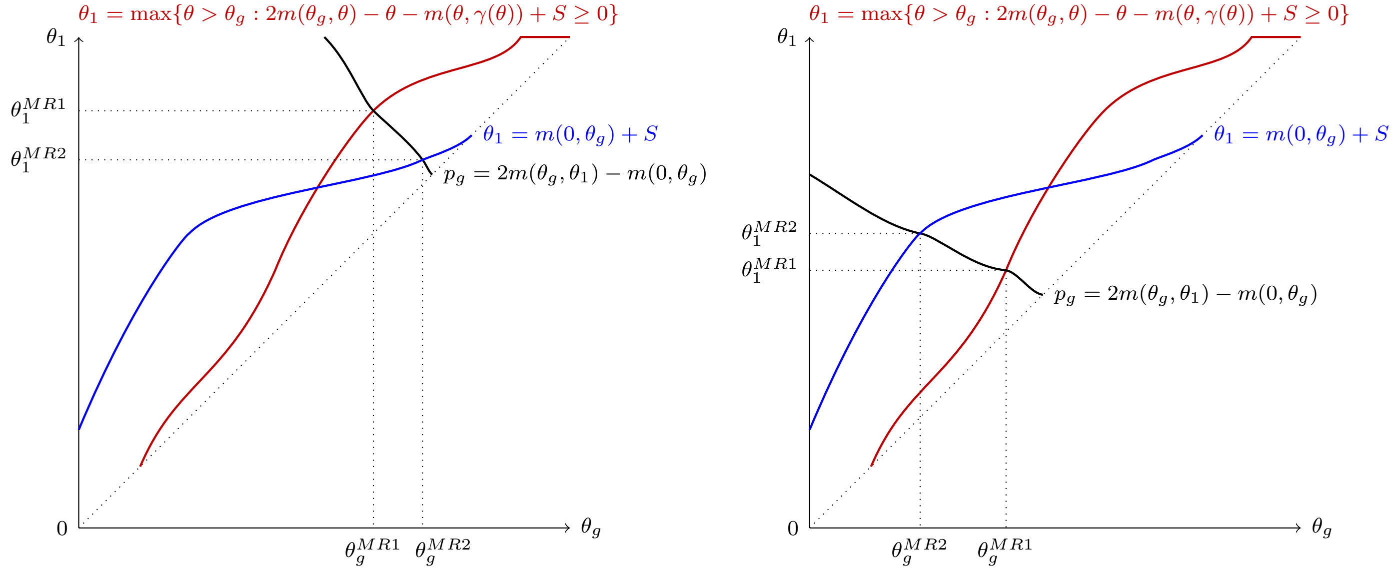
Proof. Suppose there exist θ_g, θ_1, and $\theta_{g\o}$ that satisfy (C.11) – (C.18) and also (C.20). Then we have

$$\begin{aligned}0 &= m(0, \theta_g) - \theta_1 + S \\ &= 2m(\theta_g, \theta_1) - \theta_1 - p_g + S \\ &= 2m(\theta_g, \theta_1) - \theta_1 - m(\theta_{g\o}, \gamma(\theta_{g\o})) + S \\ &< 2m(\theta_g, \theta_1) - \theta_1 - m(\theta_1, \gamma(\theta_1)) + S,\end{aligned}$$

where the first equality follows from (C.17), the second equality follows from (C.11), the third equality follows from (C.18), and the last inequality follows from (C.20). Conversely, one can also show that if θ_g, θ_1, and $\theta_{g\o}$ satisfy (C.11) – (C.18), then $2m(\theta_g, \theta_1) - \theta_1 - m(\theta_1, \gamma(\theta_1)) + S > 0$ is sufficient for them to satisfy (C.20). *Q.E.D.*

Fix a p_g at which both $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ and $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ are well-defined. Recall $\tilde{\theta}_1(\theta_g)$, $\theta_1^I(\theta_g)$, and $\theta_1^{II}(\theta_g)$ from the proofs of Lemma C.3 and Lemma C.4, which are the functions of θ_g corresponding to (C.11), (C.12), and (C.17), respectively.

If $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ violates (C.15), then we have $\theta_1^{MR1}(p_g) > m(0, \theta_g^{MR1}(p_g)) + S$ from Claim 1. This implies $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) > \theta_1^{II}(\theta_g^{MR1}(p_g))$. Furthermore, we have shown in Lemma C.4



(a) $\tilde{\theta}_1(\theta_g^{MR1}) > \theta_1^{II}(\theta_g^{MR1})$, or equivalently, $\tilde{\theta}_1(\theta_g^{MR2}) < \theta_1^{I}(\theta_g^{MR2})$

(b) $\tilde{\theta}_1(\theta_g^{MR1}) \leq \theta_1^{II}(\theta_g^{MR1})$, or equivalently, $\tilde{\theta}_1(\theta_g^{MR2}) \geq \theta_1^{I}(\theta_g^{MR2})$

Figure 3 – $p_g \in P^{MR}$ supports only one type of MR equilibria.

that $\tilde{\theta}_1(\theta_g)$ is decreasing in θ_g and $\theta_1^{II}(\theta_g)$ is increasing in θ_g if they are well-defined. Thus we have $\theta_g^{MR1}(p_g) < \theta_g^{MR2}(p_g)$ and $\theta_1^{MR1}(p_g) > \theta_1^{MR2}(p_g)$ as illustrated by Figure 3a. Moreover, we have shown in the proof of Lemma C.3 that $\theta_1^{I}(\theta_g)$ is increasing in θ_g if it is well-defined. Hence, we have $\theta_1^{I}(\theta_g^{MR2}(p_g)) > \theta_1^{I}(\theta_g^{MR1}(p_g)) = \theta_g^{MR1}(p_g) > \theta_1^{MR2}(p_g)$, which implies

$$2m(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g)) - \theta_1^{MR2}(p_g) - m(\theta_1^{MR2}(p_g), \gamma(\theta_1^{MR2}(p_g))) + S > 0.$$

Therefore, $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ satisfies (C.20) by Claim 2.

Conversely, If $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ satisfies (C.15), we have $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$ from Claim 1. This implies $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) \leq \theta_1^{II}(\theta_g^{MR1}(p_g))$. Since $\tilde{\theta}_1(\theta_g)$ is decreasing in θ_g and $\theta_1^{II}(\theta_g)$ is increasing in θ_g if $\tilde{\theta}_1$ and θ_1^{II} are well-defined, we have $\theta_g^{MR1}(p_g) \geq \theta_g^{MR2}(p_g)$ and $\theta_1^{MR1}(p_g) \leq \theta_1^{MR2}(p_g)$ as illustrated by Figure 3b. When $\theta_1^{MR1}(p_g) < 1$, we have $2m(\theta_g^{MR1}, \theta_1^{MR1}) - \theta_1^{MR1} - m(\theta_1^{MR1}, \gamma(\theta_1^{MR1})) + S = 0$. Then, by Assumption A.2-(i), we have

$$2m(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g)) - \theta_1^{MR2}(p_g) - m(\theta_1^{MR2}(p_g), \gamma(\theta_1^{MR2}(p_g))) + S \leq 0,$$

which implies that $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ violates (C.20) by Claim 2. When $\theta_1^{MR1}(p_g) = 1$, we must have $\theta_1^{MR2}(p_g) = 1$: otherwise, we have $\theta_1^{MR1}(p_g) > m(0, \theta_g^{MR2}(p_g)) + S$. This implies $1 = \theta_1^{MR2}(p_g) \geq \theta_{g\emptyset}^{MR2}(p_g)$. Then $(\theta_1^{MR2}(p_g), \theta_{g\emptyset}^{MR2}(p_g))$ violates (C.20). *Q.E.D.*

The following observation follows immediately from the above.

Corollary C.1. *At most one of MR1 and MR2 exists for any given p_g.*

In what follows, we characterize the range of bailout terms that admit either type of MR equilibria, denoted by P^{MR}. Specifically, we show that P^{MR} is a convex set.

Lemma C.6. *P^{MR} is a convex set, whenever it is non-empty.*

Proof.

Step 1. MR equilibrium cannot exist for any $p_g \geq \overline{p}_g^{MR2}$.

Fix $p_g \geq \overline{p}_g^{MR2}$. Clearly, MR2 cannot exist by Lemma C.4. Suppose $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ exists and satisfies $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$, which is equivalent to $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) \leq \theta_1^{II}(\theta_g^{MR1}(p_g))$, where $\tilde{\theta}_1$ and θ_1^{II} are functions of θ_g corresponding to (C.11) and (C.17) respectively, as in the proof of Lemma C.4. Recall also that $\tilde{\theta}_1(\theta_g)$ is decreasing in θ_g and $\theta_1^{II}(\theta_g)$ is increasing in θ_g. These properties imply that $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ also exists at the same p_g as depicted by Figure 3b, a contradiction to Corollary C.1. Therefore, if $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ exists, we must have $\theta_1^{MR1}(p_g) > m(0, \theta_g^{MR1}(p_g)) + S$. By Lemma C.5, the last inequality implies $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ violates (C.15), so MR1 cannot exist, a contradiction.

Step 2. MR equilibrium exists for every $p_g \in (\underline{p}_g^{MR1}, 1] \cap [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$.

Fix $p_g \in (\underline{p}_g^{MR1}, 1] \cap [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$. By Lemma C.3 and C.4, both $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ and $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ are well-defined. Furthermore, both $\theta_g^{MR1}(p_g)$ and $\theta_g^{MR2}(p_g)$ satisfy (C.14). If $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$, then by Claim 1, $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ satisfies (C.15), and MR1 exists. Otherwise, by Lemma C.5, $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ satisfies (C.20), and MR2 exists.

Step 3. If $\underline{p}_g^{MR1} < \underline{p}_g^{MR2}$, then MR1 exists for every $p_g \in (\underline{p}_g^{MR1}, \underline{p}_g^{MR2})$.

Fix $p_g \in (\underline{p}_g^{MR1}, \underline{p}_g^{MR2})$. Since $p_g > \underline{p}_g^{MR1}$, it follows from Lemma C.3 that there exists $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ that satisfies $m(0, \theta_g^{MR1}(p_g)) \geq I$. Suppose $\theta_1^{MR1}(p_g) > m(0, \theta_g^{MR1}(p_g)) + S$, or equivalently, $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) > \theta_1^{II}(\theta_g^{MR1}(p_g))$. Since $p_g < \overline{p}_g^{MR2}$, the inequality $\tilde{\theta}_1(\theta_g^{MR1}(p_g)) > \theta_1^{II}(\theta_g^{MR1}(p_g))$ and Lemma C.4 ensure that $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ is well-defined. The same inequality also implies $\theta_g^{MR2}(p_g) > \theta_g^{MR1}(p_g)$, as is seen in Figure 3a. However, $\theta_g^{MR2}(p_g) > \theta_g^{MR1}(p_g)$

implies $m(0,\theta_g^{MR2}(p_g)) > m(0,\theta_g^{MR1}(p_g)) \geq I$, which contradicts $p_g < \underline{p}_g^{MR2}$. We thus conclude that $\theta_1^{MR1}(p_g) \leq m(0,\theta_g^{MR1}(p_g)) + S$. By Lemma C.5, $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ satisfies (C.15), so MR1 exists.

Step 4. Suppose $\underline{p}_g^{MR2} \leq \underline{p}_g^{MR1} < \overline{p}_g^{MR2}$, then MR equilibrium exists for every $p_g \in [\underline{p}_g^{MR2}, \underline{p}_g^{MR1}]$ if and only if

$$2m(\theta_g^{MR2}(\underline{p}_g^{MR2}), \theta_1^{MR2}(\underline{p}_g^{MR2})) - \theta_1^{MR2}(\underline{p}_g^{MR2}) - m(\theta_1^{MR2}(\underline{p}_g^{MR2}), \gamma(\theta_1^{MR2}(\underline{p}_g^{MR2}))) + S \geq 0 \tag{C.21}$$

The proof is tedious and therefore omitted, but it is available from the authors.

Step 5. Suppose $\underline{p}_g^{MR1} \geq \overline{p}_g^{MR2}$. Proceeding similarly as in Step 4 and applying Step 1 and 2, one can find that every $p_g \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ supports MR equilibrium if (C.21) holds, but no $p_g \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ supports MR equilibrium otherwise.

Combining all the results above together, we conclude that: $P^{MR} = (\underline{p}_g^{MR1}, \overline{p}_g^{MR2})$ if $\underline{p}_g^{MR1} < \underline{p}_g^{MR2}$; $P^{MR} = [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ if $\underline{p}_g^{MR1} \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ and (C.21) holds; $P^{MR} = (\underline{p}_g^{MR1}, \overline{p}_g^{MR2})$ if $\underline{p}_g^{MR1} \in [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ but (C.21) does not hold; $P^{MR} = [\underline{p}_g^{MR2}, \overline{p}_g^{MR2})$ if $\underline{p}_g^{MR1} \geq \overline{p}_g^{MR2}$ and (C.21) holds; $P^{MR} = \emptyset$ if $\underline{p}_g^{MR1} \geq \overline{p}_g^{MR2}$ but (C.21) does not hold. *Q.E.D.*

The following lemma describes an important property of P^{MR}, which will be used in the proof of Theorem 3.

Lemma C.7. *If $P^{MBS} \neq \emptyset$ and $P^{MR} \neq \emptyset$, then $\sup P^{MBS} = \inf P^{MR}$.*

Proof. Recall from Lemma C.2 that $\sup P^{MBS} = \overline{p}_g^{MBS}$. Since the condition (C.10) binds at $p_g = \overline{p}_g^{MBS}$, we have

$$\theta_g^{MBS}(\overline{p}_g^{MBS}) - m(\theta_g^{MBS}(\overline{p}_g^{MBS}), \gamma(\theta_g^{MBS}(\overline{p}_g^{MBS}))) + S = 0. \tag{C.22}$$

It follows that $\theta_g^{MBS}(\overline{p}_g^{MBS}) = \underline{\theta}_g$, where $\underline{\theta}_g$, as defined in the proof of Lemma C.3-(i), is a lower bound of θ_g such that (C.12) is well-defined. Since $\theta_g^{MBS}(p_g)$ satisfies (C.7), (C.22) also implies

$$\overline{p}_g^{MBS} = 2\theta_g^{MBS}(\overline{p}_g^{MBS}) - m(0, \theta_g^{MBS}(\overline{p}_g^{MBS})) = 2m(\theta_g^{MBS}(\overline{p}_g^{MBS}), \theta_g^{MBS}(\overline{p}_g^{MBS})) - m(0, \theta_g^{MBS}(\overline{p}_g^{MBS})). \tag{C.23}$$

Since $\theta_g^{MBS}(\overline{p}_g^{MBS}) = \underline{\theta}_g$, (C.23) implies that $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$, the unique intersection between two curves $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$, exists if and only if $p_g > \overline{p}_g^{MBS}$, and thus $\overline{p}_g^{MBS} = \check{p}_g^{MR1}$.

