Prizes versus Contracts as Incentives for Innovation[*]

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Abstract

Procuring an innovation involves motivating a research effort to generate a new idea and then implementing that idea efficiently. If research efforts are unverifiable and implementation costs are private information, a trade-off arises between the two objectives. The optimal mechanism resolves the tradeoff via two instruments: a monetary prize and a contract to implement the project. The optimal mechanism favors the innovator in contract allocation when the value of the innovation is above a certain threshold, and handicaps the innovator in contract allocation when the value of the innovation is below that threshold. A monetary prize is employed as an additional incentive but only when the value of innovation is sufficiently high.

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1   Introduction

It is well-known that markets provide insufficient innovation incentives.[1] One policy remedy is to create demand for innovation via public procurement. Public buyers can use their large purchasing power as a lever to spur innovation and boost the generation and diffusion of new knowledge.[2] Not surprisingly, governments devote substantial resources to procuring innovative goods and services. In 2015, the US government spent about 21 billion dollars for public R&D contracts, and 19 billion dollars for defence R&D contracts. European countries spent about 2.6 billion euros in 2011 for non-defense R&D procurement alone (European Commission, EC 2014).[3] Indeed, history is rich with examples where public procurement has had a major effect on the development and diffusion of innovations such as supercomputers, large passenger jets, semi-conductors and the Internet.[4]

Procuring innovation presents a special challenge absent in the procurement of standard products. The value of innovation is typically realized through a follow-on good or service that embodies that innovation. Hence, successful procurement must provide *ex ante* innovation incentives and ensure *ex post* efficient selection of a supplier for the follow-on project. These two goals often conflict with each other, as the innovator need not be most adept at performing the follow-on project. Hence, awarding the follow-on contract, say, to the most efficient supplier may not adequately motivate the innovator. One solution is to shift the assignment of the follow-on contract, so as to favor or disfavor the innovator depending on the outcome of innovation. An alternative is to award a cash prize to a successful innovator. A common wisdom suggests that a cash prize would be a better instrument for incentivizing innovation, as it does not distort the assignment of the follow-on contract.

Yet, this simple wisdom is not borne out by the practice, which is mixed in the use of the two instruments. Cash prizes are indeed offered in research contests and R&D procurements. However, these methods are reserved for the circumstances in which innovations are clearly foreseeable and precisely specifiable in advance. Many innovations are not foreseen and thus are not solicited. Explicit cash rewards are rarely used for such "unsolicited proposals," even though governments do consider them and occasionally implement them.

Procurement practices also vary in the treatment of innovators in the follow-on projects. For unsolicited proposals, some countries do not treat innovators differently at the follow-on

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[1]Private incentives for innovation are insufficient because innovative activities often generate knowledge that has significant positive externalities and by its nature is difficult to protect via intellectual property.

[2]Public procurement is a significant part of economic activities; for instance, they account for about 12% of GDP in OECD countries (OECD, 2015).

[3]In parallel, the European Commission has adopted new directives (Directives 2014/24 and 2014/25) modernizing the legislative framework on public procurement, in order to incentivize a public demand for innovative goods and services.

[4]See e.g. Cabral *et al.* 2006 and Geroski, 1990 for references.

contract stage, but other countries such as Chile, Korea, Italy, and Taiwan give an advantage to the proposer/innovator at the contract awarding stage. Further, the degree to which the innovator is favored varies across cases, presumabily depending on the value of the proposed innovation. Even when a cash prize is awarded to a successful innovator, this does not preclude a special treatment of the innovator at the contract stage. For instance, R&D procurement is often bundled with the procurement of follow-on product, in which case the winner of R&D stage is guaranteed to win the production contract. A case in point is EC's "Innovation Partnerships" model, under which research and production are procured through one single tender, with the innovator also obtaining the contract for the production of the innovative project.[5]

The purpose of this paper is to study how the alternative instruments should be combined to provide incentive for innovation. We analyze this question by identifying an optimal procurement mechanism in an environment where the procurer faces a moral hazard problem *ex ante* and an adverse selection problem *ex post*. The risk-neutral innovator(s) first undertake costly effort to innovate and then a supplier is selected to perform the project that implements the chosen innovation. An innovator's effort stochastically increases the value of project, but this effort is unobserved by the outsiders, which, coupled with limited liability, gives rise to a non-trivial moral hazard problem. The value of innovation is realized when a follow-on project is performed, and there are multiple suppliers, including the innovator, who can perform that project. The cost of performing the project is private information, which gives rise to an adverse selection problem. The value of innovation is verifiable and innovators can be rewarded via monetary prizes as well as with the contract for implementing the project.

We first consider the case of a single innovator. Not only does this baseline model makes our insights transparent, but it is also often relevant as many innovative projects procured by public agencies are unsolicited, and arrive one at a time. We first find that in the absence of adverse selection – i.e., when the cost of performing the project is observed by the procurer – the common wisdom is indeed valid: the optimal mechanism relies solely on a cash prize and does not distort the assignment of the follow-on contract. Specifically, the buyer awards a prize to the innovator whenever the value of innovation is above a threshold; and she assigns the contract ex post to the supplier with the lowest cost of performing the job.

The result is quite different, however, in the presence of adverse selection, i.e., when the suppliers have private information about the cost of performing the follow-on project. The private information generates rents for the supplier who performs the project. When the

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[5]EC has an alternative model "Pre-commercial procurement" (PCP) whereby he public authority procures R&D activities from the solution exploration phase to prototyping and testing, but it reserves the right to tender competitively the newly developed products or services. See EC (2007) and https://ec.europa.eu/digital-agenda/en/pre-commercial-procurement.

rents accrue to the innovator, this can work additionally as an incentive or a disincentive, depending on the value of innovation. Specifically, for high enough value of innovation, the rents accruing to the innovator constitute an incentive, so this effectively reduces the shadow cost of awarding the contract to the innovator relative to the other suppliers. Hence, the optimal mechanism calls for distorting the assignment of the follow-on contract in favor of the innovator. By contrast, for a low value innovation, the rents accruing to the innovator constitutes a disincentive for innovation, and thus raises the shadow cost of selecting the innovator for the contract. In this case, the optimal mechanism calls for distorting the assignment of the follow-on contract against the innovator.

Cash prizes can be part of the optimal mechanism but only as a supplementary tool. Specifically, the optimal mechanism prescribes a cash prize to be awarded to the innovator (only) when the value of innovation is so high that shifting the contract right toward the innovator does not suffice to fulfil the incentive need. In a striking contrast to the common wisdom, therefore, *a contract right serves as primary tool for incentive and a cash prize serves as a supplementary tool* (when the former does not meet the required incentive need).

Comparative statics reflect the same insights. When information rents are significant (e.g., because costs are relatively heterogeneous or there are few potential suppliers), the optimal mechanism may rely solely on the contract right to incentivize innovation. By contrast, when information rents are small (e.g., because costs are relatively homogeneous or there are many potential contractors), or when the value of innovation is high compared with these rents, the optimal mechanism involves a cash prize, again because a contract right alone is not sufficient in that case.

We next extend the model to allow for multiple innovators. This situation is relevant for R&D contests or procurements wherein the buyer has a clear sense about the desired type of innovation and its feasibility. We show that the above insights carry over. First, control rights serve as a central tool for rewarding innovations. Specifically, the optimal mechanism favors the proposer of a high-value project and disfavors the proposer of a low-value project at the implementation stage. Second, as in the single innovator case, cash prizes serve as a supplementary tool, used only when an innovator's project is particularly valuable and/or when her research effort is particularly worth incentivizing. A interesting novel finding is that, when a cash prize is used, it is never split among multiple innovators. In this sense, we establish a "winner-takes-all" principle for the allocation of a cash prize.

Our findings clarify several issues that are policy relevant for public procurements. First of all, as mentioned earlier, a long-standing question is whether the contracting rights should be allocated in a way to favor the innovator/initiator of a project. Contrary to the received wisdom, we provide a clear rationale in the presence of moral hazard and adverse selection for the use of contract right as an incentive for innovation. Extreme examples of these are found in the bundling of procurement of final products with preceding R&D activities.