Since $\theta_1^I(\theta_g)$, if well-defined, is increasing in θ_g, $\theta_g^{MR1}(p_g) - m(\theta_g^{MR1}(p_g), \gamma(\theta_g^{MR1}(p_g))) + S > 0$ for all $p_g > \check{p}_g^{MR1}$. Since $\theta - m(\theta, \gamma(\theta))$ is increasing in θ, we have $\theta_g^{MR1}(p_g) > \theta_g^{MBS}(\overline{p}_g^{MBS})$ for all $p_g > \overline{p}_g^{MBS} = \check{p}_g^{MR1}$ from (C.22). By (C.8), we have $m(0, \theta_g^{MR1}(p_g)) > I$ for all $p_g > \check{p}_g^{MR1}$, which implies $\overline{p}_g^{MBS} = \underline{p}_g^{MR1}$ by Lemma C.3. Furthermore, that $\overline{p}_g^{MBS} = \check{p}_g^{MR1}$ implies that even if $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ exists for some $p_g \leq \overline{p}_g^{MBS}$, we have $\theta_g^{MR2}(p_g) - m(\theta_g^{MR2}(p_g), \gamma(\theta_g^{MR2}(p_g))) + S \leq 0$. Thus

$$2m(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g)) - \theta_1^{MR2}(p_g) - m(\theta_1^{MR2}(p_g), \gamma(\theta_1^{MR2}(p_g))) + S < 0,$$

where the strict inequality follows from Assumption A.2-(i). Hence, $(\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$ violates (C.20) by Claim 2. Therefore, we have $\inf P^{MR} = \underline{p}_g^{MR1}$, and thus $\overline{p}_g^{MBS} = \underline{p}_g^{MR1} = \inf P^{MR}$.
Q.E.D.

C.1.5 Necessary Conditions for Equilibria under the Market Shutdown in $t = 1$

In equilibrium, there exist $\theta_g \in (0, \theta_2)$ and $\theta_2 \leq 1$ such that types $\theta \in [0, \theta_g]$ sell to the government at price p_g in $t = 1$ and sell to the market at price $m(0, \theta_g \wedge \theta_0^*)$ in $t = 2$ if $m(0, \theta_g) \geq I$; and types $\theta \in (\theta_g, \theta_2)$ sell only in $t = 2$ at price $m(\theta_g, \theta_2)$.

As before, we derive conditions for such an equilibrium to exist. There are three types of equilibria: *(i)* $m(0, \theta_g) < I$; *(ii)* $m(0, \theta_g) \geq I$ and $\theta_g \leq \theta_0^*$; *(iii)* $\theta_g > \theta_0^*$.

In case *(i)*, types $\theta \in [0, \theta_g]$ cannot sell in $t = 2$. Thus, as in SBS, we must have:

$$\begin{aligned}
\theta_2 &= \gamma(\theta_g), \\
p_g &= m(\theta_g, \gamma(\theta_g)), \\
\text{s.t. } & m(0, \theta_g) < I,
\end{aligned}$$

which are same the as (C.1) – (C.3). However, the conditions (C.4) and (C.5) are no longer necessary since $t = 1$ market is shut down. With the D1 refinement, the case $\theta_g = 1$ arises if and only if $p_g \geq m(1, \gamma(1)) = 1$ and $\mathrm{E}[\theta] = m(0, 1) < I$.

In case *(ii)*, all types $\theta \in [0, \theta_g]$ sell in $t = 2$ at price $m(0, \theta_g)$. Since this equilibrium has the same structure as MBS equilibrium, θ_g and θ_2 satisfy

$$\theta_2 = \gamma(\theta_g),$$
$$p_g + m(0, \theta_g) + S = \theta_g + m(\theta_g, \gamma(\theta_g)),$$

subject to

$$m(0, \theta_g) \geq I,$$
$$\theta_g \leq \theta_0^*,$$

which are the same as (C.6) – (C.8) and (C.10). Given that the $t = 1$ market is shut down, the condition (C.9) is no longer necessary. With the D1 refinement, the case $\theta_g = 1$ can be supported if and only if $p_g \geq 1 + m(1, \gamma(1)) - m(0, 1) - S = 2 - \mathrm{E}[\theta] - S$ and $\theta_0^* = 1$.

Lastly, consider case *(iii)*, where types $\theta \in [0, \theta_g]$ sell to the government in $t = 1$, but only types $\theta \in [0, \theta_0^*]$ sell at price $m(0, \theta_0^*)$ in $t = 2$. In this equilibrium, θ_g and θ_2 satisfy

$$\theta_2 = \gamma(\theta_g),$$
$$p_g = m(\theta_g, \gamma(\theta_g)),$$
$$\text{s.t. } \theta_g > \theta_0^*.$$

With the D1 refinement, the case $\theta_g = 1$ can be supported if and only if $p_g \geq m(1, \gamma(1)) = 1$.

Using the conditions above and proceeding similarly as in the proof of Theorem 3-(ii) below, one can show that the above conditions on θ_g and θ_2 are also sufficient to support each type of the equilibria. For later use, we let $\theta_g^{sd}(p_g)$ denote the marginal types satisfying the conditions for each alternative type of equilibria. To avoid expositional complexity, $\theta_g^{sd}(p_g)$ can be occasionally abbreviated to θ_g^{sd}. Note that $\theta_g^{sd}(p_g)$ is continuous and increasing in p_g if $\theta_g^{sd}(p_g)$ is well-defined at such p_g.

C.2 Proofs of Theorem 3 and Proposition 2 – 4

Theorem 3. *There exists an interval of bailout terms $P^k \subset \mathbb{R}_+$ that supports alternative equilibrium types $k = NR, SBS, MBS, MR$, described as follows:*

(i) ***No Response (NR)****: If $p_g \in P^{NR}$, then there exists an equilibrium in which no firm accepts the government offer and the outcome in Theorem 2 prevails.*

(ii) ***No Market Rejuvenation***

- ***Severe Bailout Stigma (SBS)****: If $p_g \in P^{SBS}$, then there exists an equilibrium with $\Theta_g = \Theta_1 = \emptyset, \Theta_{g\o}, \Theta_2 \neq \emptyset$.*
- ***Moderate Bailout Stigma (MBS)****: If $p_g \in P^{MBS}$, then there exists an equilibrium with $\Theta_g, \Theta_2 \neq \emptyset, \Theta_1 = \Theta_{g\o} = \emptyset$.*

(iii) ***Market Rejuvenation (MR)****: If $p_g \in P^{MR}$, then there exists an equilibrium with $\Theta_1 \neq \emptyset$.*

Specifically, $P^{NR} = [0, p_2^*]$, $\inf P^{SBS} = p_0^*$, and $\sup P^{MBS} \leq \inf P^{MR}$, meaning an MR equilibrium requires a strictly higher p_g than does an MBS equilibrium.

Proof of Theorem 3. We first state a lemma that will be used in the proof.

Lemma C.8. *Suppose buyers in $t = 2$ believe that types $\theta \in [a, b]$ offer assets for sale. Then buyers offer the price $m(a, \gamma(a) \wedge b)$. If $m(a, \gamma(a) \wedge b) \geq I$, then types $\theta \in [a, \gamma(a) \wedge b]$ sell their assets. If $m(a, \gamma(a) \wedge b) < I$, then the $t = 2$ market fully freezes.*

Proof. See Proposition 1 of Tirole (2012). *Q.E.D.*

Proof of Theorem 3-(i).

Suppose $p_g \leq p_2^*$. Recall p_2^* is defined in Theorem 2-(i). Consider the strategies specified in Theorem 3-(i). The beliefs on the equilibrium path are given by the Bayes' rule. The off-the-path belief for firms which accept the bailout in $t = 1$ is $\theta = 0$, and such a belief satisfies D1. Given these beliefs, type θ's payoff from accepting the bailout is $p_g + S + \theta$. If $\theta \in [0, \theta_1^*]$, then we have $2p_1^* + 2S \geq \theta + p_2^* + S \geq p_g + S + \theta$. If $\theta \in (\theta_1^*, 1]$, then we have $\theta + \max\{\theta, p_2^* + S\} \geq p_g + S + \theta$. In either case, it is not profitable to accept the bailout. Hence the NR equilibrium exists for all $p_g \in [0, p_2^*]$.

Proof of Theorem 3-(ii).

Consider first the SBS equilibrium. As we saw in Section C.1.2, given $p_g \in P^{SBS}$, there exist $\theta_{g\o} = \theta_{g\o}^{SBS}(p_g)$ and $\theta_2 = \gamma(\theta_{g\o}^{SBS}(p_g))$ that satisfy (C.3) – (C.5).

We show that the prescribed equilibrium strategies are optimal for all types of firms. Consider $t = 2$. For bailout recipients, the $t = 2$ market fully freezes since $m(0, \theta_{g\o}) < I$ from (C.3). For $t = 1$ holdouts, the $t = 2$ market offer is $m(\theta_{g\o}, \gamma(\theta_{g\o}))$. Given this, types $\theta \in (\theta_{g\o}, \gamma(\theta_{g\o})]$ sell in $t = 2$, but types $\theta > \gamma(\theta_{g\o})$ do not since $\theta \leq m(\theta_{g\o}, \gamma(\theta_{g\o})) + S \iff \theta \leq \gamma(\theta_{g\o})$. Consider now $t = 1$. Type-θ firm receives payoff $p_g + S + \theta$ from accepting the bailout and $\theta + \max\{m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) + S, \theta\}$ from rejecting it. Since $p_g = m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))$ from (C.2), playing the prescribed equilibrium strategy in $t = 1$ is optimal for every type $\theta \in [0, 1]$. Given the equilibrium strategies chosen by firms, it is straightforward to see that the equilibrium price offers are also optimal and buyers break even.

Next consider the MBS equilibrium. As we saw in Section C.1.3, for every $p_g \in P^{MBS}$, the marginal types $\theta_g = \theta_g^{MBS}(p_g)$ and $\theta_2 = \gamma(\theta_g^{MBS}(p_g))$ satisfy (C.8) – (C.10). Using these conditions and proceeding similarly as in the SBS equilibrium, it is easy to show that the prescribed equilibrium strategies are optimal for all types of firms, and the equilibrium price offers are optimal for buyers in the market. What remains to show is that it is optimal for types $\theta \in [0, \theta_g]$ to sell at price $m(0, \theta_g)$ in $t = 2$ after accepting the bailout, and for buyers in $t = 2$ to offer the price $m(0, \theta_g)$ to the bailout recipients. But these follow immediately from (C.8), (C.10), and Lemma C.8.

Proof of Theorem 3-(iii).

Consider the MR1 equilibrium first. Consider $p_g \in P^{MR}$ such that there exist $\theta_g = \theta_g^{MR1}(p_g)$, $\theta_1 = \theta_1^{MR1}(p_g)$, and $\theta_2 = \gamma(\theta_1^{MR1}(p_g))$ that satisfy (C.14) and (C.15).

We first show that it is optimal for each type of firms to play the prescribed equilibrium strategies. Consider $t = 2$ on the equilibrium path. Since $m(0, \theta_g) \geq I$ from (C.14), $\theta_g < \theta_1 \leq m(0, \theta_g) + S < m(\theta_g, \theta_1) + S$ from (C.15) and Claim 1, and $\gamma(\theta_1) \leq m(\theta_1, \gamma(\theta_1)) + S$ from Definition A.1-(ii), it is optimal for types $\theta \in [0, \theta_g]$ to sell at price $m(0, \theta_g)$, types $\theta \in (\theta_g, \theta_1]$ to sell at price $m(\theta_g, \theta_1)$, and types $\theta \in (\theta_1, \gamma(\theta_1)]$ to sell at price $m(\theta_1, \gamma(\theta_1))$. However, types $\theta \in (\gamma(\theta_1), 1]$ do not sell since $\theta > m(\theta_1, \gamma(\theta_1)) + S$ for all $\theta > \gamma(\theta_1)$ from Lemma A.1-(i). Next consider $t = 1$. Accepting the bailout is optimal for types $\theta \in [0, \theta_g]$ since

$$p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S \geq \theta + m(\theta_1, \gamma(\theta_1)) + S, \tag{C.24}$$

where the first equality follows from (C.11) and the second inequality is from (C.12). From (C.24), it is also optimal for types $\theta \in (\theta_g, \theta_1]$ to sell at price $m(\theta_g, \theta_1)$. Lastly, since $2m(\theta_g, \theta_1) + 2S < \theta + \max\{\theta, m(\theta_1, \gamma(\theta_1)) + S\}$ for all $\theta > \theta_1$, it is optimal for types $\theta \in (\theta_1, 1]$ not to sell in $t = 1$.

Next, we show that the equilibrium price offers are optimal for buyers in each period. The optimality of $t = 2$ prices directly follows from Lemma C.8, so we consider only $t = 1$. We show below that no buyer benefits by deviating from offering $m(\theta_g, \theta_1)$. In $t = 1$, buyers believe that only types $\theta > \theta_g$ are available for asset sales to the market. Suppose a buyer deviates and offers $p' \neq m(\theta_g, \theta_1)$. Any firm that accepts this offer is assigned the off-the-path belief that it is the worst available type, i.e., $\theta = \theta_g$. This belief is consistent with D1. Given this belief, any such firm will be offered price $p_2 = \theta_g$ in $t = 2$. Since $\theta_g > m(0, \theta_g) \geq I$, the deviating firm enjoys the total payoff $p' + S + \max\{\theta, \theta_g + S\}$.

There are two possibilities, either $2m(\theta_g, \theta_1) - \theta_g \geq m(\theta_1, \gamma(\theta_1))$ or $2m(\theta_g, \theta_1) - \theta_g < m(\theta_1, \gamma(\theta_1))$. Consider the former case. If $p' \leq m(\theta_1, \gamma(\theta_1)) \leq 2m(\theta_g, \theta_1) - \theta_g$, we have $2m(\theta_g, \theta_1) + 2S \geq \theta_1 + p' + S$, $2m(\theta_g, \theta_1) + 2S \geq p' + \theta_g + 2S$, and $p' + S + \theta \leq \theta + m(\theta_1, \gamma(\theta_1)) + S$. Therefore, no type $\theta \in (\theta_g, 1]$ will sell at such p'. If $p' > m(\theta_1, \gamma(\theta_1))$, then we have $2m(\theta_g, \theta_1) + 2S < \theta_1 + p' + S$. Letting $\tilde{\theta} := 2m(\theta_g, \theta_1) + S - p' < \theta_1$, all types $\theta \in (\tilde{\theta}, (p' + S) \wedge 1]$ sell at p'. However, by definition of $\gamma(\cdot)$, we have $m(\tilde{\theta}, (p' + S) \wedge 1) - p' < m(\theta_1, (p' + S) \wedge 1) - p' < 0$, and thus the deviating buyer will make a loss by offering $p' \neq m(\theta_g, \theta_1)$.

Consider the latter case next. If $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_1, \gamma(\theta_1))]$, then types $\theta \leq (\theta_g, \theta' \wedge 1]$ will sell at p', where $\theta' := p' - m(\theta_1, \gamma(\theta_1)) + \theta_g + S$. Since $\lim_{p' \to 2m(\theta_g, \theta_1) - \theta_g} \theta' = \theta_1$, we have

$$\lim_{p' \to 2m(\theta_g, \theta_1) - \theta_g} (m(\theta_g, \theta' \wedge 1) - p') = m(\theta_g, \theta_1) - (m(\theta_g, \theta_1) + (m(\theta_g, \theta_1) - \theta_g)) < 0.$$

Moreover, since $\frac{d\theta'}{dp'} = 1$ and $\frac{\partial}{\partial b} m(a, b) < 1$ for any $0 \leq a \leq b \leq 1$ from Lemma A.1-(i), we have $\frac{d}{dp'}(m(\theta_g, \theta' \wedge 1) - p') \leq 0$ for all $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_1, \gamma(\theta_1))]$. As a result, it is not profitable for the buyers to offer any $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_1, \gamma(\theta_1))]$. If $p' > m(\theta_1, \gamma(\theta_1))$, then all types $\theta \in (\theta_g, (p' + S) \wedge 1]$ will sell at p'. Since $p' > m(\theta_1, \gamma(\theta_1)) > m(\theta_g, \gamma(\theta_g))$, we have $m(\theta_g, (p' + S) \wedge 1) - p' < 0$ for all $p' > m(\theta_1, \gamma(\theta_1))$. Hence, the deviating buyer will make a loss by offering p'. If $p' \leq 2m(\theta_g, \theta_1) - \theta_g$, then, as shown before, types $\theta > \theta_g$ will not sell at such p'. Consequently, all buyers in $t = 1$ optimally offer $m(\theta_g, \theta_1)$.

We now turn to the MR2 equilibrium. Suppose MR2 exists at $p_g \in P^{MR}$. Then, by Lemma C.6, there exist $\theta_g = \theta_g^{MR2}(p_g), \theta_1 = \theta_1^{MR1}(p_g), \theta_{g\o} = \theta_{g\o}^{MR2}(p_g)$, and $\theta_2 = \gamma(\theta_{g\o}^{MR2}(p_g))$ satisfying (C.14) and (C.20).

First, we show that it is optimal for each type of firms to play the prescribed equilibrium strategies. Consider $t = 2$ first. Since $\theta_1 = m(0, \theta_g) + S$ from (C.17), we have $\theta_g < \theta_1 = m(0, \theta_g) + S$, $\theta_1 = m(0, \theta_g) + S < m(\theta_g, \theta_1) + S$, and $\theta > m(0, \theta_g) + S$ for all $\theta > \theta_1$. Therefore it is optimal for types $\theta \in [0, \theta_g]$ to sell at price $m(0, \theta_g)$, but types $\theta \in (\theta_1, \theta_{g\o}]$ not to sell at that price. Furthermore, it is optimal for types $\theta \in (\theta_g, \theta_1]$ to sell at price $m(\theta_g, \theta_1)$. Finally, from the definition of $\gamma(\cdot)$, it is optimal for types $\theta \in (\theta_{g\o}, \gamma(\theta_{g\o})]$ to sell at price $m(\theta_{g\o}, \gamma(\theta_{g\o}))$, but types $\theta \in (\gamma(\theta_{g\o}), 1]$ not to sell at that price. Consider next $t = 1$. From (C.11), (C.17), and (C.18), we have

$$p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S \geq \theta + p_g + S = \theta + m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) + S,$$

where the inequality holds strictly if and only if $\theta \in [0, \theta_1)$. This inequality implies the prescribed equilibrium strategies are optimal for all types $\theta \in [0, \theta_1]$. For all $\theta > \theta_1$, the above inequality is reversed. Thus it is optimal for types $\theta \in (\theta_1, \theta_{g\emptyset}]$ to accept the bailout and for types $\theta \in (\theta_{g\emptyset}, 1]$ not to sell in $t = 1$.

Next, we show that the equilibrium price offers are also optimal for buyers in the market. Consider the $t = 2$ market first. On the equilibrium path, buyers believe that types $\theta \in [0, \theta_g] \cup (\theta_1, \theta_{g\emptyset}]$ accept the bailout in $t = 1$, types $\theta \in (\theta_g, \theta_1]$ sell to the $t = 1$ market, and types $\theta \in (\theta_{g\emptyset}, 1]$ do not sell in $t = 1$. By Lemma C.8, it is optimal for the $t = 2$ buyers to offer $m(\theta_g, \theta_1)$ to the second types and $m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))$ to the the third types. It remains to show that it is optimal for the $t = 2$ buyers to offer $m(0, \theta_g)$ to types $\theta \in [0, \theta_g] \cup (\theta_1, \theta_{g\emptyset}]$. Suppose a buyer deviates and offers $p' \neq m(0, \theta_g)$. By Lemma C.8, it can be shown that $p' < m(0, \theta_g)$ cannot be an equilibrium strategy. Suppose now $p' > m(0, \theta_g)$. If $p' \leq \theta_1 - S$, then p' can attract types $\theta \in [0, \theta_g]$ at most, resulting in a loss to the deviating buyer. If $p' > \theta_1 - S$, then types $\theta \in [0, \theta_g] \cup (\theta_1, \theta']$ sell assets at p', where $\theta' := (p' + S) \wedge \theta_{g\emptyset}$. Thus the deviating buyer's payoff is $\mathring{m}(0, \theta_g, \theta_1, \theta') - p'$ (see Definition A.1-(iii) for the definition of $\mathring{m}(a, b, c, d)$). From the definition of θ', we have $\lim_{p' \to m(0, \theta_g)} \theta' = \theta_1$. Since $\frac{\partial}{\partial \theta'} \mathring{m}(0, \theta_g, \theta_1, \theta') < 1$ from Lemma A.1-(iii), we have $\mathring{m}(0, \theta_g, \theta_1, \theta') - p' < 0$ for any $p' > m(0, \theta_g)$. Thus any deviation $p' \neq m(0, \theta_g)$ results in a loss to the deviating buyer, and thus the offer $m(0, \theta_g)$ to the firms accepting the bailout in the previous period is optimal for buyers.

To complete the proof, we need to show that it is optimal for buyers to offer $m(\theta_g, \theta_1)$ in $t = 1$. Suppose a buyer deviates to $p' \neq m(\theta_g, \theta_1)$. The off-the-path belief assigned to the firms accepting p' (consistent with the D1 refinement) is that they are the type $\theta = \theta_g$ with probability 1. Since $\theta_g > m(0, \theta_g) \geq I$ from (C.14), these firms can sell in $t = 2$ at price $\theta_g > I$ after selling at the deviation price p' in $t = 1$, and thus get the payoff $p' + S + \max\{\theta, \theta_g + S\}$.

If $p' > m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))$, then all types $\theta \in (\theta_{g\emptyset}, (p' + S) \wedge 1]$ will sell at price p' in $t = 1$ since $\theta + p' + S > \theta + m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) + S$. Moreover, some types $\theta \in (\theta_g, \theta_1]$ may also sell at p'. Since $(2m(\theta_g, \theta_1) + 2S) - (\theta + p' + S)$ is decreasing in θ, there exists a $\tilde{\theta}$ such that types $\theta \in (\tilde{\theta}, \theta_1]$ will sell at p'. The deviating buyer's payoff is then $\mathring{m}(\tilde{\theta}, \theta_1, \theta_{g\emptyset}, (p' + S) \wedge 1) - p'$. However, we have $\mathring{m}(\tilde{\theta}, \theta_1, \theta_{g\emptyset}, (p' + S) \wedge 1) - p' < m(\theta_{g\emptyset}, (p' + S) \wedge 1) - p' < 0$, where the last inequality follows from the definition of $\gamma(\cdot)$ and the condition $p' > m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))$. If $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset}))]$ (and if such an interval is not empty), we have $p' + \theta_g + 2S > 2m(\theta_g, \theta_1) + 2S$. This inequality implies that types $\theta \le (\theta_g, \theta_1] \cup (\theta_{g\o}, \theta']$ sell at p' in $t = 1$, where $\theta' := p' + \theta_g + S - m(\theta_{g\o}, \gamma(\theta_{g\o}))$. Thus the deviating buyer's payoff is $\mathring{m}(\theta_g, \theta_1, \theta_{g\o}, \theta') - p'$. However, since $\lim_{p' \to 2m(\theta_g,\theta_1)-\theta_g} \theta' = \theta_1$, $\frac{d\theta'}{dp'} = 1$, and $\frac{\partial}{\partial \theta'} \mathring{m}(\theta_g, \theta_1, \theta_{g\o}, \theta') < 1$, we have $\mathring{m}(\theta_g, \theta_1, \theta_{g\o}, \theta') - p' < 0$ for any $p' \in (2m(\theta_g, \theta_1) - \theta_g, m(\theta_{g\o}, \gamma(\theta_{g\o}))]$, resulting in a loss to the deviating buyer. If $p' \le 2m(\theta_g, \theta_1) - \theta_g$, then we have $p' + \theta_g + 2S \le 2m(\theta_g, \theta_1) + 2S$, so p' will not be accepted by any type. Put together, we conclude that it is optimal for the $t = 1$ market buyers to offer $m(\theta_g, \theta_1)$.

The proof of $\sup P^{MBS} \le \inf P^{MR}$ for the case $P^{MR}, P^{MBS} \ne \emptyset$ is referred to Lemma C.7. Finally, it follows immediately from Corollary C.1 that any $p_g \in P^{MR}$ can support at most one type of MR equilibrium. *Q.E.D.*

Proposition 2.

(i) (Dampened initial responses) Fix $p_g \ge \max\{p_0^*, I\}$. In any equilibrium, the trade volume in $t = 1$ is (weakly) smaller than the trade volume $F(p_g + S)$ in the one-shot model.

(ii) (Positive net gains) The total trade volume is higher with a bailout than without, if either MBS, MR1, or MR2 equilibrium would prevail under a bailout. The same holds even when an SBS equilibrium arises from a bailout if the $t = 1$ market fully freezes without a bailout.

(iii) (Delayed benefits) The $t = 2$ trade volume is higher with a bailout than without, if either MBS or MR1 equilibrium would prevail under a bailout.

(iv) (Discontinuous effects) Let $\Phi(p_g)$ denote the set of total trade volumes that would result from some equilibrium given bailout $p_g \in [I, 1]$. The correspondence $\Phi(\cdot)$ does not admit a selection that is continuous in p_g.

Proof of Proposition 2.

Proof of Proposition 2-(i).

First consider the SBS equilibrium. Since $\theta_{g\o}^{SBS} < \gamma(\theta_{g\o}^{SBS}) \le m(\theta_{g\o}^{SBS}, \gamma(\theta_{g\o}^{SBS})) + S = p_g + S$ from (C.2), we have $\theta_{g\o}^{SBS} < (p_g + S) \wedge 1$. The statement thus follows. Next consider the MBS equilibrium. Since $\theta_g^{MBS} > 0$, we have $m(0, \theta_g^{MBS}) < m(\theta_g^{MBS}, \gamma(\theta_g^{MBS}))$, which implies $\theta_g^{MBS} < (p_g + S) \wedge 1$ by (C.7). Third, consider the MR1 equilibrium. By Claim 1 within the proof of Lemma C.5, we have $\theta_1^{MR1} \le m(0, \theta_g^{MR1}) + S$. Moreover, it follows from (C.11) that $p_g = m(\theta_g^{MR1}, \theta_1^{MR1}) + (m(\theta_g^{MR1}, \theta_1^{MR1}) - m(0, \theta_g^{MR1})) > m(\theta_g^{MR1}, \theta_1^{MR1}) > m(0, \theta_g^{MR1})$. Therefore,

$\theta_1^{MR1} \leq m(0, \theta_g^{MR1}) + S < (p_g + S) \wedge 1$, as was to be shown. Lastly, in the MR2 equilibrium, $\theta_{g\o}^{MR2} < (p_g + S) \wedge 1$ since $\theta_{g\o}^{MR2} < m(\theta_{g\o}^{MR2}, \gamma(\theta_{g\o}^{MR2})) + S$.

Proof of Proposition 2-(ii).

Recall from Theorem 2 that θ_1^* is the marginal type selling in $t = 1$ in the equilibrium without bailout.

First, consider the MBS equilibrium. By (C.9), the MBS equilibrium can exist only if $\theta - m(\theta, \gamma(\theta)) + S < 0$ for some $\theta > 0$, thereby implying $0 - m(0, \gamma(0)) + S < 0$ since $\theta - m(\theta, \gamma(\theta)) + S$ is increasing in θ. Thus we have $2m(0, \theta) - \theta - m(\theta, \gamma(\theta)) + S < \theta - m(\theta, \gamma(\theta)) + S < 0$ for all $\theta > 0$, hence $\theta_1^* = 0$ from Assumption A.2-(i) (i.e., $t = 1$ freezes absent bailout). Thus the total volume of trade in the absence of bailout is $F(\gamma(0)) = F(\theta_0^*)$ (i.e., the trade occurs only in $t = 2$). Since $F(\theta_g^{MBS}) + F(\gamma(\theta_g^{MBS})) > F(0) + F(\gamma(0)) = F(\theta_0^*)$, the MBS equilibrium, if it exists, yields larger total trade than without bailout.

Second, consider the MR1 equilibrium which yields the total trade volume$F(\theta_1^{MR1}) + F(\gamma(\theta_1^{MR1}))$. Since θ_1^{MR1} is determined by (C.12), we have $\Delta(\theta_g^{MR1}, \theta_1^{MR1}; S) = m(\theta_g^{MR1}, \theta_1^{MR1}) - \theta_1^{MR1} + (m(\theta_g^{MR1}, \theta_1^{MR1}) - m(\theta_1^{MR1}, \gamma(\theta_1^{MR1}))) + S \geq 0$. Furthermore, θ_1^* must satisfy $\Delta(0, \theta_1^*; S) \geq 0$ since $\theta_1^* = \max\{\theta > 0 : \Delta(0, \theta; S) = 2m(0, \theta) - \theta - m(\theta, \gamma(\theta)) + S \geq 0\}$. Since $\theta_g^{MR1} > 0$, we have $\Delta(0, \theta; S) < \Delta(\theta_g^{MR1}, \theta; S)$, so we have $\theta_1^{MR1} \geq \theta_1^*$, where the equality holds for the case $\theta_1^* = 1$. Therefore, $F(\theta_1^{MR1}) + F(\gamma(\theta_1^{MR1})) \geq F(\theta_1^*) + F(\gamma(\theta_1^*))$, as was to be shown, where the equality holds for the case $\theta_1^* = 1$.