The US Congress expanded recently the use of Other Transaction Authority ("OTA").[6] OTA allows a prototype project to transition to award of a follow-on production contract. The Department of Defence may make such award without the use of competitive procedures provided (i) competitive procedures were used in the initial prototype transaction award and (ii) the OTA contractor successfully completed the prototype project. Likewise, as mentioned earlier, the "Innovation Partnerships" model introduced by EC allows for a similar bundling. In 2016, this model was announced by Transport for London for the development of a new composite conductor rail system to fit constrained areas of the undergraund network and to improve the energy efficiency.[7]

Second, not only does our theory rationalize the use of contract right as an incentive for innovation, it provides some insight for the specific way in which the contract right allocation should be designed. Specifically, our analysis suggests that the degree to which the innovator should be favored at the contract awarding stage should vary monotonicially with the value of her innovation and the significance of the adverse selection problem at the contract stage. Many observed practices are consistent with these. In Chile and Korea, bonus points are given to the innovator in the bidding competition for contract allocation. A given percentage is more valuable the more significant the more severe the adverse selection problem. Further, there is some evidence, the procurer adjusts the bonus points based on the perceived value of the innovation. For example, the first two unsolicited proposals for airport concessions in Chile obtained a bidding bonus equal to 20 percent points of the allowed score, whilst the third airport proposal received 10 percent points.[8] Other methods of favoring the innovator, such as best-and-final offers and swiss challenge systems, to be discussed in detail later, also allows the bidding advantage accorded to the innovator to vary with the severity of adverse selection.

Finally, our analysis also clarifies the role of monetary prizes for promoting innovation and their optimal design. Specifically, we identify their roles as supplementary to the contract rights. This explains the relative paucity/insignificance of explicit monetary reward given to unsolicited projects. Except for research contests (when the projects are clearly defined in advance), we are not aware of explicit prizes given to unsolicited projects. In the case monetary prizes are used for example in research contests, the prizes are often never split across multiple winners. This feature is in turn consistent with the winner-take-all principle we establish.

The paper is organized as follows. In Section 2, we study the case of a single innovator.

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[6]Section 815, Amendments to Other Transaction Authority, of the National Defense Authorization Act for Fiscal Year 2016, signed into law on November 25, 2015.

[7]London: Transport equipment and auxiliary products to transportation, Periodic indicative notice – utilities Supplies, number 2016/S 217-395943.

[8]These projects concerned respectively the expansion of the airports of Puerto Montt (June 1995), Iquique (August 1995) and Calama (Ocober 1997).

Section 2.1 sets-up the model, Section 2.2 presents a number of benchmarks, and section 2.3 develops the main analysis. In Section 3, we extend the analysis to the case of multiple innovators. In Section 4, we discuss the insights that our analysis offers for the approaches used in practice for unsolicited proposals and innovation procurement. In Section 5, we discuss the related literature. In Section 6, we make some concluding remarks.

2   Procurement with a Single Innovator

We consider here the case in which a single innovator may propose a project. This case serves to clarify the main results in a simple way, but it is also practically relevant for the case with unsolicited proposals. The decision facing the buyer is whether to adopt the project and, if so, select a contractor for its implementation.

2.1   Model

A buyer – representing a public agency – wishes to procure an innovative project through two stages: *innovation* and *implementation*. In the first stage, an innovator, say firm 1, exerts effort $e \geq 0$ to come up with a project. The effort e costs the innovator $c(e) \geq 0$ but affects the value v of the project stochastically. The innovation project has elements of nonexcludability and nonrivalrly, which makes it non-commercializable. Hence, intellectual property rights are not effective for incentivizing the research effort. Examples of such projects include information technologies for traffic management systems or a power plant for carbon capture and storage preventing the release of large quantities of CO2 into the atmosphere.

We assume that $c(\cdot)$ is increasing, strictly convex, twice differentiable and such that $c'(0) = 0$. The value v is drawn from $V := [\underline{v}, \bar{v}]$ according to a c.d.f. $F(\cdot|e)$, which admits a twice-differentiable density $f(\cdot|e)$ in the interior. An increase in e shifts the distribution $F(\cdot|e)$ in the sense of the Monotone Likelihood Ratio Property, that is:

$$\frac{f(v'|e')}{f(v|e')} > \frac{f(v'|e)}{f(v|e)}, \text{ for any } v' > v \text{ and } e' > e. \qquad (MLRP)$$

The innovator's effort e is unobservable. The project value v is instead publicly observable and verifiable. The verifiability of v is a reasonable assumption in many procurement contexts, where projects can be described using precise functional and performance terms. For example, in the case of technological improvements for faster medical tests, transport units with lower energy consumption, or for information and communication technology (ICT) systems with interoperability characteristics, v may capture respectively the speed increase for the medical test, the degree of energy efficiency of the transport unit, or the

technical functionalities of the ICT system verified in submitted prototypes. Later on, we explore the case where the project value v is not contractible and discuss how our insights can be transposed to such situations (see Section 4.2).

In the second stage, n potential firms, including the innovator, compete to implement the project. Each firm $i \in N := \{1, ..., n\}$ faces a cost θ_i, which is privately observed and drawn from $\Theta := [\underline{\theta}, \overline{\theta}]$ according to a c.d.f. $G_i(\cdot)$, which admits density $g_i(\cdot)$ in the interior. We assume that $\underline{\theta} < \overline{v}$ and $G_i(\theta_i)/g_i(\theta_i)$ is nondecreasing in θ_i, for each $i \in N$.

If the project is not implemented, all parties obtain zero payoff. If instead a project of value v is implemented at cost t for the principal, then the principal's welfare is given by:

$$v - t.$$

By the revelation principle, we can formulate the problem facing the principal as that of choosing a direct revelation mechanism that is incentive-compatible. A direct mechanism is denoted by: $(x, t) : V \times \Theta^n \to \Delta^n \times \mathbb{R}^n$, which specifies the probability $x_i(v, \theta)$ that firm i implements the project and the transfer payment $t_i(v, \theta)$ that it receives, when the project proposed by firm 1 has value v and firms report types $\theta := (\theta_1, ..., \theta_n)$; by construction, the assignment probabilities must lie in $\Delta^n := \{(x_1, ..., x_n) \in [0, 1]^n | \sum_{i \in N} x_i \in [0, 1]\}$. The dependence of the mechanism on the project value v reflects its verifiability, whereas the absence of the argument e arises from its unobservability to the principal.

The timing of the game is as follows:

1. The principal offers a direct revelation mechanism specifying the allocation decision (i.e., whether the project will be implemented and, if so, by which firm) and a payment to each firm, as functions of firms' reports on their costs.

2. The innovator chooses e; the value v is realized and observed by all parties.

3. Firms observe their costs and decide whether to participate.

4. Participating firms report their costs, the project is implemented (or not), and transfers are made according to the mechanism.

For each $v \in V$, let

$$u_i(v, \theta_i'|\theta_i) := \mathbb{E}_{\theta_{-i}} \left[t_i(v, (\theta_i', \theta_{-i})) - \theta_i x_i(v, (\theta_i', \theta_{-i})) \right]$$

denote the *interim* expected profit that firm i could obtain by reporting a cost θ_i' when it actually faces a cost θ_i, and let

$$U_i(v, \theta_i) := u_i(v, \theta_i|\theta_i)$$

denote firm i's expected payoff under truthful revelation of its type θ_i. The revelation principle requires the direction mechanism (x, t) to satisfy *incentive compatibility:*

$$U_i(v, \theta_i) \geq u_i(v, \theta_i'|\theta_i), \quad \forall i \in N, \forall v \in V, \forall (\theta_i, \theta_i') \in \Theta^2. \qquad (IC)$$

Note that the principal cannot force the firms to participate before the project is developed by the innovator, as they decide whether to participate only after learning their cost. This is a natural assumption in many settings. For example, in the case of unsolicited proposals the identities of the candidates capable of executing the project are unknown until the nature of the project — its value and the costs of implementing it – is determined. This makes it difficult for the principal to approach prospective suppliers for early buy-in. More generally, procurers are reluctant to enter firms into loss making contracts, as this would typically discourage participation of risk-averse or small firms, and also cause severe service disruptions when a firm chooses to default rather than honor the contract. Limited liability on the firm's side is therefore reasonable in public procurement settings.

The principal cannot therefore solicit the relevant firms and force them to buy in. This feature requires the direct mechanism (x, t) to satisfy *individual rationality*:

$$U_i(v, \theta_i) \geq 0, \quad \forall i \in N, \forall v \in V, \forall \theta_i \in \Theta, \qquad (IR)$$

As we shall see, together with (IC), this requirement will cause the principal to leave information rents to the selected supplier.[9]

We also assume that the principal must at least break even for each realized value v of the project. In other words, a feasible mechanism (x, t) must satisfy *limited liability*:

$$\mathbb{E}_\theta \left[w(v, \theta) \right] \geq 0, \quad \forall v \in V, \qquad (LL)$$

where

$$w(v, \theta) := \sum_{i \in N} \left[x_i(v, \theta) v - t_i(v, \theta) \right]$$

denotes the principal's surplus upon realizing the value v of the project. Limited liability may arise from political constraints. Public projects are scrutinized by various stake-holders such as legislative body, project evaluation authority, consumer advocacy groups, and media, who might reject projects likely to run a loss. We note however that it is not crucial that the constraint is of the particular form assumed in (LL); the general thrust of our analysis carries through as long as there there is some cap on either the maximum loss the principal can

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[9]In the absence of this individual rationality constraint, the principal could leave no rents to the firms by requiring them to "buy-in" to a contract via an upfront fee. As a result, the first-best could be achieved at the implementation stage, and there would be no gain from using contract rights to reward the innovator; monetary prizes would indeed be preferable.

sustain or the maximum payment she can make to the firm.[10] Indeed, public agencies and local authorities tend to operate within the boundaries of well-defined budgets. Procurement tenders typically include a contract value which specifies the maximum payment allowed for the supplier.