Next, consider the MR2 equilibrium, which yields the total trade volume $F(\theta_1^{MR2}) + F(\gamma(\theta_{g\o}^{MR2}))$. If $\theta_1^* = 0$, then the MR2 equilibrium yields larger total trade than without bailout since $\theta_1^* = 0 < \theta_1^{MR2}$ and $\gamma(\theta_1^*) = \gamma(0) < \gamma(\theta_{g\o}^{MR2})$. Suppose $\theta_1^* > 0$. If $S \geq 1$, then it follows from (C.17) that $\theta_1^{MR2} = 1$, implying $\Theta_{g\o} = \emptyset$. Thus, $S < 1$ for the MR2 equilibrium to exist. From Assumption A.2-(v), we have

$$\begin{aligned}
& m(0, \theta) \leq \theta - m(0, \theta) \text{ for all } \theta \in [0, 1], \\
\Longrightarrow\ & m(0, S) \leq S - m(0, S), \\
\Longrightarrow\ & 2m(0, S) \leq m(S, \gamma(S)), \\
\Longrightarrow\ & 2m(0, S) - S - m(S, \gamma(S)) + S \leq 0, \\
\Longrightarrow\ & \theta_1^* \leq S.
\end{aligned}$$

Therefore, we have $m(0, \theta) + S > \theta_1^*$ for all $\theta \in (0, \theta_0^*)$. By (C.17) and (C.20), we have $\theta_{g\o}^{MR2} > \theta_1^{MR2} > \theta_1^*$, and thus $F(\theta_1^{MR2}) + F(\gamma(\theta_{g\o}^{MR2})) > F(\theta_1^*) + F(\gamma(\theta_1^*))$.

Lastly, consider the SBS equilibrium, which yields the total trade volume $F(\gamma(\theta_{g\o}^{SBS}(p_g)))$. If $\theta_1^* = 0$, then the total trade volume without bailout is $F(\gamma(0))$. Since $\theta_{g\o}^{SBS}(p_g) > 0$ for all $p_g \in P^{SBS}$, we have $F(\gamma(\theta_{g\o}^{SBS}(p_g))) > F(\gamma(0))$.

Proof of Proposition 2-(iii).

First, consider the MBS equilibrium, which yields the trade volume $F(\gamma(\theta_g^{MBS}))$ in $t = 2$. As shown in the proof of Proposition 2-(ii), the MBS equilibrium exists only if $\theta_1^* = 0$, and thus the trade volume in $t = 2$ in the absence of bailout is $F(\gamma(0)) = F(\theta_0^*)$. This implies $F(\gamma(\theta_g^{MBS})) > F(\gamma(0)) = F(\theta_0^*)$.

Next, consider the MR1 equilibrium, which yields the trade volume $F(\gamma(\theta_1^{MR1}))$ in $t = 2$. Similar to the proof of Proposition 2-(ii), we have $\theta_1^{MR1} \geq \theta_1^*$ where the equality holds for the case $\theta_1^* = 1$. This inequality implies $F(\gamma(\theta_1^{MR1})) \geq F(\gamma(\theta_1^*))$ with the equality when $\theta_1^* = 1$.

Proof of Proposition 2-(iv).

Let $\phi : [I, 1] \to \mathbb{R}_+$ be a function such that $\phi(p_g) \in \Phi(p_g)$. In what follows, we show that $\phi(\cdot)$ cannot be continuous at every $p_g \in [I, 1]$ if $\theta_1^* > 0$. The argument for the case $\theta_1^* = 0$ is similar but with a slight modification.[4]

Step 1. For $p_g = I$, the only possible equilibrium is NR-type and thus $\Phi(I) = \{F(\theta_1^*) + F(\gamma(\theta_1^*))\}$.

Suppose either an MR1 or MR2 equilibrium exists for $p_g = I$. By (C.11), (C.14), and (C.15), we have $I \leq \max\{m(0, \theta_g^{MR1}), m(0, \theta_g^{MR2})\} < p_g = I$, a contradiction. Moreover, since $2m(0, \theta_1^*) - \theta_1^* - m(\theta_1^*, \gamma(\theta_1^*)) + S \geq 0$, it follows from Assumption A.2-(i) that $2m(0, 0) - 0 - m(0, \gamma(0)) + S = 0 - m(0, \gamma(0)) + S > 0$, which implies $\theta - m(\theta, \gamma(\theta)) + S > 0$ for all $\theta \in [0, 1]$. From (C.9), MBS equilibrium cannot exist for any $p_g \in [I, 1]$. Lastly, since $I \leq m(0, \theta_1^*) \leq p_0^* = m(0, \gamma(0))$, we have $p_0^* = \inf P^{SBS} \geq I$. Since $p_g > \inf P^{SBS}$ for all $p_g \in P^{SBS}$ by Lemma C.1, SBS equilibrium cannot exist for $p_g = I$.

Step 2. If $\phi(\cdot)$ is a continuous selection from $\Phi(\cdot)$, then $\phi(p_g) = F(\theta_1^*) + F(\gamma(\theta_1^*))$ for all $p_g \in [I, p_2^*]$ (Recall $p_2^* = m(\theta_1^*, \gamma(\theta_1^*))$ from Theorem 2).

Suppose there exists an SBS equilibrium for some $p_g \in (I, p_2^*]$. We have $m(0, \theta_{g\o}^{SBS}(p_g)) < I \leq m(0, \theta_1^*)$ by (C.3), which implies $\theta_{g\o}^{SBS}(p_g) < \theta_1^*$. Since the total trade volume is $F(\theta_1^*) + F(\gamma(\theta_1^*))$ under the NR equilibrium and the equilibrium absent bailout and $F(\gamma(\theta_{g\o}^{SBS}(p_g)))$ under the SBS equilibrium, the NR equilibrium yields strictly larger overall trade than the SBS type. Suppose next there exists either an MR1 or MR2 equilibrium for some $p_g \in (I, p_2^*]$. By

[4] In the case $\theta_1^* = 0$, we have to take into account the possibility that the MBS equilibrium can exist for some $p_g \in [I, 1]$. But the MBS equilibrium cannot exist in the case $\theta_1^* > 0$, in which we have $\theta - m(\theta, \gamma(\theta)) + S > 0$ for any $\theta > 0$, so any $\theta_g > 0$ cannot satisfy (C.9).

Proposition 2-(ii), either of the two yields strictly larger overall trade than the NR equilibrium. Since $\phi(p_g) = F(\theta_1^*) + F(\gamma(\theta_1^*))$ at $p_g = I$ from Step 1, continuity of ϕ requires that $\phi(p_g) = F(\theta_1^*) + F(\gamma(\theta_1^*))$ for all $p_g \in [I, p_2^*]$.

Step 3. Any $\phi \in \Phi$ such that $\phi(p_g) = F(\theta_1^*) + F(\gamma(\theta_1^*))$ for all $p_g \in [I, p_2^*]$ is discontinuous at $p_g = p_2^*$.

Suppose there exists an SBS equilibrium for some $p_g \geq p_2^* = m(\theta_1^*, \gamma(\theta_1^*))$. Then, it follows from (C.2) that $\theta_{g\emptyset}^{SBS}(p_g) \geq \theta_1^*$, which implies $m(0, \theta_{g\emptyset}^{SBS}(p_g)) \geq m(0, \theta_1^*) \geq I$. However, this inequality contradicts (C.3). Therefore, there cannot exist SBS equilibrium for any $p_g \geq p_2^*$, and thus $\phi(p_g)$ must be equal to the total trade volume under either MR1 or MR2 equilibrium for any $p_g \in [p_2^*, 1]$. By Proposition 2-(ii), both MR1 and MR2 equilibria yield strictly larger total trade than $F(\theta_1^*) + F(\gamma(\theta_1^*))$ for any $p_g \in [p_2^*, I]$. Thus we must have $\phi(p_g) > F(\theta_1^*) + F(\gamma(\theta_1^*))$ for all $p_g \in [p_2^*, 1]$. By Step 2, the last inequality implies $\lim_{p_g \to p_2^*+} \phi(p_g) > \lim_{p_g \to p_2^*-} \phi(p_g)$.

Combining Step 2 and 3, we conclude that Φ does not admit a continuous selection. *Q.E.D.*

Proposition 3. *Suppose that an MR (either MR1 or MR2) equilibrium arises given p_g. In that case, offering a bailout at the same p_g,* but with the $t = 1$ market shut down, *would (at least weakly) increase the total trade volume.*

Proof of Proposition 3. First, suppose p_g admits an MR1 equilibrium. By Claim 1 within the proof of Lemma C.5, we have $\theta_1^{MR1} \leq m(0, \theta_g^{MR1}) + S$. Since $\theta_1^{MR1}(p_g) > \theta_g^{MR1}(p_g)$, we also have $\theta_1^{MR1}(p_g) \leq \theta_0^*$, where the equality holds when $\theta_1^{MR1}(p_g) = 1$. Since $p_g \leq m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g)))$ from (C.15), we have $p_g \leq m(\theta_0^*, \gamma(\theta_0^*))$. This result implies there exists $\theta_g^{sd}(p_g) \leq \theta_0^*$ such that $p_g = (\theta_g^{sd}(p_g) - m(0, \theta_g^{sd}(p_g)) - S) + m(\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g)))$. Since $p_g + m(0, \theta_g^{MR1}(p_g)) + S \geq \theta_1^{MR1}(p_g) + m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g)))$ from (C.11) and (C.12), we have $p_g > \theta_1^{MR1}(p_g) - m(0, \theta_1^{MR1}(p_g)) - S + m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g)))$. By Lemma A.1, $\theta - m(0, \theta) + m(\theta, \gamma(\theta))$ is increasing in θ, so we have $\theta_g^{sd}(p_g) \geq \theta_1^{MR1}(p_g)$, which implies $m(0, \theta_g^{sd}(p_g)) \geq I$ from (C.14). Therefore, when the market is shut down in $t = 1$, the same p_g admits an equilibrium which yields the total trade volume $F(\theta_g^{sd}(p_g)) + F(\gamma(\theta_g^{sd}(p_g))) \geq F(\theta_1^{MR1}(p_g)) + F(\gamma(\theta_1^{MR1}(p_g)))$, as was to be shown, where the equality holds for the case $\theta_1^{MR1}(p_g) = \theta_0^* = \theta_g^{sd}(p_g) = 1$.

Next, suppose p_g admits an MR2 equilibrium. Since $\theta_1^{MR2}(p_g) = m(0, \theta_g^{MR2}(p_g)) + S$ from (C.17) and $\theta_g^{MR2}(p_g) < \theta_1^{MR2}(p_g)$, we have $\theta_1^{MR2}(p_g) < m(0, \theta_1^{MR2}(p_g)) + S$, which implies $\theta_1^{MR2}(p_g) < \theta_0^*$. If $\theta_{g\emptyset}^{MR2}(p_g) > \theta_0^*$, then there exists $\theta_g^{sd}(p_g)$ such that $p_g = m(\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g)))$.

By (C.18), we have $\theta^{MR2}_{g\o}(p_g) = \theta^{sd}_g(p_g)$. Hence, under the market shutdown in $t = 1$, p_g admits the equilibrium which yields the total trade volume $F(\theta^*_0) + F(\gamma(\theta^{sd}_g(p_g))) > F(\theta^{MR2}_1(p_g)) + F(\gamma(\theta^{MR2}_{g\o}(p_g)))$, as was to be shown. If $\theta^{MR2}_{g\o}(p_g) \leq \theta^*_0$, we have $p_g \leq m(\theta^*_0, \gamma(\theta^*_0))$ by (C.18). Thus there exists $\theta^{sd}_g(p_g) \leq \theta^*_0$ such that $p_g + m(0, \theta^{sd}_g(p_g)) + S = \theta^{sd}_g(p_g) + m(\theta^{sd}_g(p_g), \gamma(\theta^{sd}_g(p_g)))$. Since $\theta^{sd}_g(p_g) \leq \theta^*_0 \iff \theta^{sd}_g(p_g) \leq m(0, \theta^{sd}_g(p_g)) + S$, we have $p_g \leq m(\theta^{sd}_g(p_g), \gamma(\theta^{sd}_g(p_g)))$. Since $p_g = m(\theta^{MR2}_{g\o}(p_g), \gamma(\theta^{MR2}_{g\o}(p_g)))$ and $\theta^{MR2}_{g\o}(p_g) > \theta^{MR2}_g(p_g)$, we have $m(0, \theta^{sd}_g(p_g)) \geq I$ from (C.14). Therefore, under the market shutdown in $t = 1$, p_g admits the equilibrium which yields the total trade volume $F(\theta^{sd}_g(p_g)) + F(\gamma(\theta^{sd}_g(p_g)) > F(\theta^{MR2}_1(p_g) + F(\gamma(\theta^{MR2}_{g\o}(p_g))$, as was to be shown. *Q.E.D.*

Proposition 4. *Suppose that the market remains closed in $t = 1$, and let p_g be a given bailout offer. The total trade volume decreases when the government adds another bailout offer $p'_g \in [I, p_g)$.*

Proof of Proposition 4. Fix p_g and p'_g such that $p_g > p'_g \geq I$. For the bailout that has two options $\{p'_g, p_g\}$, there are two possibilities. First, only one of the offers is accepted by a positive measure of firms. Second, both offers are accepted by a positive measure of firms. Since the first possibility is identical to the case with a single offer, we hereafter restrict our focus on the second possibility.

Let θ^{do}_g denote the highest type selling to the government at either p_g or p'_g. In this equilibrium, types $\theta \in (\theta^{do}_g, \gamma(\theta^{do}_g)]$ sell at price $m(\theta^{do}_g, \gamma(\theta^{do}_g))$ in $t = 2$ by Lemma C.8. Since both offers are accepted by positive measures of types, we have the following observations. First, all types choosing p'_g must also sell in $t = 2$; otherwise, they will get a higher payoff by selling at price $p_g > p'_g$ in $t = 1$. Second, the $t = 2$ price for types choosing p_g (denoted by p_2) should be strictly lower than the $t = 2$ price for types choosing p'_g (denoted by p'_2), which follows from the fact that types $\theta \in (\theta^{do}_g, \gamma(\theta^{do}_g)]$ are indifferent between selling at p_g and selling at p'_g, i.e., $p'_g + p'_2 + 2S = p_g + p_2 + 2S$. These observations imply that there are two types of equilibria: all types selling at price p_g in $t = 1$ also sell in $t = 2$; a positive measure of types selling at price p_g in $t = 1$ do not sell in $t = 2$. We show below that any type of equilibria given the bailout with a single offer p_g yields larger overall trade than both types of equilibria above given the bailout with double offers.

First, consider the equilibrium in which all firms choosing p_g in $t = 1$ also sell in $t = 2$, which yields the total trade volume $F(\theta^{do}_g) + F(\gamma(\theta^{do}_g))$. Since buyers in $t = 2$ earn zero profit and $p_2 < p'_2$, there exists $\underline{\theta}^{do}_g < \theta^{do}_g$ such that $p_2 = m(0, \underline{\theta}^{do}_g)$. Since type-$\theta^{do}_g$ firm prefers selling at price p_2 in $t = 2$ to not selling, we have $\theta_g^{do} \leq m(0, \underline{\theta}_g^{do}) + S$, thereby implying $\theta_g^{do} \leq \theta_0^*$. Furthermore, since type-θ_g^{do} firm prefers selling at price p_g in $t = 1$ to not selling, we have

$$p_g + p_2 + 2S = p_g + m(0, \underline{\theta}_g^{do}) + 2S \geq \theta_g^{do} + m(\theta_g^{do}, \gamma(\theta_g^{do})) + S.$$

Since $\underline{\theta}_g^{do} < \theta_g^{do}$, we have $p_g > \theta_g^{do} - m(0, \theta_g^{do}) - S + m(\theta_g^{do}, \gamma(\theta_g^{do}))$. Therefore, there always exists θ_g^{sd} determined by either $p_g \geq \theta_g^{sd} - m(0, \theta_g^{sd}) - S + m(\theta_g^{sd}, \gamma(\theta_g^{sd}))$ (the inequality holds strictly for the case $\theta_g^{sd} = 1$) subject to $\theta_g^{sd} \leq \theta_0^*$ or $p_g = m(\theta_g^{sd}, \gamma(\theta_g^{sd}))$ subject to $\theta_g^{sd} > \theta_0^*$. If $\theta_g^{sd} \leq \theta_0^*$, then $\theta_g^{sd} \geq \theta_g^{do}$ since $\theta - m(0, \theta) + m(\theta, \gamma(\theta))$ is increasing in θ; if $\theta_g^{sd} > \theta_0^*$, then $\theta_g^{sd} > \theta_g^{do}$ since $\theta_g^{do} \leq \theta_0^*$. These observations imply that there exists an equilibrium given the bailout with the single offer p_g, which yields the overall trade $F(\theta_g^{sd} \wedge \theta_0^*) + F(\gamma(\theta_g^{sd})) \geq F(\theta_g^{do}) + F(\gamma(\theta_g^{do}))$, as was to be shown, where the equality holds for the case $\theta_g^{sd} = \theta_g^{do} = \theta_0^* = 1$.

Next, consider the equilibrium where a positive measure of types selling at price p_g in $t = 1$ do not sell in $t = 2$. By playing this strategy, a type-θ firm earns the total payoff $p_g + S + \theta$, which is increasing in θ. Hence, θ_g^{do} is determined by $p_g + S + \theta_g^{do} = \theta_g^{do} + m(\theta_g^{do}, \gamma(\theta_g^{do})) + S$, which is equivalent to $p_g = m(\theta_g^{do}, \gamma(\theta_g^{do}))$. Let $\hat{\theta}_g^{do} \in [\underline{\theta}_g^{do}, \theta_g^{do})$ be the highest type that sells in both periods. Then the total trade volume under this equilibrium is $F(\hat{\theta}_g^{do}) + F(\gamma(\theta_g^{do}))$. Since type $\hat{\theta}_g^{do}$ must be indifferent between selling in both periods and selling only in $t = 1$, we have

$$p_g + p_2 + 2S = \hat{\theta}_g^{do} + p_g + S \iff \hat{\theta}_g^{do} = m(0, \underline{\theta}_g^{do}) + S.$$

Since $\hat{\theta}_g^{do} \geq \underline{\theta}_g^{do}$, the condition above implies $\hat{\theta}_g^{do} \leq \theta_0^*$. Furthermore, the same condition also implies $p_g > \hat{\theta}_g^{do} - m(0, \hat{\theta}_g^{do}) - S + m(\hat{\theta}_g^{do}, \gamma(\hat{\theta}_g^{do}))$ since $\underline{\theta}_g^{do} \leq \hat{\theta}_g^{do} < \theta_g^{do}$. From these results, one can find that the bailout with the single offer p_g admits one of the following types of equilibria: either $\theta_g^{sd} \leq \theta_0^*$ and $m(0, \theta_g^{sd}) \geq I$ or $\theta_g^{sd} > \theta_0^*$. First, suppose $\theta_g^{sd} \leq \theta_0^*$ and $m(0, \theta_g^{sd}) \geq I$. If $\theta_g^{sd} < 1$, then $p_g = (\theta_g^{sd} - m(0, \theta_g^{sd}) - S) + m(\theta_g^{sd}, \gamma(\theta_g^{sd})) \leq m(\theta_g^{sd}, \gamma(\theta_g^{sd}))$, which implies $\theta_g^{do} \leq \theta_g^{sd}$. If $\theta_g^{sd} = \theta_0^* = 1$, then $\theta_g^{do} \leq \theta_g^{sd}$. Next, suppose $\theta_g^{sd} > \theta_0^*$, then we have $\theta_g^{do} = \theta_g^{sd} > \theta_0^* \geq \hat{\theta}_g^{do}$. Putting all the results altogether, we have shown that $F(\theta_g^{sd} \wedge \theta_0^*) + F(\gamma(\theta_g^{sd})) \geq F(\hat{\theta}_g^{do}) + F(\gamma(\theta_g^{do}))$, that is, the bailout with the single offer p_g always yields (weakly) larger total trade than the bailout with double offers $\{p'_g, p_g\}$. *Q.E.D.*

D Proofs for Section 5

Proceeding similarly as in Section C.1, we derive conditions characterizing the marginal types θ_g, θ_1, and $\theta_{g\emptyset}$ of SMR equilibrium. Next, we find the set of bailout terms that support the SMR equilibrium, denoted by P^{SMR}.

D.1 Necessary Conditions for SMR Equilibrium

In this equilibrium, types $\theta \in \Theta_g = [0, \theta_g]$ sell to the government in $t = 1$ and to the market in $t = 2$ at price $m(0, \theta_g)$, types $\theta \in \Theta_1 = (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods, types $\theta \in \Theta_{g\o} = (\theta_1, \theta_{g\o}]$ sell only in $t = 1$ to the government, and the rest do not sell in either period. The firms' total payoffs are $p_g + m(0, \theta_g) + 2S$ if $\theta \in [0, \theta_g]$, $2m(\theta_g, \theta_1) + 2S$ if $\theta \in (\theta_g, \theta_1]$, $p_g + S + \theta$ if $\theta \in (\theta_1, \theta_{g\o}]$, and 2θ if $\theta \in (\theta_{g\o}, 1]$. From these payoffs, the marginal types must satisfy the following indifference conditions:

$$p_g + m(0, \theta_g) = 2m(\theta_g, \theta_1), \tag{D.1}$$

$$\theta_1 \leq m(0, \theta_g) + S, \tag{D.2}$$

$$\theta_{g\o} = (p_g + S) \wedge 1. \tag{D.3}$$

Note that the inequality in (D.2) is strict for the case $\theta_1 = 1$.

In addition to the above conditions, we need the conditions that guarantee that the $t = 2$ price covers the investment cost and that the sets Θ_g, Θ_1, and $\Theta_{g\o}$ must be non-empty, except for the boundary case. Thus

$$m(0, \theta_g) \geq I, \tag{D.4}$$

$$\theta_g < \theta_1 \leq \theta_{g\o}. \tag{D.5}$$

From (D.1), we have $m(0, \theta_g) < p_g$. Applying this to (D.3) shows that $\theta_1 < \theta_{g\o}$ always holds for the interior case $\theta_1 < 1$. The weak inequality in (D.5) holds as equality for the boundary case $\theta_1 = 1$.

Lemma D.1. *There exist $\underline{p}_g^{SMR} \leq \overline{p}_g^{SMR}$ such that (D.1) – (D.3) admit a unique $(\theta_g, \theta_1, \theta_{g\o})$ that satisfies (D.4) and (D.5) if and only if $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$.*

Proof. Note that (D.1) and (D.2) are equivalent to (C.11) and (C.17), respectively. Thus, using these conditions and proceeding similarly to the proof of Lemma C.4, one can show that there exist $\underline{p}_g^{SMR} \leq \overline{p}_g^{SMR}$ such that (D.1) and (D.2) define a unique (θ_g, θ_1) that satisfies $0 < \theta_g < \theta_1$ and (D.4) if and only if $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$. Furthermore, since $\theta_1 = m(0, \theta_g) + S < m(\theta_g, \theta_1) + S < p_g + S$, where the second inequality follows from (D.1), $\theta_{g\o}$ determined by (D.3) satisfies (D.5) for all $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$. *Q.E.D.*

For any $p_g \in [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$, let $\theta_g^{SMR}(p_g)$, $\theta_1^{SMR}(p_g)$, and $\theta_{g\o}^{SMR}(p_g)$ denote the marginal types θ_g, θ_1, and $\theta_{g\o}$ determined by (D.1) – (D.3). For expositional convenience, we may abbreviate $\theta_g^{SMR}(p_g)$, $\theta_1^{SMR}(p_g)$, and $\theta_{g\o}^{SMR}(p_g)$ to θ_g^{SMR}, θ_1^{SMR}, and $\theta_{g\o}^{SMR}$, respectively. In addition, define $P^{SMR} := [\underline{p}_g^{SMR}, \overline{p}_g^{SMR})$.

D.2 Proofs of Theorem 4 and Proposition 5 and 6

Theorem 4. *There exists a nonempty interval $P^{SMR} \subset (I, 1]$ of bailout terms such that (i) an SMR equilibrium exists with cutoffs $\theta_1 < \theta_0^*$ and $\theta_{g\o} = p_g + S$ if $p_g \in P^{SMR}$; and (ii) no other equilibria exist.*

Proof of Theorem 4.

Proof of Theorem 4-(i).

Consider $p_g \in P^{SMR}$ such that there exist $\theta_g = \theta_g^{SMR}(p_g), \theta_1 = \theta_1^{SMR}(p_g)$, and $\theta_{g\o} = \theta_{g\o}^{SMR}(p_g)$ that satisfy (D.4) and (D.5).

First, we show that it is optimal for each type of firms to play the prescribed equilibrium strategies. Consider $t = 2$ first. For firms accepting the bailout or holding out in $t = 1$, the price offer in $t = 2$ is $m(0, \theta_g)$. By (D.2), we have $\theta_1 \leq m(0, \theta_g) + S$, where the inequality holds for the case $\theta_1 = 1$. Thus, after accepting the bailout in $t = 1$, types $\theta \in [0, \theta_g]$ sell at price $m(0, \theta_g)$ in $t = 2$, but types $\theta > \theta_1$ do not sell in $t = 2$. For types selling to the market in $t = 1$, the price offer in $t = 2$ is $m(\theta_g, \theta_1)$. Since $\theta_1 \leq m(0, \theta_g) + S < m(\theta_g, \theta_1) + S$, types $\theta \in (\theta_g, \theta_1]$ sell in $t = 2$ after selling in $t = 1$. Consider $t = 1$ next. By (D.1), we have $p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S$. If $\theta_1 < 1$, then, by (D.2), we have $p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S < \theta + p_g + S \iff \theta > \theta_1$. If $\theta_1 = 1$, then $\theta + p_g + S \leq p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S$ for all $\theta \in [0, 1]$, where the inequality follows from (D.2). Thus, it is optimal for types $\theta \in [0, \theta_g] \cup (\theta_1, 1]$ to accept the bailout and types $\theta \in (\theta_g, \theta_1]$ to sell to the market. Lastly, since $\theta_{g\o} = (p_g + S) \wedge 1$ from (D.3), it is optimal for types $\theta > \theta_{g\o}$ not to sell.

Second, we show that it is optimal for buyers to make the equilibrium price offers. Consider $t = 2$ first. For firms not selling at price $m(\theta_g, \theta_1)$ in $t = 1$, buyers believe that their types are $\theta \in [0, \theta_g] \cup (\theta_1, 1]$. By Lemma C.8, any $p' < m(0, \theta_g)$ cannot be an equilibrium price offer in $t = 2$. Next, suppose a buyer in $t = 1$ deviates and offers $p' > m(0, \theta_g)$. Since $p' + S > \theta_1$ from (D.2), types $\theta \in [0, \theta_g] \cup (\theta_1, (p' + S) \wedge 1]$ will sell at price p'. Then, the deviating buyer's payoff is $\mathring{m}(0, \theta_g, \theta_1, (p' + S) \wedge 1) - p'$. However, since $\lim_{p' \searrow (\theta_1 - S)} \mathring{m}(0, \theta_g, \theta_1, (p' + S) \wedge 1) = m(0, \theta_g)$ and $\frac{\partial}{\partial \theta} \mathring{m}(0, \theta_g, \theta_1, \theta) \leq 1$ for all $\theta \geq \theta_1$ from Lemma A.1-(iii), we have $\mathring{m}(0, \theta_g, \theta_1, (p' + S) \wedge 1) - p' < 0$

for any $p' > \theta_1 - S$. Therefore, buyers optimally offer $m(0, \theta_g)$ to all firms that do not sell to the market in $t = 1$. For firms selling to the market in $t = 1$, buyers believe their types are $\theta \in (\theta_g, \theta_1]$. Since $\theta_1 \leq m(0, \theta_g) + S < m(0, \theta_1) + S$, we have $\theta_1 \leq \theta_0^* = \gamma(0) < \gamma(\theta_g)$. Hence, by Lemma C.8, buyers optimally offer $m(\theta_g, \theta_1)$. Lastly, proceeding similarly as in the characterization of MR2 equilibrium in the proof of Theorem 3, one can show that it is optimal for buyers to offer $m(\theta_g, \theta_1)$ in $t = 1$.

Proof of Theorem 4-(ii). We show below that no cutoff structure but the SMR type is possible in equilibrium.

Case 1. $\Theta_g \neq \emptyset$, $\Theta_1 \neq \emptyset$, $\Theta_{g\o} \neq \emptyset$, *and* $\Theta_2 \neq \emptyset$

This equilibrium is characterized by $0 < \theta_g < \theta_1 < \theta_{g\o} < \theta_2 \leq 1$. Let p_2 be the $t = 2$ price offer to the firms that do not sell to the market in $t = 1$. Under the secret bailout, such p_2 will be offered to types $\theta \in [0, \theta_g] \cup (\theta_1, 1]$. Since types $\theta \in (\theta_1, \theta_{g\o}]$ choose not to sell in $t = 2$, we must have $\theta_{g\o} \geq p_2 + S$. But types $\theta \in (\theta_{g\o}, \theta_2]$ prefer selling in $t = 2$, hence $\theta_2 \leq p_2 + S$, a contradiction.