Finally, as the innovator chooses effort e in its best interest, the mechanism must also satisfy the following *moral hazard* condition:

$$e \in \arg\max_{\tilde{e}} \left\{ \mathbb{E}_{v,\theta} \left[U_1(v, \theta_1) \mid \tilde{e} \right] - c(\tilde{e}) \right\}. \tag{MH}$$

The principal's problem is to choose an optimal mechanism satisfying these constraints. More formally, she solves the problem:

$$[P] \qquad \max_{x,t,e} \qquad \mathbb{E}_{v,\theta} \left[w\left(v, \theta\right) \mid e \right],$$

$$\text{subject to } (IR), (IC), (LL) \text{ and } (MH)$$

2.2 Benchmarks

Before solving $[P]$, it is useful to begin with two benchmarks.

No adverse selection *ex post*. In this benchmark, we shut off the adverse selection problem by assuming that the principal observes the firms' implementation costs. Formally, the principal's problem is the same as $[P]$, except that the constraint (IC) is now absent. We label such relaxed problem $[P - FB]$, where "FB" refers to first-best implementation efficiency. In this problem, once the principal approves the project, she can implement it by paying the true cost θ_i to firm i, without giving up any information rent.

We show now that incentivizing the research effort with contract rights is suboptimal. In line with conventional wisdom, cash prizes are the best instrument as they do not distort the selection of a supplier whereas contract rights do. Thus, the solution to $[P - FB]$ is characterized as follows:

PROPOSITION 1. *(First-Best) There exist $\lambda^{FB} > 0$ and $e^{FB} > 0$ such that the optimal mechanism solving $[P - FB]$ awards firm i a contract to implement the project with probability:*

$$x_i^{FB}(v, \theta) := \begin{cases} 1 & \text{if } \theta_i < \min\left\{v, \min_{j \neq i} \theta_j\right\}, \\ 0 & \text{otherwise}, \end{cases}$$

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[10]Without any such constraint, the optimal mechanism would not be well defined: the principal would find it desirable to pay an arbitrarily large bonus to the innovator only for a vanishingly set of projects with values close to $\bar{v}$. Such a scheme may be of theoretical interest but is unreasonable and unrealistic.

with a transfer that simply compensates the winning firm's cost, except that firm 1 is paid additionally a monetary prize equal to

$$\rho_1^{FB}(v) := \begin{cases} \mathbb{E}_\theta \left[\sum_{i \in N} x_i^{FB}(v, \theta) (v - \theta_i) \right] > 0 & \text{if } v > \hat{v}^{FB}, \\ 0 & \text{if } v < \hat{v}^{FB}, \end{cases}$$

where $\hat{v}^{FB}$ is such $\underline{v} < \hat{v}^{FB} < \bar{v}$ and solves $\beta^{FB}(v) = 1$, where

$$\beta^{FB}(v) := \lambda^{FB} \frac{f_e(v|e^{FB})}{f(v|e^{FB})},$$

and e^{FB} satisfies (MH).

PROOF. See Appendix A. □

Notice that the contract right is assigned efficiently to the firm with the lowest cost as long as it is less than the value v of the project. Incentive for innovation is provided solely by the cash prize, in a manner familiar from the moral hazard literature (e.g., Mirrlees (1975); Holmstrom (1979)). The realized project value v is an informative signal about the innovator's effort, and paying an additional dollar to the innovator for a project with value v relaxes (MH) by $\frac{f_e(v|e^{FB})}{f(v|e^{FB})}$. Multiplied by the shadow value λ^{FB} of relaxing (MH), $\beta^{FB}(v) = \lambda^{FB} \frac{f_e(v|e^{FB})}{f(v|e^{FB})}$ measures the *incentive benefit* to the principal. Naturally, the optimal mechanism calls for awarding the maximal feasible prize to the innovator if $\beta^{FB}(v) > 1$ and zero prize otherwise. Given (MLRP), the incentive benefit $\frac{f_e(v|e^{FB})}{f(v|e^{FB})}$ is strictly increasing in v, so the threshold value $\hat{v}^{FB}$ is well-defined. Simply put, the optimal mechanism calls for paying as much as possible to the innovator whenever the project value v is high enough to indicate that the incentive benefit exceeds the cost, and nothing otherwise. In the former case, (LL) must be binding, so the maximal feasible prize is given by the net surplus that the project generates after reimbursing the implementing firm.

In sum, the innovator's incentive payment has the structure of the bang-bang solution: there exists a threshold value of the project so that the innovator is paid nothing when the value of project falls short of that threshold and the entire value of the project when the value of the project exceeds that threshold.[11] Importantly, absent adverse selection, the principal never uses contracting rights to motivate the innovating firm.

No moral hazard *ex ante*. In this benchmark, we shut off the moral hazard problem by assuming that the project value follows some exogenous distribution $F(v)$ which does not depend on effort. Formally, the problem facing the principal in this benchmark is the

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[11]This feature of the bang-bang solution is reminiscent of the several well known results such as Mirrelees (1975) and Innes (1990).

same as $[P]$, except that the moral hazard constraint (MH) is absent and the distribution function $F(v|e)$ is replaced by the exogenous distribution $F(v)$. The resulting problem, labeled $[P-SB]$, conforms to the standard optimal auction design problem, except for the (LL) constraint. Ignoring the latter, the optimal auction solution, labeled "the second-best mechanism," is familiar from Myerson (1981). One can easily see that this solution satisfies (LL), and thus constitutes a solution to $[P-SB]$ as well. As the associated analysis is standard, we provide the characterization of the solution without a proof.

PROPOSITION 2. *(Myerson) The second-best mechanism awards firm i the contract to implement the project with probability:*

$$x_i^{SB}(v, \theta_i) := \begin{cases} 1 & \text{if } J_i(\theta_i) \leq \min\left\{v, \min_{j \neq i} J_j(\theta_j)\right\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $J_i(\theta_i) := \theta_i + \frac{G_i(\theta_i)}{g_i(\theta_i)}$ is firm i's virtual cost.

2.3   Optimal Mechanism

We now consider problem $[P]$, in which the principal faces *ex post* adverse selection with respect to firms' implementation costs as well as *ex ante* moral hazard with respect to the innovator's effort. Throughout the analysis, we assume that an optimal mechanism exists, which induces an interior effort level e^*. The following Proposition characterizes this optimal mechanism:

PROPOSITION 3. *There exists $\lambda^* > 0$ such that the optimal mechanism solving $[P]$ is characterized as follows:*

(i) *The mechanism assigns a contract to firm $i = 1, ..., n$ to implement the project with probability*

$$x_i^*(v, \theta) = \begin{cases} 1 & \text{if } K_i^*(v, \theta_i) \leq \min\left\{v, \min_{j \neq i} K_j^*(v, \theta_j)\right\}, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$K_i^*(v, \theta_i) := \begin{cases} J_i(\theta_i) - \min\left\{\beta^*(v), 1\right\} \frac{G_i(\theta_i)}{g_i(\theta_i)} & \text{if } i = 1 \\ J_i(\theta_i) & \text{if } i \neq 1 \end{cases}, \text{ and } \beta^*(v) := \lambda^* \frac{f_e(v|e^*)}{f(v|e^*)}.$$

(ii) *The mechanism awards firm i an expected transfer, $T_i^*(v, \theta_i) = \mathbb{E}_{\theta_{-i}}[t_i^*(v, \theta)]$, equal to*

$$T_i^*(v, \theta_i) := \rho_i^*(v) + \int_{\theta_i}^{\overline{\theta}} X_i^*(v, s) ds + \theta_i X_i^*(v, \theta_i),$$

where:

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- the second term reflects the information rent generated by the expected probability of awarding the contract, $X_i^*(v, \theta_i) = \mathbb{E}_{\theta_{-i}}[x_i^*(v, (\theta_i, \theta_{-i}))]$, and

- the first term corresponds to a "cash prize," which is zero for a non-innovator (i.e., $\rho_i^*(v) := 0$ for $i \neq 1$) and, for the innovator ($i = 1$), is equal to

$$\rho_1^*(v) := \begin{cases} \mathbb{E}_\theta\left[\sum_{i \in N} x_i^*(v, \theta)\left[v - J_i(\theta_i)\right]\right] > 0 & \text{if } \beta^*(v) > 1, \\ 0 & \text{if } \beta^*(v) < 1. \end{cases}$$

(iii) The effort e^* satisfies $e^* > 0$ and

$$\int_v \int_\theta \left[\rho_1^*(v) + \frac{G_1(\theta)}{g_1(\theta)} x_1^*(v, \theta)\right] g(\theta) d\theta f_e(v|e^*) dv = c'(e^*).$$

PROOF. See Appendix B. □

To gain more intuition about this characterization, it is useful to decompose the principal's payment to each firm into two components. The first component is the information rent that must be paid to elicit the firm's private information about its cost. By a standard envelope theorem argument, this component is uniquely tied to – and should therefore be interpreted as being necessitated by – the awarding of the contract to a firm. We can thus call it the *contract payment*. The second component is a constant amount paid to the firm regardless of its cost. As this component is not related to the contract assignment, we call it the *cash prize* and denote it by $\rho_i^*(v)$. Obviously, the principal would never pay any cash prize to a non-innovating firm (i.e., $i \neq 1$). For the innovating firm (i.e., $i = 1$), however, a cash prize may be necessary. The question is how the principal should combine these two types of payments to encourage innovation.