Case 2. $\Theta_g \neq \emptyset$, $\Theta_1 = \emptyset$, $\Theta_{g\o} = \emptyset$, *and* $\Theta_2 \neq \emptyset$

This equilibrium is characterized by $0 < \theta_g < \theta_2 \leq 1$. In this case, types $\theta \in [0, \theta_g]$ accept the bailout in $t = 1$ and types $\theta \in [0, \theta_2]$ sell to the $t = 2$ market at price $m(0, \theta_2)$. When the $t = 1$ market is open after the bailout, buyers believe that types $\theta \in (\theta_g, 1]$ are available for asset trade. That is, a buyer at the $t = 1$ market can make a positive profit by offering $p' = m(\theta_g, \gamma(\theta_g)) - \varepsilon$ for some $\varepsilon > 0$, a contradiction.

Case 3. $\Theta_g \neq \emptyset$, $\Theta_1 \neq \emptyset$, $\Theta_{g\o} = \emptyset$, *and* $\Theta_2 = \emptyset$

This equilibrium is characterized by $0 < \theta_g < \theta_1 \leq 1$, where types $\theta \in [0, \theta_g]$ accept the bailout in $t = 1$ and sell at price $m(0, \theta_g)$ in $t = 2$, and types $\theta \in (\theta_g, \theta_1]$ sell at price $m(\theta_g, \theta_1)$ in both periods. For these strategies to be optimal, we must have $p_g + m(0, \theta_g) + 2S = 2m(\theta_g, \theta_1) + 2S$, or equivalently $p_g = 2m(\theta_g, \theta_1) - m(0, \theta_g)$. Since type θ_1 firm weakly prefers selling in both periods to not selling in either period, we must have $2m(\theta_g, \theta_1) + 2S = 2\theta_1$, or equivalently, $\theta_1 = m(\theta_g, \theta_1) + S$. However, since $p_g > m(\theta_g, \theta_1)$, we have $p_g + S > m(\theta_g, \theta_1) + S \geq \theta_1$. That is, type θ_1 firm will get a strictly higher payoff by accepting the bailout, a contradiction.

Case 4. $\Theta_g \neq \emptyset$, $\Theta_1 = \emptyset$, $\Theta_{g\o} \neq \emptyset$, *and* $\Theta_2 \neq \emptyset$

This case is similar to Case 1.

Case 5. $\Theta_g \neq \emptyset$, $\Theta_1 = \emptyset$, $\Theta_{g\o} \neq \emptyset$, *and* $\Theta_2 = \emptyset$

This equilibrium is characterized by $0 < \theta_g < \theta_{g\o} < 1$, where types $\theta \in [0, \theta_{g\o}]$ accept the bailout, and only a subset of these types $\theta \in [0, \theta_g]$ sell to the market at price $m(0, \theta_g)$ in $t = 2$. When the market is open in $t = 1$ after the bailout, types $\theta \in (\theta_{g\emptyset}, 1]$ are available for asset trade. Thus, a buyer can make a positive profit by deviating and offering a price $p' = m(\theta_{g\emptyset}, \gamma(\theta_{g\emptyset})) - \varepsilon$ for some $\varepsilon > 0$, a contradiction.

Case 6. $\Theta_g \neq \emptyset$, $\Theta_1 \neq \emptyset$, $\Theta_{g\emptyset} = \emptyset$, and $\Theta_2 \neq \emptyset$:

This equilibrium is characterized by $0 < \theta_g < \theta_1 < \theta_2 \leq 1$, where types $\theta \in [0, \theta_g]$ accept the bailout in $t = 1$ and sell at price $\mathring{m}(0, \theta_g, \theta_1, \theta_2)$ in $t = 2$, types $\theta \in (\theta_g, \theta_1]$ sell to the market at price $m(\theta_g, \theta_1)$ in both periods, and types $\theta \in (\theta_1, \theta_2]$ sell only in $t = 2$ at price $\mathring{m}(0, \theta_g, \theta_1, \theta_2)$. Thus the indifference conditions for the marginal types are

$$p_g + \mathring{m}(0, \theta_g, \theta_1, \theta_2) = 2m(\theta_g, \theta_1), \tag{D.6}$$

$$2m(\theta_g, \theta_1) + 2S = \theta_1 + \mathring{m}(0, \theta_g, \theta_1, \theta_2) + S, \tag{D.7}$$

$$\theta_2 = \mathring{\gamma}(0, \theta_g, \theta_1). \tag{D.8}$$

From (D.6) and (D.7), we have

$$\theta_1 = p_g + S. \tag{D.9}$$

To show that this equilibrium cannot exist, we prove that there exists $p'_1 \neq m(\theta_g, \theta_1)$ which gives a positive profit to market buyers in $t = 1$. When the market is open in $t = 1$, types $\theta \in (\theta_g, 1]$ are available for asset sales. Thus firms accepting p'_1 offered by a deviating buyer are assigned the off-the-path belief consistent with D1 equal to $\theta = \theta_g$. This implies that, after selling at p'_1 in $t = 1$, these firms can sell at price θ_g in $t = 2$ if $\theta_g \geq I$, leading to the total payoff $p'_1 + S + \max\{\theta, \theta_g + S\}$. If $\theta_g < I$, then they do not sell in $t = 2$, hence the total payoff $p'_1 + S + \theta$.

There are three possibilities: $\theta_g < I$, $\theta_g \in [I, \theta_1 - S)$, or $\theta_g \geq \theta_1 - S$. First, suppose $\theta_g < I$. Since type-θ firm sells at price p'_1 if and only if $p'_1 + S + \theta \geq p_g + \mathring{m}(0, \theta_g, \theta_1, \theta_2) + 2S$, types $\theta \in [\theta'_1, (p'_1 + S) \wedge 1]$ sell at price p'_1, where

$$\theta'_1 := (\mathring{m}(0, \theta_g, \theta_1, \theta_2) - p'_1 + (p_g + S)) \vee \theta_g = (\mathring{m}(0, \theta_g, \theta_1, \theta_2) - p'_1 + \theta_1) \vee \theta_g.$$

By offering p'_1, the buyer earns $m(\theta'_1, (p'_1 + S) \wedge 1) - p'_1$. Since $m(\theta_1, \theta_2) - \mathring{m}(0, \theta_g, \theta_1, \theta_2) > 0$, $\lim_{p'_1 \to \mathring{m}(0,\theta_g,\theta_1,\theta_2)+} (p'_1 + S) \wedge 1 = \theta_2$ from (D.8), and $\lim_{p'_1 \to \mathring{m}(0,\theta_g,\theta_1,\theta_2)+} \theta'_1 = \theta_1$ from (D.9), there exists $p'_1 > \mathring{m}(0, \theta_g, \theta_1, \theta_2)$ such that $m(\theta'_1, (p'_1 + S) \wedge 1) - p'_1 > 0$.

Next, suppose $\theta_g \geq I$ but $\theta_g + S < \theta_1$. Since $\theta_g + S < \theta_1 = p_g + S$, we have $p'_1 + S + \theta \geq$

$p_g + \mathring{m}(0, \theta_g, \theta_1, \theta_2) + 2S \iff \theta \geq (\mathring{m}(0, \theta_g, \theta_1, \theta_2) - p_1') + \theta_1$. Proceeding similarly as in the previous case, one can show that there exists $p_1' > \mathring{m}(0, \theta_g, \theta_1, \theta_2)$ that gives a positive profit to the deviating buyer.

Lastly, suppose $\theta_g + S \geq \theta_1$, which is equivalent to $\theta_g \geq p_g$ by (D.9). Types $\theta \in (\theta_g, \theta_1]$ sell at p_1' if and only if

$$\mathring{m}(0, \theta_g, \theta_1, \theta_2) + p_g + 2S \leq p_1' + \theta_g + 2S \iff p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2) - (\theta_g - p_g).$$

Similarly, types $\theta \in (\theta_1, \theta_g + S]$ sell at p_1' if and only if

$$\mathring{m}(0, \theta_g, \theta_1, \theta_2) + S + (\theta_g + S) \leq p_1' + \theta_g + 2S \iff p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2).$$

Furthermore, types $\theta > \theta_g + S$ sell at p_1' if and only if

$$p_1' + S + \theta \geq \theta + \mathring{m}(0, \theta_g, \theta_1, \theta_2) + S \iff p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2).$$

Thus, by offering $p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2)$, the buyer's expected payoff is $m(\theta_g, (p_1' + S) \wedge 1) - p_1'$. Since $\mathring{m}(0, \theta_g, \theta_1, \theta_2) < m(\theta_g, \theta_2)$ and $\lim_{p_1' \to m(0,\theta_g,\theta_1,\theta_2)+}(p_1' + S) \wedge 1 = \theta_2$ from (D.8), there exists $p_1' \geq \mathring{m}(0, \theta_g, \theta_1, \theta_2)$ such that $m(\theta_g, (p_1' + S) \wedge 1) - p_1' > 0$. Thus the prescribed equilibrium strategy cannot be optimal for buyers in $t = 1$. *Q.E.D.*

Proposition 5.

(i) (Front-loading of trade) An SMR equilibrium, if it exists, supports a larger trade volume in $t = 1$ but a smaller trade volume in $t = 2$ than an MR equilibrium for the same p_g.

(ii) Given $p_g \in P^{SMR}$, the total trade volume supported in the SMR equilibrium is the same as that in the MR2 equilibrium if p_g admits the MR2 equilibrium; but the comparison is ambiguous if p_g admits the MR1 equilibrium.

Proof of Proposition 5.

Proof of Proposition 5-(i).

First, suppose $p_g \in P^{SMR}$ admits the MR1 equilibrium under transparency. Since $\theta_1^{MR1}(p_g) \leq m(0, \theta_g^{MR1}(p_g)) + S$ from Claim 1 within the proof of Lemma C.5 and $p_g > m(0, \theta_g^{MR1}(p_g))$ from (C.11), we have $\theta_1^{MR1}(p_g) < p_g + S$, and thus $F(\theta_1^{MR1}(p_g)) \leq F(\theta_{g\o}^{SMR}(p_g))$. Therefore, the

SMR equilibrium arising from the same p_g yields a larger total trade volume in $t=1$ than the MR1 equilibrium. Furthermore, since $p_g \leq m(\theta_1^{MR1}, \gamma(\theta_1^{MR1}))$ from (C.15), we have

$$\theta_1^{SMR}(p_g) < \theta_{g\o}^{SMR}(p_g) = (p_g + S) \wedge 1 \leq (m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g))) + S) \wedge 1 = \gamma(\theta_1^{MR1}(p_g)).$$

Hence, we have $F(\theta_1^{SMR}(p_g)) < F(\gamma(\theta_1^{MR1}(p_g)))$, so the SMR equilibrium yields a smaller trade volume in $t=2$ than the MR1 equilibrium.

Second, suppose $p_g \in P^{SMR}$ admits the MR2 equilibrium under transparency. Since (C.11) and (C.17) are equivalent to (D.1) and (D.2), respectively, we have $(\theta_g^{SMR}(p_g), \theta_1^{SMR}(p_g)) = (\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$. Since $p_g = m(\theta_{g\o}^{MR2}(p_g), \gamma(\theta_{g\o}^{MR2}(p_g)))$ from (C.18) and $\theta_{g\o}^{SMR}(p_g) = (p_g + S) \wedge 1$ from (D.3), we have $\theta_{g\o}^{SMR}(p_g) = \gamma(\theta_{g\o}^{MR2}(p_g))$. Hence, we have $F(\theta_{g\o}^{SMR}) \geq F(\theta_{g\o}^{MR2})$ and $F(\theta_1^{SMR}) \leq F(\theta_1^{MR2}) + (F(\gamma(\theta_{g\o}^{MR2})) - F(\theta_{g\o}^{MR2}))$, which is the desired result.

Proof of Proposition 5-(ii).

First, suppose $p_g \in P^{SMR}$ admits the MR2 equilibrium under transparency. As shown in the proof of Proposition 5-(i), we have $(\theta_g^{SMR}(p_g), \theta_1^{SMR}(p_g)) = (\theta_g^{MR2}(p_g), \theta_1^{MR2}(p_g))$. Moreover, $\theta_{g\o}^{SMR}(p_g) = \gamma(\theta_{g\o}^{MR2}(p_g))$ by (C.18) and (D.3). Since the total trade volume is $F(\theta_{g\o}^{SMR}(p_g)) + F(\theta_1^{SMR}(p_g))$ in SMR equilibrium and $F(\gamma(\theta_{g\o}^{MR2}(p_g))) + F(\theta_1^{MR2}(p_g))$ in MR2 equilibrium, we have the desired result.

Second, suppose some $p_g \in P^{SMR}$ admits the MR1 equilibrium. Recall the functions $\tilde{\theta}_1(\theta_g)$, $\theta_1^I(\theta_g)$, and $\theta_1^{II}(\theta_g)$ corresponding to (C.11), (C.12), and (C.17), respectively. As shown in the proof of Lemma C.3, $(\theta_g^{MR1}(p_g), \theta_1^{MR1}(p_g))$ is determined as a unique point of intersection between two curves $\tilde{\theta}_1(\theta_g)$ and $\theta_1^I(\theta_g)$. Given the equivalence of (D.1) and (D.2) with (C.11) and (C.17), $(\theta_g^{SMR}(p_g), \theta_1^{SMR}(p_g))$ is defined as a unique point of intersection between two curves $\tilde{\theta}_1(\theta_g)$ and $\theta_1^{II}(\theta_g)$, as seen in the proof of Lemma C.4. Since p_g supports the MR1 equilibrium, we have $\theta_1^{MR1}(p_g) \leq \theta_1^{II}(\theta_g^{MR1}(p_g)) = \theta_1^{SMR}(p_g)$, as shown in the proof of Lemma C.5. Moreover, by (C.15), we have $\theta_{g\o}^{SMR}(p_g) \leq (p_g + S) \wedge 1 \leq (m(\theta_1^{MR1}(p_g), \gamma(\theta_1^{MR1}(p_g))) + S) \wedge 1 \leq \gamma(\theta_1^{MR1}(p_g))$. From these observations, the comparison of the total trade volume is ambiguous. *Q.E.D.*

Proposition 6. *Suppose that the government offers a secret bailout at $p_g \geq \max\{I, p_1^*\}$ and further shuts down the $t=1$ market. Then in equilibrium,*

(i) firms with types $\theta \leq p_g + S$ accept the bailout in $t=1$ and those with $\theta \leq \theta_0^*$ sell to the market in $t=2$;

(ii) the total trade volume in this equilibrium is larger than in the SMR equilibrium, whenever the latter exists for the same p_g.

Proof of Proposition 6.

Proof of Proposition 6-(i).