The key observation in answering this question hinges on the incentive benefit $\beta^*(v) = \lambda^* \frac{f_e(v|e^*)}{f(v|e^*)}$. As explained earlier, this term represents the value of paying a dollar to the innovator for developing a project worth v – more precisely, the effect $\frac{f_e(v|e^*)}{f(v|e^*)}$ of relaxing (MH) multiplied by its worth λ^* to the principal. If moral hazard were not a concern, we would have $\lambda^* = 0$ and thus $\beta^*(v) = 0$, and the optimal mechanism would reduce to the second-best auction mechanism described in Proposition 2. But, at this second-best solution, the innovator does not fully internalize the surplus that her effort generates for the buyer.[12] Hence, (MH) is binding, implying that the optimal mechanism prescribes a stronger incentive for effort than the second-best mechanism.

Given $\lambda^* > 0$, the incentive benefit $\beta^*(\cdot)$ is nonzero and the optimal mechanism departs from the second-best mechanism. In particular, the contract assignment now depends on the realized value of the project, through the shadow cost $K_i^*(v, \theta_i)$. For a non-innovating firm

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[12]The innovator does have some incentives, for her rents increase with the effort. Yet, the resulting incentives are not sufficient, as these rents understate the surplus accruing to the buyer.

(i.e., $i \neq 1$), the shadow cost is the same as its virtual cost, $J_i(\theta_i)$, just as in the second-best benchmark. Instead, for the innovator (i.e., $i = 1$), the shadow cost differs from its virtual cost by a term, $\beta^*(v)\frac{G_1(\theta_1)}{g_1(\theta_1)}$, which reflects the need to incentivize its research effort.[13]

Intuitively, rewarding the innovator for a low-value project (evidence of low effort) weakens its innovation incentives, whereas rewarding it for a high-value project (evidence of high effort) strengthens them. Indeed, by $(MLRP)$, $\beta^*(v) = \lambda^* \frac{f_e(v|e^*)}{f(v|e^*)}$ increases in v and there exists a unique $\tilde{v} \in (\underline{v}, \overline{v})$ such that $\beta^*(\tilde{v}) = 0$. Thus, $\beta^*(v) < 0$ when $v < \tilde{v}$, so that rewarding the innovator indeed weakens its innovation incentives. Awarding a cash prize to the innovator is never optimal in this case. For the same reason, each dollar paid as information rents weakens the innovator's incentive, causing the shadow cost $K_1^*(v; \theta_1)$ of assigning the contract to the innovator to *exceed* its virtual cost $J_1(\theta_1)$, by $-\beta^*(v)\, G_1(\theta_1)/g_1(\theta_1)\, (> 0)$. Hence, compared with the second-best mechanism, the optimal mechanism calls for biasing the contract allocation against the innovator.

When instead $v > \tilde{v}$, there are two possibilities. If $v < \hat{v} := \sup\{v \in V \mid \beta^*(v) \leq 1\}$, the incentive benefit $\beta^*(v)$ of paying a dollar to the innovator is positive but less than a dollar. In this case, it is still optimal not to award a cash prize, as it would entail a net loss for the principal. However, a fraction $\beta^*(v)$ of the information rent accruing to the innovator goes toward its innovation incentive, which *reduces* the shadow cost $K_1^*(v, \theta_1)$ of assigning the contract to the innovator below its virtual cost $J_1(\theta_1)$ by $\beta^*(v)\, G_1(\theta_1)/g_1(\theta_1)$. Compared with the second-best benchmark, the optimal mechanism distorts allocation of the contract in favor of the innovator.

If $v > \hat{v}$,[14] a dollar payment to the innovator yields more than a dollar incentive benefit. A cash prize is then beneficial, which is why $\rho_1^*(v) > 0$. Hence, the principal transfers any surplus she collects, either through the cash prize or through the information rent; that is, (LL) is binding. Furthermore, any increase in information rents for the innovator simply crowds out the cash prize by an equal amount. It follows that the incentive benefit of the information rent paid to the innovator is at most one 1 (and not $\beta^*(v) > 1$), and the shadow cost $K_1^*(v, \theta_1)$ reduces to the actual production cost θ_1. Compared with the second-best, the optimal mechanism distorts the allocation of the contract in favor of the innovator to such an extent that the innovator is treated as an "in-house" supplier. Any further distortion in favor of the innovator is suboptimal, because it reduces the total "pie," and thus the cash prize to the innovator, more than it increases the information rent to that firm.

We can state these observations more formally as follows:

COROLLARY 1. *There exists $\tilde{v}$ and $\hat{v}$, with $\underline{v} < \tilde{v} < \hat{v} \leq \overline{v}$, such that the optimal mechanism*

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[13]Awarding the contract to the innovator with type θ_1 with an additional probability unit necessitates giving information rent of a dollar to types below θ_1—so $\frac{G_1(\theta_1)}{g_1(\theta_1)}$ *ex ante* —and each dollar paid to the innovator yields the incentive benefit of $\beta^*(v)$.

[14]This case occurs only when $\beta^*(\overline{v}) > 1$, so that $\hat{v} < \overline{v}$.

has the following characteristics:

- *If $v < \tilde{v}$, then no prize is awarded and $x_1^*(v, \theta) \leq x_1^{SB}(v, \theta)$, whereas $x_i^*(v, \theta) \geq x_i^{SB}(v, \theta)$ for all $i \neq 1$;*

- *If $\tilde{v} < v < \hat{v}$, then no prize is awarded but $x_1^*(v, \theta) \geq x_1^{SB}(v, \theta)$, whereas $x_i^*(v, \theta) \leq x_i^{SB}(v, \theta)$ for all $i \neq 1$;*

- *If $v > \hat{v}$ (which only occurs if $\hat{v} < \bar{v}$), then a prize is awarded to the innovator and $x_1^*(v, \theta) \geq x_1^{SB}(v, \theta)$, whereas $x_i^*(v, \theta) \leq x_i^{SB}(v, \theta)$ for all $i \neq 1$.*

Whether it is optimal to award a monetary prize (i.e., $\hat{v} < \bar{v}$) depends on how much effort needs to be incentivized and on how much incentive would have been provided by the information rents under a standard second-best auction. We show in Online Appendix A that awarding a prize can be optimal when the range of project values is large and when there is either little cost heterogeneity or a large number of firms. The former implies that innovation incentives matter a lot, whereas the latter implies that the procurement auction does not generate much in information rents.

Corollary 1 shows that the optimal mechanism departs from a standard second-best auction in different ways for high-value and low-value projects. In fact, this mechanism can be easily implemented as a variant of common procurement designs.

- $v > \tilde{v}$: *Bidding credit.* In this range, the contract allocation is biased in favor of the innovator, who may be selected to implement the project even when it is not the most efficient firm. In practice, this could be achieved by giving the innovator a bidding credit in the tendering procedure. Bidding credits can take many forms, but most commonly, they consist of additional points in the score of the firm's bid. Such a system is adopted in Chile and Korea to incentivize unsolicited proposals.

- $v < \tilde{v}$: *Handicap.* In this range, the contract allocation is biased against the innovator, who may not be selected to implement the project despite being the most efficient firm. We are not aware of the use of such a bias for procuring innovative projects; however, handicap systems are used, for example, when governments want to favor domestic industries.[15] We discuss this further below (see section 4.1).

We note further that in the region where a cash prize is optimal, the mechanism can be implemented in a very familiar and simple manner:

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[15]Under "preferential price margins", purchasing entities accept bids from domestic suppliers over foreign suppliers as long as the difference in price does not exceed a specific margin of preference. The price preference margin can result from an explicit "buy local policy," e.g., the "Buy America Act."

- $v > \hat{v}$: *Full delegation.* In this range, the innovator is awarded a monetary prize $\rho_1^*(v)$ equal to the full value of the project (net of information rents) and it obtains the contract if $\theta_1 < \min\{v, \min_{i \neq 1} J_i(\theta_i)\}$. This can be achieved by delegating the procurement to the innovator for a fixed price equal to the value of the project. Indeed, suppose that the principal offers a payment v to the innovator to deliver the project either by itself or by subcontracting it to a different firm. The innovator then acts as a prime contractor with the authority to assign production. Under this regime, facing the price $v > \hat{v}$ and given θ_1, the innovator chooses $(x(v, \cdot), t(v, \cdot)) : \Theta^n \to \Delta \times \mathbb{R}^{n-1}$ so as to solve:

$$\max_{x,t} \ \mathbb{E}_{\theta_{-1}} \left[(v - \theta_1) x_1(v, \theta_1, \theta_{-1}) + \sum_{i \neq 1} \{ v x_i(v, \theta_1, \theta_{-1}) - t_i(v, \theta_1, \theta_{-1}) \} \right],$$

subject to (IR) and (IC)

The standard procedure of using the envelope theorem and changing the order of integration results in the optimal allocation x solving

$$\max_{x,t} \mathbb{E}_{\theta_{-1}} \left[(v - \theta_1) x_1(v, \theta_1, \theta_{-1}) + \sum_{i \neq 1} \left[v - J_i(\theta_i) \right] x_i(v, \theta_1, \theta_{-1}) \right],$$

which is exactly the allocation x^* for the case of $v > \hat{v}$.

The above results also have implications for the project adoption itself. For instance, when only the innovator can implement the project ($n = 1$), our results simplify to:

COROLLARY 2. *For $n = 1$, we have:*

- *If $v < \tilde{v}$, then $K(v, \theta) > J(\theta) (> \theta)$: Compared with the first-best, there is a downward distortion – under-implementation of the project – which is even more severe than in the standard second-best.*

- *If $\tilde{v} < v < \hat{v}$, then $J(\theta) < K(v, \theta) < \theta$: There is still a downward distortion compared with the first-best, but it is less severe than in the standard second-best.*

- *If $v \geq \hat{v}$, then $J(\theta) < K(v, \theta) = \theta$: There is no distortion anymore; the project is implemented whenever it should be, from a first-best standpoint.*

Illustration. To illustrate the above insights, consider the following example: (i) implementation costs are uniformly distributed over $\Theta = [0, 1]$; (ii) the innovator can exert an effort $e \in [0, 1]$ at cost $c(e) = \gamma e$; and (iii) the value v is distributed on $[0, 1]$ according to the density $f(v|e) = e + (1 - e) 2 (1 - v)$: exerting effort increases value in the MLRP sense,
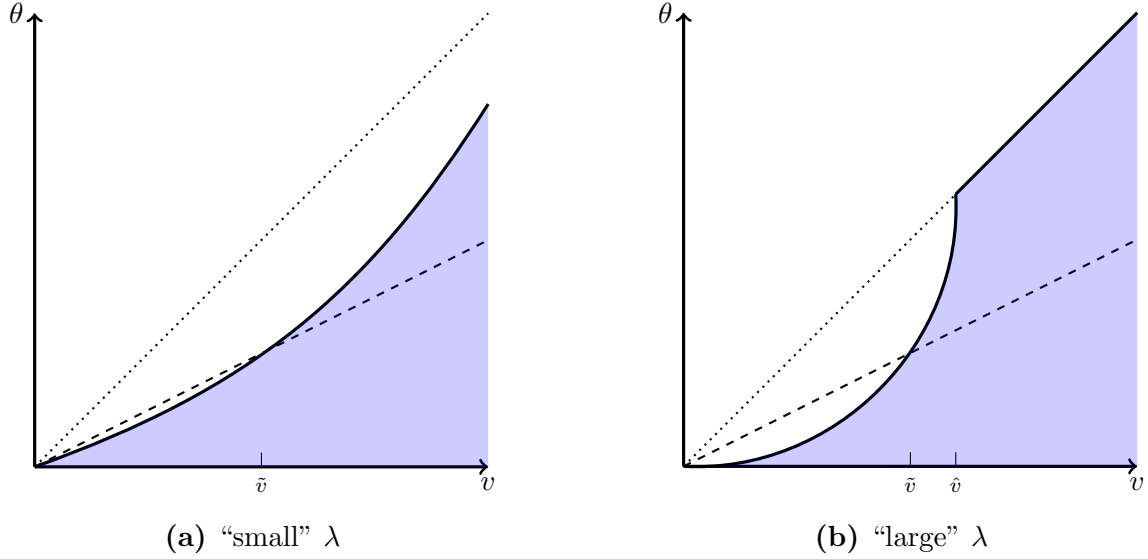
Figure 1 – Project implementation under different values of λ.

from a triangular density peaked at $v = 0$ for $e = 0$ (in particular, $f(1|0) = 0$) to a better (in fact, the uniform) distribution for $e = 1$. Note that $f_e(v|e) = 2v - 1 \gtreqless 0 \iff v \gtreqless \tilde{v} = 1/2$.

The linearity of the cost and benefits ensures that it is optimal to induce maximal effort ($e^* = 1$) as long as the unit cost γ is not too high. Conversely, as long as $e^* = 1$, the Lagrangian multiplier λ^* increases with the cost γ. For exposition purposes, we will use different values of λ^* (reflecting different values of γ) to illustrate the role of innovation incentives.

Consider first the case in which only the innovator can implement its project (i.e., $n = 1$). Figure 1 depicts the range of the firm's costs for which the project is implemented under the optimal contract for different project values v. Figure 1-(a) depicts the case of $\lambda^* = 0.8$, where $\hat{v} = \overline{v}$, implying that a monetary prize is never awarded, whereas Figure 1-(b) shows the case of $\lambda^* = 4$, where $0 = \underline{v} < \tilde{v} < \hat{v} = 5/8 < \overline{v} = 1$. As the cost is uniformly distributed, the highest cost for which the project is implemented also equals the probability of the project being implemented, $p^*(v) := \mathbb{E}_\theta[x_1^*(v, \theta)]$. Compared with the second best, depicted by the dashed line, the optimal mechanism implements the project for a smaller range of costs (thus with a lower probability) when the project has a low value ($v < \tilde{v} = 1/2$), but implements it for a larger range of costs (thus with a higher probability) when the project has a high value ($v > \tilde{v}$). When $\lambda^* = 4$ (Figure 1-(b)), stronger innovation incentives are required for large project values: there is a range of values $v > \hat{v}$ for which (LL) is binding, so that the principal exhausts the use of contract rights as incentive for innovation and she starts offering a cash prize. As noted, in such a case, the optimal assignment coincides with the first-best, depicted by the 45-degree line.

Focusing on the case $\lambda^* = 4$, Figure 2 illustrates the case of $n = 2$ when the cost of each firm is distributed uniformly over $\Theta = [0, 1]$):

- For $v < \tilde{v}$ (see Figure 2-(a), where $v = 1/4$): Compared with the first-best or the standard second-best, it is again optimal to bias the allocation of the contract against the innovator. This is now achieved in two ways. As before, the project is implemented less often than in the second-best (and thus, *a fortiori*, than in the first-best): the optimal mechanism shifts the vertical boundary of project implementation to the left of the second best (depicted by the dashed line). But in addition, when the project is implemented, the innovator obtains the contract less often than in the first-best or the standard second-best, where the more efficient supplier would be selected; graphically, this is reflected by the triangular shaded area.

- For $\tilde{v} < v < \hat{v}$ (see Figure 2-(b), where $v = 7/12$): Compared with the standard second-best (depicted by the dashed lines) it is now optimal to reward the innovator, both by implementing the project more often (rectangular shaded area) and by favoring the innovator in the competition with its rival (triangular shaded area).

- Finally, for $v \geq \hat{v}$ (see Figure 2-(c), where $v = 4/5$), the innovator's shadow cost reduces to its actual cost. The allocation of the contract thus favors the innovator even more, and the project is implemented substantially more often than in the standard second-best (in particular, it is now implemented whenever $\theta_1 < v$), although it is implemented less often than in the first-best (e.g., when $\theta_2 < v < \theta_1$ and $J(\theta_2) = 2\theta_2$). Graphically, the rectangular and triangular shaded areas further expand.