Fix any $p_g \geq \max\{p_1^*, I\}$. Since the market is shut down in $t=1$, firms have only two choices available in $t=1$: either accepting the bailout or rejecting it. In $t=2$, buyers have the same belief as the prior regardless of the firms' action taken in $t=1$, and thus they offer the price p_0^* in $t=2$. Given p_0^*, only types $\theta \leq (p_0^*+S)\wedge 1 = \theta_0^*$ sell in $t=2$. Since this $t=2$ price is independent of firms' actions in $t=1$, firms accept the bailout if and only if $\theta \leq (p_g+S)\wedge 1$.

Proof of Proposition 6-(ii).

Fix any $p_g \in P^{SMR} \cap [p_1^* \vee I, 1]$. Since $\theta_{g\o}^{SMR}(p_g) = (p_g+S)\wedge 1$ from (D.3), the SMR equilibrium yields the same trade volume in $t=1$ as that under the market shutdown in $t=1$. On the other hand, since $\theta_1^{SMR}(p_g) \leq \theta_0^*$ from (D.2) and (D.5), the SMR equilibrium yields smaller total trade volume in $t=2$ than when the market is shut down in $t=1$. *Q.E.D.*

E Proofs for Section 6

Theorem 5. *Let $\mathcal{M}$ denote the set of mechanisms that satisfy the restrictions imposed above. Then, the following holds:*

(i) If $M=(q,t)\in\mathcal{M}$, then $q(\cdot)$ is nonincreasing, and $q(\theta)\leq 1$ for all $\theta > \theta_0^*$, where θ_0^* is the highest type that sells its asset in the one-shot model without a bailout.

(ii) [Revenue Equivalence] If $M=(q,t)$ and $M'=(q',t')$ both in $\mathcal{M}$ have $q=q'$, then $W(M)=W(M')$. In other words, an equilibrium allocation pins down the welfare, expressed as follows:

$$\int_0^1 \left[J(\theta)q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda\frac{F(\theta)}{f(\theta)}\right)\right] f(\theta)d\theta, \tag{E.1}$$

where

$$J(\theta) := (1+\lambda)S - \lambda\frac{F(\theta)}{f(\theta)}.$$

(iii) Consider two possible mechanisms, labeled A and B, (possibly associated with different levels of p_g or by different disclosure policies) such that equilibrium $i=A,B$ induces trade

volume $q_i(\cdot)$ across the two periods. Suppose

$$\int_0^1 q_A(\theta)f(\theta)d\theta = \int_0^1 q_B(\theta)f(\theta)d\theta$$

but there exists $\tilde{\theta} \in (0,1)$ such that $q_A(\theta) \geq q_B(\theta)$ for $\theta \leq \tilde{\theta}$ and $q_A(\theta) \leq q_B(\theta)$ for $\theta \geq \tilde{\theta}$. Then, equilibrium A yields higher welfare than equilibrium B, strictly so if $q_A(\theta) \neq q_B(\theta)$ for a positive measure of θ's.

Proof of Theorem 5.

Proof of Theorem 5-(i). As is standard, the monotonicity of $q(\cdot)$ follows from (IC). Fix any mechanism $\mathcal{M}$ that satisfies (IC) and (IR) for every $\theta \in [0,1]$ and all restrictions stated in the main text. For convenience of exposition, let $t_1(\theta)$ and $t_2(\theta)$ be respective transfers given to the firms in period 1 and 2 if they report their types as θ. Recall that $t_1(\cdot)$ can be made by the government or the private buyers, but $t_2(\cdot)$ is made only by the buyers. Moreover, notice that $\mathrm{E}[\theta|t_2(\theta) = t] = t$ for any $t \geq I$ such that $Pr(\theta|t_2(\theta) = t) > 0$, which follows from the zero-profit condition.

Define $\hat{\theta} := \sup\{\theta : q(\theta) = 2\}$ and $\check{\theta} := \sup\{\theta : q(\theta) = 1\}$. In addition, define $\bar{t} \geq I$ as total transfer offered to the firms reporting their types as $\theta \in (\hat{\theta}, \check{\theta}]$. By the non-rationing restriction, $q(\theta) = 1$ for all $\theta \in (\hat{\theta}, \check{\theta}]$. Hence, from ($IC$), $t(\theta) = \bar{t}$ for all $\theta \in (\hat{\theta}, \check{\theta}]$. Moreover, define $\underline{t} := \hat{\theta} - S$. Since $\check{\theta} \leq \bar{t} + S$ in equilibrium, we have $\bar{t} \geq \underline{t}$.

Before proving $q(\theta) \leq 1$ for all $\theta \geq \theta_0^*$, we first observe

$$t_1(\theta) \leq \bar{t} \;\text{ for all }\; \theta \in [0, \hat{\theta}]. \tag{E.2}$$

To prove this, suppose there exists $\tilde{\theta} \in [0, \hat{\theta}]$ such that $t_1(\tilde{\theta}) > \bar{t}$. Since $t_1(\tilde{\theta}) + S + \theta > \bar{t} + S + \theta$, every type $\theta \in (\hat{\theta}, \check{\theta}]$ will misreport its type as $\tilde{\theta}$, a contradiction.

We now prove $q(\theta) \leq 1$ for all $\theta \geq \theta_0^*$. By definition of $\hat{\theta}$, it suffices to show $\hat{\theta} \leq \theta_0^*$, or equivalently, $\hat{\theta} \leq m(0, \hat{\theta}) + S$. To show $\hat{\theta} \leq \theta_0^*$, we decompose the set $[0, \hat{\theta}] = \{\theta : q(\theta) = 2\}$ into two disjoint subsets $\hat{\Theta}_l$ and $\hat{\Theta}_h$ such that types $\theta \in \hat{\Theta}_l$ receive $t_2(\theta) < \bar{t}$ in $t = 2$, and types $\theta \in \hat{\Theta}_h$ receive $t_2(\theta) \geq \bar{t}$.

<u>Step 1.</u> $\underline{t} \leq \mathrm{E}[\theta|\theta \in \hat{\Theta}_l]$.

Fix any $\theta \in \hat{\Theta}_l$. By truthful reporting, type θ earns the payoff $t_1(\theta) + t_2(\theta) + 2S$. Since this type can get the payoff $t_1(\hat{\theta}) + t_2(\hat{\theta}) + 2S$ by reporting its type as $\hat{\theta}$, (IC) requires $t_1(\theta) + t_2(\theta) + 2S \geq t_1(\hat{\theta}) + t_2(\hat{\theta}) + 2S$. Since type $\hat{\theta}$'s (IC) implies the reverse inequality, we have

$t_1(\theta)+t_2(\theta)=t_1(\hat{\theta})+t_2(\hat{\theta})$ for any $\theta\in\hat{\Theta}_l$. Since type $\hat{\theta}$ is indifferent between $q=2$ and $q=1$, we must have $t_1(\hat{\theta})+t_2(\hat{\theta})+2S=\hat{\theta}+\bar{t}+S$. Given $\hat{\theta}=\underline{t}+S$, we have $t_1(\hat{\theta})+t_2(\hat{\theta})=\underline{t}+\bar{t}$, which implies $t_1(\theta)+t_2(\theta)=\underline{t}+\bar{t}$. From (E.2), it then follows $t_2(\theta)=\underline{t}+(\bar{t}-t_1(\theta))\geq\underline{t}$. Furthermore, the zero-profit condition for the buyers in $t=2$ implies $\mathrm{E}[\theta|\theta\in\hat{\Theta}_l]=\mathrm{E}[t_2(\theta)|\theta\in\hat{\Theta}_l]$. Hence, we have $\mathrm{E}[\theta|\theta\in\hat{\Theta}_l]=\mathrm{E}[t_2(\theta)|\theta\in\hat{\Theta}_l]\geq\mathrm{E}[\underline{t}|\theta\in\hat{\Theta}_l]=\underline{t}$.

Step 2. $\underline{t}\leq\mathrm{E}[\theta|\theta\in\hat{\Theta}_h]$.

We can decompose $\hat{\Theta}_h$ further into the two disjoint subsets $\hat{\Theta}_h^{>\bar{t}}:=\{\theta\in\hat{\Theta}_h|t_2(\theta)>\bar{t}\}$ and $\hat{\Theta}_h^{=\bar{t}}:=\{\theta\in\hat{\Theta}_h|t_2(\theta)=\bar{t}\}$. We show below $\underline{t}\leq\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{>\bar{t}}]$ and $\underline{t}\leq\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}]$.

First, consider $\hat{\Theta}_h^{>\bar{t}}$. Since $t_2(\theta)\leq\bar{t}$ if $\theta\in\hat{\Theta}_l\cup\hat{\Theta}_h^{=\bar{t}}$ and $t_2(\theta)=\bar{t}$ if $\theta\in(\hat{\theta},\check{\theta}]$ (if such θ sells in $t=2$), only types $\theta\in\hat{\Theta}_l^{>\bar{t}}$ will sell at a price higher than $\bar{t}$ in $t=2$. From the zero-profit condition for the buyers in $t=2$, it then follows $\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{>\bar{t}}]=\mathrm{E}[t_2(\theta)|\theta\in\hat{\Theta}_h^{>\bar{t}}]$. Since $t_2(\theta)>\bar{t}\geq\underline{t}$ for all $\theta\in\hat{\Theta}_h^{>\bar{t}}$, we have $\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{>\bar{t}}]=\mathrm{E}[t_2(\theta)|\theta\in\hat{\Theta}_h^{>\bar{t}}]>\bar{t}\geq\underline{t}$.

Next, consider $\hat{\Theta}_h^{=\bar{t}}$. Suppose to the contrary $\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}]<\underline{t}$. Let $\check{\Theta}$ be the set of types $\theta\in(\hat{\theta},\check{\theta}]$ selling in $t=2$. Since these types receive $t_2(\theta)=\bar{t}$, we have $\mathrm{E}[t_2(\theta)|\hat{\Theta}_h^{=\bar{t}}\cup\check{\Theta}]=\bar{t}$. Moreover, since $0\leq Pr(\theta\in\check{\Theta})\leq Pr(\theta\in(\hat{\theta},\check{\theta}])$, there exists $\overline{\theta}\in(\hat{\theta},\check{\theta}]$ such that $Pr(\theta\in\check{\Theta})=Pr(\theta\in[\overline{\theta},\check{\theta}])$ and $\mathrm{E}[\theta|\theta\in[\overline{\theta},\check{\theta}]]\geq\mathrm{E}[\theta|\theta\in\check{\Theta}]$. By Lemma A.1-(i), we have

$$
\begin{aligned}
\frac{\partial}{\partial y}\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}\cup[\overline{\theta},y]] &= \frac{f(y)}{\{F(y)-F(\overline{\theta})\}+Pr(\theta\in\hat{\Theta}_h^{=\bar{t}})}(y-m(\overline{\theta},y)) \\
&\quad -\frac{f(y)}{[\{F(y)-F(\overline{\theta})\}+Pr(\theta\in\hat{\Theta}_h^{=\bar{t}})]^2}\int_{\theta\in\hat{\Theta}_h^{=\bar{t}}}\theta dF(\theta) \\
&\leq \frac{f(y)}{F(y)-F(\overline{\theta})}(y-m(\overline{\theta},y)) \\
&= \frac{\partial}{\partial y}m(\overline{\theta},y) \\
&< 1.
\end{aligned}
\tag{E.3}
$$

Since $\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}]<\underline{t}$ and $\underline{t}+S=\hat{\theta}\leq\overline{\theta}$, we have $\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}]+S<\overline{\theta}$. Combining this inequality with (E.3), we have $\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}\cup[\overline{\theta},\check{\theta}]]+S<\check{\theta}$. Since $\check{\theta}\leq\bar{t}+S$, we have $\bar{t}+S\geq\check{\theta}>\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}\cup[\overline{\theta},\check{\theta}]]+S\geq\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}\cup\check{\Theta}]+S$, where the last inequality follows from $\mathrm{E}[\theta|\theta\in\check{\Theta}]\leq\mathrm{E}[\theta|\theta\in[\overline{\theta},\check{\theta}]]$. This implies $\bar{t}=\mathrm{E}[t_2(\theta)|\theta\in\hat{\Theta}_h^{=\bar{t}}\cup\check{\Theta}]>\mathrm{E}[\theta|\theta\in\hat{\Theta}_h^{=\bar{t}}\cup\check{\Theta}]$, which contradicts the zero-profit condition for the buyers in $t=2$.

Step 3. $\hat{\theta}\leq\theta_0^*$.

Since $\underline{t}\leq\mathrm{E}[\theta|\theta\in\hat{\Theta}_l]$ from Step 1, $\underline{t}\leq\mathrm{E}[\theta|\theta\in\hat{\Theta}_h]$ from Step 2, and $\hat{\Theta}_l\cup\hat{\Theta}_h=[0,\hat{\theta}]$, we have $\underline{t}\leq\mathrm{E}[\theta|\theta\leq\hat{\theta}]=m(0,\hat{\theta})$, and thus $\hat{\theta}=\underline{t}+S\leq m(0,\hat{\theta})+S$, or equivalently, $\hat{\theta}\leq\theta_0^*$.

Proof of Theorem 5-(ii). First, recall

$$t(\theta) = u^M(\theta) - \theta(2 - q(\tilde{\theta})) - Sq(\tilde{\theta}). \tag{E.4}$$

Next, the envelope theorem applied to (IC) along with $u^M(1) = 2$ gives us

$$u^M(\theta) = u^M(1) - \int_\theta^1 (2 - q(s))ds = 2 - \int_\theta^1 (2 - q(s))ds. \tag{E.5}$$

Substituting (E.4) and (E.5) into the welfare and integrating by parts leads to

$$\begin{aligned}
W(M) &= \int_0^1 \left[u^M(\theta) + (1+\lambda)\theta q(\theta) - (1+\lambda)t(\theta)\right] f(\theta)d\theta. \\
&= \int_0^1 \left[u^M(\theta) + (1+\lambda)\theta q(\theta) - (1+\lambda)(u^M(\theta) - \theta(2 - q(\theta)) - Sq(\theta))\right] f(\theta)d\theta \\
&= \int_0^1 \left[(1+\lambda)Sq(\theta) - \lambda u^M(\theta) + 2(1+\lambda)\theta\right] f(\theta)d\theta \\
&= \int_0^1 \left[\left((1+\lambda)S - \lambda\frac{F(\theta)}{f(\theta)}\right) q(\theta) - 2\lambda + 2\left((1+\lambda)\theta + \lambda\frac{F(\theta)}{f(\theta)}\right)\right] f(\theta)d\theta.
\end{aligned}$$

This gives us (E.1). Revenue equivalence follows also from the observation that the welfare depends only on the allocation rule q.