3 Procurement with Multiple Innovators

We now assume that several firms may innovate and propose projects, as well as implement them. This case captures the problem of a buyer who wishes to procure innovative projects, products or services which several firms are capable of developing. The buyer has a clear sense of what she needs but an innovation is necessary in order to fulfil her demand. Examples include the Norwegian Department of Energy procuring a new technology for carbon capture and storage;[16] or the Scottish Government procuring low-cost, safe and effective methods of locating, securing and protecting electrical array cables in Scottish sea conditions.[17] In both instances, the public authority called for projects by means of Request for Proposals (RFP) with detailed specifications, and multiple firms responded by submitting different projects.
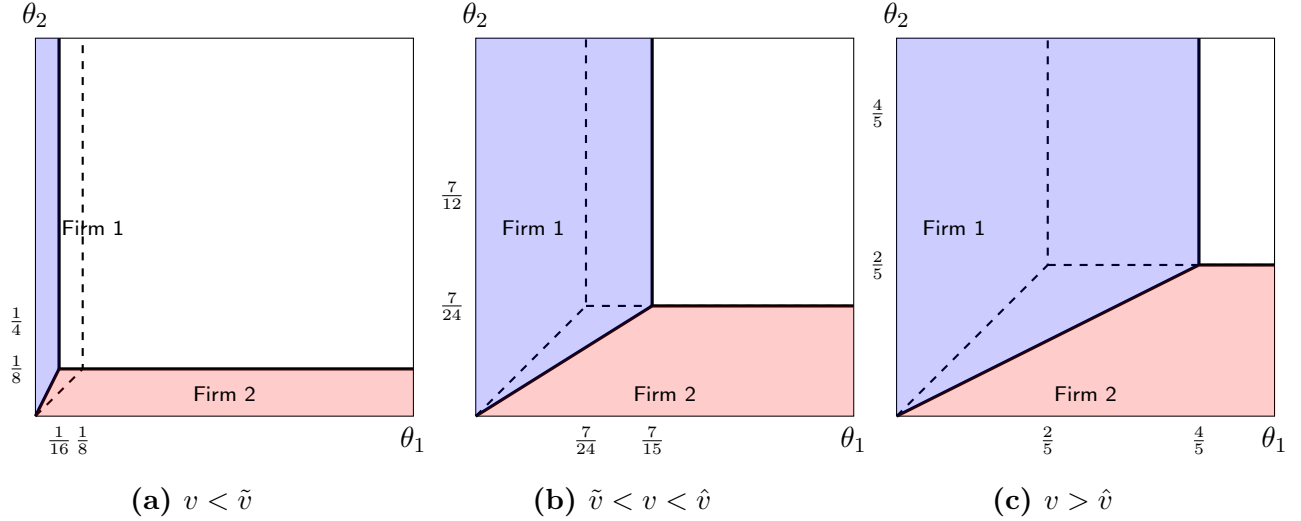
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[16]http://ted.europa.eu/udl?uri=TED:NOTICE:214787-2011:TEXT:EN:HTML&src=0
[17]http://ted.europa.eu/udl?uri=TED:NOTICE:436615-2013:TEXT:EN:HTML&src=0

Figure 2 – Contract assignment under different values of λ.

For the sake of exposition, we will suppose from now on that each firm $k = 1, ..., n$ can develop a project of value v^k, which is publicly observable and distributed over V according to a c.d.f. $F^k(v^k|e^k)$ with density $f^k(v^k|e^k)$, where e^k denotes firm k's innovation effort.[18] We assume that firms decide on these efforts simultaneously, and we denote by $\mathbf{e} = (e^1, ..., e^n)$ the profile of efforts chosen by them. The alternative projects correspond to competing ways to fulfil the same need, so they are substitutes in the sense that the buyer will choose at most one project. The previous setting corresponds to the special case where F^k is concentrated on $\underline{v}$ for all $k \neq 1$.

In practice, a firm's cost of implementing a project may depend on the nature of innovation, including the identity of the innovator. In some cases, the innovator may have cost advantages in implementing the project, for example because of its superior knowledge of the proposed solution. In other cases, the innovator may have cost disadvantages, for example because it is specialized in R&D and lacks the manufacturing capabilities necessary for implementing the developed prototype. To accommodate such an interdependency between innovation and implementation, we assume that firm i's cost of implementing project k is given by $\theta_i + \psi_i^k$, where:

- as before, θ_i is an idiosyncratic shock, privately observed by firm i and distributed according to the c.d.f. G_i;

- ψ_i^k represents an additional cost, potentially both project- and firm-specific, which for simplicity is supposed to be common knowledge.

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[18]While formally all implementors are also innovators, the case of "pure contractors" can be accommodated by setting the density to zero for $v > \underline{v}$.

Without loss of generality, we consider a direct revelation mechanism that specifies an allocation and a payment to each firm as a function of realized project values, $\mathbf{v} = (v^1, ..., v^n)$, and of reported costs. Note that an allocation involves a decision as to which project is selected as well as who implements that project.

A mechanism is thus of the form $(x, t) : V^n \times \Theta^n \to \Delta^{n^2} \times \mathbb{R}^n$. The objective of the principal can now be expressed as:

$$\max_{x, t, \mathbf{e}} \mathbb{E}_{\mathbf{v}, \theta} \left[w\left(\mathbf{v}, \theta\right) \mid \mathbf{e} \right],$$

where the *ex post* net surplus is now equal to

$$w\left(\mathbf{v}, \theta\right) = \sum_{i \in N} \left[\sum_{k \in N} v^k x_i^k\left(\mathbf{v}, \theta\right) - t_i\left(\mathbf{v}, \theta\right) \right].$$

The individual rationality and incentive compatibility constraints become

$$U_i(v, \theta_i) \geq 0, \quad \forall i \in N, \forall \mathbf{v} \in V^n, \forall \theta_i \in \Theta,$$

$$U_i(v, \theta_i) \geq u_i(v, \theta_i'|\theta_i), \quad \forall i \in N, \forall \mathbf{v} \in V^n, \forall (\theta_i, \theta_i') \in \Theta^2,$$

where firm i's *interim* expected profits, when lying and when reporting the truth, are respectively given by:

$$u_i(\mathbf{v}, \theta_i'|\theta_i) = \mathbb{E}_{\theta_{-i}}[t_i(\mathbf{v}, \theta_i', \theta_{-i}) - \sum_{k \in N}(\theta_i + \psi_i^k)x_i^k(\mathbf{v}, \theta_i', \theta_{-i})] \text{ and } U_i(\mathbf{v}, \theta_i) = u_i(\mathbf{v}, \theta_i|\theta_i).$$

Finally, the buyer's limited liability and firms' moral hazard constraints can be expressed as:

$$\mathbb{E}_\theta \left[w\left(\mathbf{v}, \theta\right) \mid \mathbf{e} \right] \geq 0, \quad \forall \mathbf{v} \in V^n,$$

$$e^i \in \arg\max_{\tilde{e}^i} \mathbb{E}_{\mathbf{v}, \theta} \left[U_i(\mathbf{v}, \theta_i) \mid \tilde{e}^i, e^{-i} \right] - c_i(\tilde{e}^i), \quad \forall i \in N.$$

As in the previous section, we assume that an optimal mechanism exists, which induces an interior profile of efforts $\mathbf{e}^*$. The following Proposition then partially characterizes this optimal mechanism:

PROPOSITION 4. *There exists* $\lambda^* = (\lambda^{1*}, ..., \lambda^{n*}) \geq 0$ *such that the optimal mechanism solving* [P]:

- *selects firm i to implement project k with probability*

$$x_i^{k*}(\mathbf{v}, \theta) = \begin{cases} 1 & \text{if } v^k - K_i^*(\mathbf{v}, \theta_i) - \psi_i^k \geq \max\left\{0, \max_{(l,j) \neq (k,i)} v^l - K_j^*(\mathbf{v}, \theta_j) - \psi_j^l\right\}, \\ 0 & \text{otherwise,} \end{cases}$$

  where

$$K_i^*(\mathbf{v}, \theta_i) := J_i(\theta_i) - \left(\frac{\beta^i(v^i)}{\max\{\max_k \beta^k(v^k), 1\}} \right) \left(\frac{G_i(\theta_i)}{g_i(\theta_i)} \right), \text{ and } \beta^i(v^i) := \lambda^{i*} \frac{f_e^i(v^i|e^{i*})}{f^i(v^i|e^{i*})}.$$

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- *awards each firm i an expected transfer*

$$T_i^*(\mathbf{v}, \theta_i) := \rho_i^*(\mathbf{v}) + \sum_{k \in N} \left(\psi_i^k + \theta_i \right) X_i^{k*}(\mathbf{v}, \theta_i) + \int_{\theta_i}^{\overline{\theta}} \sum_{k \in N} X_i^{k*}(\mathbf{v}, s) ds,$$

where $X_i^{k*}(\mathbf{v}, \theta_i) = \mathbb{E}_{\theta_{-i}}[x_i^{k*}(\mathbf{v}, \theta_i, \theta_{-i})]$; and the transfer includes a cash prize

$$\rho_i^*(\mathbf{v}) := \mathbb{E}_\theta \left[\sum_{k,j \in N} x_j^{k*}(\mathbf{v}, \theta) \left\{ v^k - \psi_j^k - J_j(\theta_j) \right\} \right],$$

which is positive only if $\beta^i(v^i) > \max \left\{ \max_{j \in N} \beta^j(v^j), 1 \right\}$.

PROOF. See Online Appendix B. □

To interpret this characterization, consider first the case in which known differences in implementation cost are additively separable across suppliers and projects: $\psi_i^k = \psi_i + \psi^k$ for all i and k. Then, the project selection is simply based on the "net values" of the projects, $v^k - \psi^k$, without regard to who implements the chosen project.[19] Hence, there is no need to wait until the realization of the costs before selecting the project. Even in this case, the realized project values still affect the choice of the supplier through their impact on virtual costs: $K_i^*(v, \theta_i)'$s depend on all realized values, including those of unselected projects.[20] In particular, a higher v^k calls for increasing not only the probability that project k is selected, but also the probability that firm k (the innovator) is selected to implement the chosen project even when project k is not selected.

If the separability condition is not satisfied, the choices of the project and of the supplier are more closely linked. Suppose for instance that $\psi_k^k = 0 < \psi_i^k = \bar{\psi}$ for all k and $i \neq k$: that is, each firm has a cost advantage of $\bar{\psi}$ for the project it proposes. If two firms i and j are such that $v^i > v^j$, but θ_j is significantly lower than θ_i, the desire to exploit this cost advantage may lead the principal to choose project j over project i.

A few other observations are worth making. First, as intuition suggests, the optimal allocation $x_i^{k*}(\mathbf{v}, \theta)$ is nondecreasing in (v^i, θ_{-i}) and nonincreasing in $(\mathbf{v}^{-i}, \theta_i)$. In addition, as all firms are now potential innovators, each virtual cost $K_i^*(\mathbf{v}, \theta_i)$ is characterized by two cutoffs, $\tilde{v}^i$ and $\hat{v}^i$, defined as in the previous section but with somewhat different implications. As before, each innovator is favored by a bias at the implementation stage when $v^i > \tilde{v}^i := \beta_i^{-1}(0)$ and is instead handicapped when $v^i < \tilde{v}^i$. To what extend a firm will be favored or

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[19]To see this, note that the difference in surplus when a contractor i implements project k or project l is given by

$$\left(v^k - K_i - \psi^k - \psi_i \right) - \left(v^l - K_i - \psi^l - \psi_i \right) = \left(v^k - \psi^k \right) - \left(v^l - \psi^l \right),$$

and thus does not depend on which contractor i is selected.

[20]Note that for a "pure contractor," $K_i(\mathbf{v}, \theta_i) = J_i(\theta_i)$, as in a standard second-best auction.

handicapped depends on the relative magnitude of the shadow values $\beta^i(v^i)$ across firms and thus on the values brought by the other projects, $\mathbf{v}^{-i}$.

Second, a "winner-takes-all" principle holds in the sense that at most one firm is awarded a cash prize. As in the case of a single innovator, a cash prize is worth giving only when the incentive benefit $\beta^i(v^i)$ exceeds one. But with multiple innovators, there may be several firms i for which $\beta^i(v^i) > 1$. Due to the limited liability of the buyer, an additional dollar paid to a firm is one less dollar available to reward another firm. As the incentive benefit of a dollar is proportional to $\beta^i(v^i)$, the marginal benefit of the prize is maximized by giving the prize only to the firm with the highest $\beta^i(v^i)$. Splitting the available cash across firms is never optimal for the same reason that it was never optimal to give less than the maximal prize to the innovator in the single innovator case. In the same vein, even if $\beta^i(v^i) > 1$ for several firms, only firm $\hat{i} := \arg\max_{i\in N}\{\beta^i(v^i)\}$ will face undistorted virtual cost $K_{\hat{i}}^*(\mathbf{v}, \theta_{\hat{i}}) = \theta_{\hat{i}}$; the others will face a distorted virtual cost equal to

$$K_i^*(\mathbf{v}, \theta_i) = \theta_i + \left[1 - \frac{\beta^i(v^i)}{\beta^{\hat{i}}(v^{\hat{i}})}\right] \frac{G_i(\theta_i)}{g_i(\theta_i)} > \theta_i.$$

Third, in case a prize is used, it should be given to the firm whose effort was worth incentivizing most (i.e., the firm with the highest $\beta^i(v^i)$), even if it is not the one with the best project (i.e., the highest v^i). In the same vein, the recipient of the prize is not necessarily the firm whose project is selected. For instance, if innovators are better placed to implement their own projects, but firms are otherwise *ex ante symmetric* (so that $\beta^i(\cdot) = \beta(\cdot)$), then the firm with the best project may receive a prize (if the value of its projects exceeds $\hat{v} = \beta^{-1}(1)$), and yet cost considerations may lead the principal to select another project.[21]

Finally, Proposition 4 does not explicitly characterize the set of firms that are induced to make efforts for innovation. Since losing projects are never implemented, innovation efforts are "duplicated." If there is little uncertainty in the outcome of innovation (e.g., the support of f_i is highly concentrated on a narrow support), therefore it may be optimal to induce only one firm to exert efforts. This is entirely consistent with Proposition 4: λ^{i*} will be strictly positive for only one firm i in that case. Asymmetric treatment of firms may arise even when firms are *ex ante* symmetric (i.e., $\psi_i^k = \psi$ and $f^k(\cdot) = f(\cdot)$), since $\lambda^* = (\lambda^{1*}, ..., \lambda^{n*})$, and thus $\beta = (\beta^1, ..., \beta^n)$, could be asymmetric endogenously. In this case, the mechanism would call for treating even *ex ante* symmetric firms asymmetrically.

In practice, however, there are a couple of reasons why multiple firms may be induced to innovate. First, if the outcomes of innovation efforts are sufficiently uncertain and stochastic, then there is a "sampling" benefit from inducing multiple firms to make efforts and generate

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[21]Consider for example the case $n = 2$, and suppose that $\psi_1^1 = \psi_2^2 = 0 < \psi_2^1 = \psi_1^2 = +\infty$ (that is, a firm can only implement its own project). In this case, if $v^1 > \max\{v^2, \hat{v}\}$ but $\theta_1 < \theta_2$, firm 1 receives a prize but firm 2's project is selected if the cost difference is large enough.

favorable draws. Second, a procurer, particularly in the public sector, is often prohibited from discriminating firms especially they are ex ante symmetrically. Proposition 4 is valid even for this case, with $\lambda^{1*} = ... = \lambda^{n*}$. In that case, the best project is selected and, from MLRP, the highest $\beta^i(v^i)$ corresponds to the highest v^i; hence, only that project can ever be awarded a cash prize.

REMARK 1 (Self-serving innovation strategies.). *We have so far assumed that firms' R&D efforts only affect the values of their projects. In practice, firms may have an incentive to target an innovation project that they will be best positioned to implement. For instance, a firm may entrench itself by pursuing an innovation project which no other firm can implement. Such a targeting possibility would further reinforce the main thrust of our results. While the second best auction would actually encourage such self-serving innovation strategies, favoring innovators with high-value projects, as prescribed by our optimal mechanism, would mitigate these incentives and encourage instead the adoption of more valuable innovation strategies.]*

4 Discussion

In this section we discuss how our insights relate to the mechanisms used in practice. We first consider some feasibility issues with respect to handicaps (Section 4.1) and the verifiability of the value of proposals (Section 4.2). We then discuss the implications of our analysis for current practice in the management of unsolicited proposals (Section 4.3) and in the procurement of innovation (Section 4.4).

4.1 On the feasibility of handicaps

The optimal mechanism relies on a "stick and carrot" approach: it rewards good proposals by conferring an advantage in the procurement auction (possibly together with a monetary prize) and punishes weak proposals with a handicap in the procurement auction. In practice, many innovation procurement mechanisms involve cash prizes or distort the contract allocation in favor of the innovators, but handicaps for weak projects do not appear to be used. This may stem from the risk of manipulation: An innovator with a low value project may for instance get around the handicapping by setting up a separate corporate entity to participate in the implementation tender.

To get some sense of how the mechanism would need to be adjusted if handicaps were explicitly ruled out, in Online Appendix C we extend our baseline model by assuming that the innovator cannot be left worse off than under the standard second best mechanism.[22]

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[22]It can be checked that this indeed ensures that the innovator is never worse off than a pure contractor – see Online Appendix C.