Proof of Theorem 5-(iii). The welfare difference between the two equilibria is

$$\begin{aligned}
W_A - W_B &= \int_0^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&= \int_0^{\tilde{\theta}} J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta + \int_{\tilde{\theta}}^1 J(\theta)[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&> \int_0^{\tilde{\theta}} J(\tilde{\theta})[q_A(\theta) - q_B(\theta)]f(\theta)\theta + \int_{\tilde{\theta}}^1 J(\tilde{\theta})[q_A(\theta) - q_B(\theta)]f(\theta)\theta \\
&= J(\tilde{\theta}) \int_0^1 (q_A(\theta) - q_B(\theta))f(\theta)d\theta = 0,
\end{aligned}$$

where the inequality follows from the fact that $q_A(\theta) \geq q_B(\theta)$ for $\theta \leq \tilde{\theta}$ and $q_A(\theta) \leq q_B(\theta)$ for $\theta \geq \tilde{\theta}$ and that J is decreasing. The inequality must be strict if $q_A(\theta)$ and $q_B(\theta)$ differ on a positive measure of θ's, since J is strictly decreasing. *Q.E.D.*

Proposition 7. *The equilibria are compared as follows.*

(i) Given a transparent bailout policy, an equilibrium with $t = 1$ market shutdown dominates in welfare an equilibrium without $t = 1$ market shutdown.

(ii) Given a secret bailout policy, an equilibrium with $t = 1$ market shutdown dominates in welfare an equilibrium without $t = 1$ market shutdown.

(iii) With $t = 1$ market shutdown, an equilibrium under secrecy dominates in welfare an equilibrium under transparency.

Proof of Proposition 7.

Proof of Proposition 7-(i).

Let $q_T(\theta) \in \{0, 1, 2\}$ denote the total quantity of assets sold by type-θ firm across the two periods in each alternative type of equilibria $T \in \{SBS, MBS, MR1, MR2, sd\}$ (we use notation $T = sd$ to label each alternative type of equilibria under the shutdown of the market in $t = 1$).

We first show that the $t = 1$ market shutdown yields at least the same welfare as either SBS or MBS does. From Section C.1.5, any $p_g \in P^{SBS}$ admits an equilibrium with the $t = 1$ market shutdown characterized by the marginal type $\theta_g^{sd}(p_g) = \theta_{g\emptyset}^{SBS}(p_g)$, where we have $q_{sd}(\theta) = q_{SBS}(\theta)$ for all $\theta \in [0, 1]$. Similarly, any $p_g \in P^{MBS}$ yields an equilibrium with the $t = 1$ market shutdown characterized by $\theta_g^{sd}(p_g) = \theta_g^{MBS}(p_g)$, where $q_{sd}(\theta) = q_{MBS}(\theta)$ for all $\theta \in [0, 1]$.

We next compare welfare under either an MR1 or MR2 equilibrium with that under the $t = 1$ market shutdown. First, fix $p_g \in P^{MR}$ that admits an MR1 equilibrium. As shown in the proof of Proposition 3, the same p_g admits an equilibrium with the $t = 1$ market shutdown characterized by $\theta_g^{sd}(p_g) \geq \theta_1^{MR1}(p_g)$, where (C.7) defines $\theta_g^{sd}(p_g)$. Since $\theta_g^{sd}(p_g)$ is increasing in p_g, there exists $p_g' \leq p_g$ such that $\theta_g^{sd}(p_g') = \theta_1^{MR1}(p_g)$, and thus $q_{sd}(\theta) = q_{MR1}(\theta)$ for all $\theta \in [0, 1]$.

Next, fix $p_g \in P^{MR}$ that admits an MR2 equilibrium, which yields

$$q_{MR2}(\theta) = \begin{cases} 2 & \text{if } \theta \leq \theta_1^{MR2}, \\ 1 & \text{if } \theta \in (\theta_1^{MR2}, \gamma(\theta_{g\emptyset}^{MR2})], \\ 0 & \text{if } \theta > \gamma(\theta_{g\emptyset}^{MR2}). \end{cases}$$

As shown in the proof of Proposition 3, the same p_g admits the equilibrium with the $t = 1$ market shutdown characterized by $\theta_g^{sd}(p_g) \geq \theta_{g\emptyset}^{MR2}(p_g)$, where $\theta_g^{sd}(p_g)$ is determined by either (C.7) or

$p_g = m(\theta_g, \gamma(\theta_g))$. This equilibrium yields

$$q_{sd}(\theta) = \begin{cases} 2 & \text{if } \theta \le \theta_g^{sd} \wedge \theta_0^*, \\ 1 & \text{if } \theta \in (\theta_g^{sd} \wedge \theta_0^*, \gamma(\theta_g^{sd})], \\ 0 & \text{if } \theta > \gamma(\theta_g^{sd}). \end{cases}$$

As shown in Section C.1.5, $\theta_g^{sd}(p_g)$ is increasing in p_g. This implies that there exists an equilibrium under the $t = 1$ market shutdown for some $p_g' \le p_g$ such that $\int_0^1 q_{sd}(\theta)dF(\theta) = \int_0^1 q_{MR2}(\theta)dF(\theta)$. Since $\theta_1^{MR2}(p_g) < \theta_0^*$ and $\theta_1^{MR2}(p_g) < \theta_{g\emptyset}^{MR2}(p_g)$ from (C.17) and (C.20), we have $\theta_1^{MR2}(p_g) < \theta_g^{sd}(p_g') < \theta_{g\emptyset}^{MR2}(p_g)$. These inequalities imply that $q_{MR2}(\theta) \le q_{sd}(\theta)$ if $\theta \le \theta_g^{sd}(p_g')$, where $q_{MR2}(\theta) < q_{sd}(\theta)$ for all $\theta \in (\theta_1^{MR2}(p_g), \theta_g^{sd}(p_g')]$; and $q_{MR2}(\theta) \ge q_{sd}(\theta)$ otherwise, where $q_{MR2}(\theta) > q_{sd}(\theta)$ for all $\theta \in (\gamma(\theta_g^{sd}(p_g')), \gamma(\theta_{g\emptyset}^{MR2}(p_g))]$. Hence, by Theorem 5-(iii), the equilibrium with the $t = 1$ market shutdown arising from p_g' yields higher welfare than the MR2 equilibrium arising from p_g.

Proof of Proposition 7-(ii).

Similar to the proof of Proposition 7-(i), let $q_T(\theta) \in \{0, 1, 2\}$ denote the total quantity of assets sold by type-θ firm over the two periods in each alternative type of equilibria $T \in \{SMR, sd\}$.

First, suppose $p_g \ge p_0^*$ admits an SMR equilibrium. In this equilibrium, we have $q_{SMR}(\theta) = 2$ for all $\theta \in [0, \theta_1^{SMR}(p_g)]$, $q_{SMR}(\theta) = 1$ for all $\theta \in (\theta_1^{SMR}(p_g), \theta_{g\emptyset}^{SMR}(p_g)]$, and $q_{SMR}(\theta) = 0$ otherwise. When the market is shut down in $t = 1$, the same p_g admits an equilibrium which yields $q_{sd}(\theta) = 2$ for all $\theta \in [0, \theta_0^*]$, $q_{sd}(\theta) = 1$ for all $\theta \in (\theta_0^*, (p_g + S) \wedge 1]$, and $q_{sd}(\theta) = 0$ otherwise. Since $\theta_{g\emptyset}^{SMR}(p_g) = (p_g + S) \wedge 1$ from (D.3) and $\theta_1^{SMR}(p_g) \le \theta_0^*$ from (D.2), there exists a $p_g' \le p_g$ which makes the total trade volume with the $t = 1$ market shutdown equal to that in the SMR equilibrium arising from p_g. Since $\theta_1^{SMR}(p_g) \le \theta_0^* \le (p_g' + S) \wedge 1 \le \theta_{g\emptyset}^{SMR}(p_g)$, Theorem 5-(iii) implies that the equilibrium with the $t = 1$ market shutdown arising from p_g' yields higher welfare than the SMR equilibrium arising from p_g.

Next, suppose $p_g < p_0^*$ admits an SMR equilibrium. Since $\theta_1^{SMR}(p_g) < \theta_{g\emptyset}^{SMR}(p_g) < \theta_0^*$, the same p_g admits an equilibrium with the $t = 1$ market shutdown, which yields $q_{sd}(\theta) > q_{SMR}(\theta)$ if $\theta \in (\theta_1^{SMR}, \theta_0^*]$ and $q_{sd}(\theta) = q_{SMR}(\theta)$ otherwise. By the assumption $J(\theta) > 0$ if and only if $\theta < \hat{\theta}^*$ and $\hat{\theta}^* > \theta_0^*$, the equilibrium with the $t = 1$ market shutdown yields strictly higher welfare than the SMR equilibrium, as was to be shown.

Proof of Proposition 7-(iii).

First, suppose p_g admits an equilibrium under transparency characterized by $\theta_g^{sd}(p_g)$, where (C.2) defines $\theta_g^{sd}(p_g)$ subject to $m(0, \theta_g) < I$. In this equilibrium, we have $q(\theta) = 1$ if $\theta \in [0, \gamma(\theta_g^{sd}(p_g))]$ and $q(\theta) = 0$ if $\theta > \gamma(\theta_g^{sd}(p_g))$. Under secrecy, the same p_g admits an equilibrium, in which $q(\theta) = 2$ if $\theta \in [0, \theta_0^*]$, $q(\theta) = 1$ if $\theta \in (\theta_0^*, (p_g + S) \wedge 1]$, and $q(\theta) = 0$ if $\theta > (p_g + S) \wedge 1$. Since $(p_g + S) \wedge 1 = \gamma(\theta_g^{sd}(p_g))$ and $J(\theta) > 0$ for all $\theta \leq \theta_0^*$, the equilibrium under secrecy yields higher welfare than the equilibrium under transparency.

Next, suppose p_g admits an equilibrium under transparency characterized by $\theta_g^{sd}(p_g)$, where (C.7) defines $\theta_g^{sd}(p_g)$ subject to (C.8) and (C.10). In this equilibrium, we have $q(\theta) = 2$ if $\theta \in [0, \theta_g^{sd}(p_g)]$, $q(\theta) = 1$ if $\theta \in (\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g))]$, and $q(\theta) = 0$ if $\theta > \gamma(\theta_g^{sd}(p_g))$. Since $\theta_g^{sd}(p_g) \leq \theta_0^* < \gamma(\theta_g^{sd}(p_g))$ and $p_g \leq m(\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g)))$ from (C.7) and (C.10), there exists an equilibrium under secrecy for some $p_g' \in (\theta_0^* - S, m(\theta_g^{sd}(p_g), \gamma(\theta_g^{sd}(p_g))))$, which yields the same total trade volume as that under transparency with p_g. In this equilibrium, we have $q(\theta) = 2$ if $\theta \in [0, \theta_0^*]$, $q(\theta) = 1$ if $\theta \in (\theta_0^*, (p_g + S) \wedge 1]$, and $q(\theta) = 0$ if $\theta > (p_g + S) \wedge 1$. Hence, by Theorem 5-(iii), the equilibrium under secrecy arising from p_g' yields higher welfare than that under transparency arising from p_g.

Lastly, suppose p_g admits an equilibrium under transparency characterized by $\theta_g^{sd}(p_g)$, where $\theta_g^{sd}(p_g)$ is determined by $p_g = m(\theta_g, \gamma(\theta_g))$ subject to $\theta_g > \theta_0^*$. In this equilibrium, we have $q(\theta) = 2$ if $\theta \in [0, \theta_0^*]$, $q(\theta) = 1$ if $\theta \in (\theta_0^*, \gamma(\theta_g^{sd}(p_g))]$, and $q(\theta) = 0$ if $\theta > \gamma(\theta_g^{sd}(p_g))$. Under secrecy, the same p_g admits an equilibrium in which $q(\theta) = 2$ if $\theta \in [0, \theta_0^*]$, $q(\theta) = 1$ if $\theta \in (\theta_0^*, (p_g + S) \wedge 1]$, and $q(\theta) = 0$ if $\theta > (p_g + S) \wedge 1$. Since $(p_g + S) \wedge 1 = \gamma(\theta_g^{sd}(p_g))$, each type θ sells the exactly same quantity of assets under secrecy as it would under transparency. Hence, by Theorem 5-(iii), the equilibrium under secrecy induces the same welfare as the equilibrium under transparency. *Q.E.D.*

Proposition 8. *The optimal bailout mechanism has*

$$q^*(\theta) = \begin{cases} 2 & \text{if } \theta \leq \theta_0^*, \\ 1 & \text{if } \theta \in (\theta_0^*, \hat{\theta}^*], \\ 0 & \text{if } \theta > \hat{\theta}^*. \end{cases}$$

The optimal policy is implemented by a secret bailout policy with $p_g = \hat{\theta}^* - S$ accompanied by the shutdown of the market in $t = 1$.

Proof of Proposition 8. Let q be an arbitrary feasible allocation rule satisfying $[P]$. Then,

$$\begin{aligned}
&W(q^*) - W(q) \\
&= \int_0^1 J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta \\
&= \int_0^{\min\{\hat{\theta}^*,\theta_0^*\}} J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta + \int_{\min\{\hat{\theta}^*,\theta_0^*\}}^{\max\{\hat{\theta}^*,\theta_0^*\}} J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta \\
&\quad + \int_{\max\{\hat{\theta}^*,\theta_0^*\}}^1 J(\theta)[q^*(\theta) - q(\theta)]f(\theta)d\theta.
\end{aligned}$$

The first integral is nonnegative since $J(\theta) \geq 0$ and $q(\theta) \leq 2 = q^*(\theta)$ for $\theta < \min\{\hat{\theta}^*, \theta_0^*\} \leq \hat{\theta}^*$. The last integral is also nonnegative since $J(\theta) \leq 0$ and $q(\theta) \geq 0 = q^*(\theta)$ for $\theta > \max\{\hat{\theta}^*, \theta_0^*\} \geq \hat{\theta}^*$. Finally, we show below that the middle integral is also nonnegative. Suppose first $\hat{\theta}^* < \theta_0^*$. Then, for any $\theta \in (\min\{\hat{\theta}^*, \theta_0^*\}, \max\{\hat{\theta}^*, \theta_0^*\}] = (\hat{\theta}^*, \theta_0^*]$, $J(\theta) \leq 0$ and $q(\theta) \geq 1 = q^*(\theta)$, so the middle integral is nonnegative. Suppose next $\hat{\theta}^* < \theta_0^*$. Then, for $\theta \in (\min\{\hat{\theta}^*, \theta_0^*\}, \max\{\hat{\theta}^*, \theta_0^*\}] = (\theta_0^*, \hat{\theta}^*]$, $J(\theta) \geq 0$ and $q(\theta) \leq 1 = q^*(\theta)$, so the middle integral is nonnegative. Since all three integrals are nonnegative, the allocation rule q^* is optimal.

The last statement follows from Proposition 6-(i). *Q.E.D.*

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