That is, the mechanism must take into account the additional constraint:

$$x_1(v, \theta) \geq x_1^{SB}(v, \theta).$$

We show that, keeping constant the multiplier λ for the innovator's incentive constraint, ruling out handicaps has no impact on the contract right for high-value projects (namely, those with $v \geq \tilde{v}$), as $x_1^*(v, \theta) > x_1^{SB}(v, \theta)$ in this case. By contrast, for low-value projects (i.e., those with $v < \tilde{v}$), the no-handicap constraint is binding and the innovator's probability of obtaining the contract is increased, from $x_1^*(v, \theta)$ to $x_1^{SB}(v, \theta)$. Interestingly, the no-handicap constraint does not affect the size of the prize. Of course, removing the "stick" raises the cost of providing innovation incentives, and thus we would expect an increase in the multiplier of the incentive constraint λ (implying that the favorable bias for a high-value project is larger and that the monetary prize is more often awarded) and a reduction in the optimal innovation effort.

4.2   Robust mechanisms with respect to v

The optimal mechanism allocates the project on the basis of its value. In practice, this value may be difficult to measure objectively or costly to verify, which in turn calls for more robust rules. Even in this case, biasing the implementation tender still provides an effective way of incentivizing innovators. To see this, in Online Appendix D we consider a variant of our baseline model in which: (i) the buyer, having observed its value, remains free to decide whether to implement the project or not; and (ii) the implementation tender cannot depend on the value of the project (that is, $x(v, \theta) = x(\theta)$ and $\rho(v) = \rho$ for all v). Obviously, the innovator has no incentives to exert any research effort when the project is never implemented, or when it is always implemented (in this latter case, the innovator gets the same expected information rent, regardless of the value of its proposal). However, if the project is implemented only when it is sufficiently valuable, then it is always optimal to bias the tender in favor of the innovator.[23] Interestingly, handicaps are never optimal in this case. In addition, as long as the principal observes the value of the project, such mechanism can be used regardless of whether this value is also observed by the firms, or can be verified by third parties such as courts.

4.3   Current practice on Unsolicited Proposals

Public authorities are sometimes approached directly by private companies with proposals for developing projects even without any formal solicitation. To allow the contracting authority to make a proper evaluation of the technical and economic feasibility of the project

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[23]Specifically, the shadow costs are of the form $K_1(\theta_1) < J_1(\theta_1)$ and $K_i(\theta_i) = J_i(\theta_i)$ for $i > 1$.

and to determine whether the project is likely to be successfully implemented, the proposer must typically submit a technical and economic feasibility study, an environmental impact study and satisfactory information regarding the concept or technology contemplated in the proposal.[24] Despite the often significant cost involved in the proposal submission, some countries do not allow public authorities to directly reward these unsolicited proposals. Our analysis suggests instead that it can be optimal to reward valuable proposals through contract rights and possibly by monetary prizes. Hodges and Dellacha (2007) describe three alternative ways used in practice:

∘ Bonus system. The system gives the original project proponent a bonus in the tendering procedure. A bonus can take many forms but most commonly involves additional points in the score of the original proponent's technical or financial offer. This system is, for example, adopted in Chile and Korea. **In the former, the bonus points are linked to the value of the proposed project.**

∘ Swiss challenge system. The Swiss challenge system gives the original project proponent the right to counter-match any better offers. It is most common in the Philippines and is also used in Guam, India, Italy, and Taiwan. Under this procedure, the original proposer will counter-match the lowest rival bid and win the contract whenever its cost is less than that bid. Anticipating this, rival bidders will respond by shading their bids but still bid above their costs. Hence, the system distorts the contract allocation in favor of the proposer (who wins the contract for sure when its cost is less than the rivals' costs, but may also win when its cost is above theirs).[25]

∘ Best and final offer system. Here, the key element is multiple rounds of tendering, in which the original proponent is given the advantage of automatically participating in the final round. It is used in Argentina and South Africa.

Our analysis suggests that these mechanisms have some merit, as biasing the implementation stage in favor of the innovator may indeed promote innovation. The bonus system has the additional merit of allowing the advantage to be linked to the value of the proposed project, with higher project values resulting in greater advantages. Furthermore, as we discuss in Section 4.2, the unconditional advantage granted to the innovator under the Swiss challenge system and the best and final offer system can be rationalized when the value of the project is hard to verify. Note that none of these systems provides for explicit handicapping.

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[24]See for example UNICITRAL Model Legislative Provisions on Privately Financed Infrastructure Projects.

[25]See Burguet and Perry (2009) for the formal analysis of the right of first refusal in a procurement context. Their model does not involve *ex ante* investment, however.

4.4   On the Optimality of Bundling R&D and Implementation

In the practice of innovation procurement, we observe two polar cases.

First, under *pure bundling,* the firm whose project is selected also implements it. This approach was, for instance, followed in US Defense Procurement in the 1980s, where the winner of the technical competition for the best prototype was virtually assured of being awarded the follow-on defense contract (see Lichtenberg, 1990; and Rogerson, 1994). More recently, the European Procurement Directive 2014/24/EU has introduced the so-called "Innovation Partnerships" for the joint procurement of R&D services and large-scale production. As mentioned in the Introduction, the US has recently expanded the use of Other Transaction Authority with the National Defence Authorization Act for 2016, thus enhancing the possibily to direct award the production contract to the firm that was awarded the contract for the R&D services.

Second, under *unbundling,* the selection of the project and its implementation are kept entirely separate; therefore, the firm whose project is selected is treated exactly in the same way as any other firm at the implementation stage. Examples include research contests or the European Pre-commercial Procurement (PCP) model,[26] or the standard approach to R&D procurement under the US federal laws and regulations that apply to government procurement contracts).[27] In both cases, firms compete for innovative solutions at the R&D stage, and the best solution(s) may receive a prize. The procurer does not commit itself to acquire the resulting innovations.

Our analysis identifies specific circumstances in which the two extreme cases can be optimal.

COROLLARY 3.   *1. Pure bundling is optimal if, for each $i, k \in N$, $\psi_k^k = 0$ and $\psi_i^k = \infty$ if $i \neq k$.*

    2. Unbundling is optimal if there exists $N_1, N_2 \subset N$ with $N_1 \cup N_2 = N$ and $N_1 \cap N_2 = \emptyset$ such that, for each $i, k \in N$, $\psi_i^k = 0$ if $k \in N_1$ and $i \in N_2$ and $\psi_i^k = \infty$ otherwise. In this case, the optimal mechanism selects the project k from N_1 with the highest value v^k if $v^k \geq \min_j J_j(\theta_j)$, rewards the innovator $i \in N_1$ with the highest $\beta^i(v^i) > 1$, and awards the implementation contract to a firm $j \in N_2$ with the lowest virtual cost $J_j(\theta_j) < v^k$.

Pure bundling can be optimal when there are large economies of scope between R&D and implementation as described by the condition in 3-(1). For example, in the procurement of complex IT systems, the knowledge acquired by the software developer typically confers a

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[26]See EC (2007) and https://ec.europa.eu/digital-agenda/en/pre-commercial-procurement.

[27]Part 35, Federal Acquisition Regulation (FAR), https://www.acquisition.gov/browsefar.

considerable cost advantage for the management and upgrading of the software. In this case, selecting the same firm for both R&D and implementation is likely to be better. However, even in that case, our results stress that the selection of the project should be based on both value and cost considerations.

By contrast, unbundling is optimal when firms specialize in either innovation or in implementation (e.g., manufacturing or construction). Corollary 3-(2) describes such a case: firms are partitioned into two groups so that one specializes in innovation and the other specializes in implementation.[28] In that case, the optimal mechanism selects the project and rewards the innovator from the former group, according to the first-best scheme in Proposition 1, and awards the implementation contract to a firm in the second group according to the second-best scheme in Proposition 2.

Unbundling is sometimes prescribed as an affirmative action policy toward *small and medium enterprises (SMEs)*. In both Europe and the US, procurement programs aimed at stimulating R&D investment from *SMEs* provide for separation between the R&D stage and the implementation stage. Funding is provided based on firms' project proposals. The Small Business Innovation Research (SBIR) program in the US or the Small Business Research Initiative (SBRI) in the UK are characterized by this separation between project selection and implementation.[29] Such a policy can be justified based on Corollary 3-(2) on the ground that small or medium R&D firms often lack manufacturing capabilities and thus would be at a clear disadvantage when the R&D competition is bundled with the contract implementation. For instance, if SMEs constitute group N_1 and non-SMEs constitute N_2, it is then desirable to promote research effort specifically from SMEs and ban non-SMEs from proposing a project (as under SBIR and SBRI). And indeed, a study commissioned by the European Commission[30] finds empirical evidence that PCP (i.e., unbundling) increases both participation by and awarding to SMEs, compared to conventional joint procurement of R&D services and supply (i.e., bundling).

A similar reasoning suggests that when base university research plays a key role in R&D activities, separation between selection and implementation may help to promote universities' participation. When instead innovators are also likely to play a role at the implementation stage, unbundling is never optimal.

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[28]While Corollary 3 portrays pure implementors (firms $j \in N_2$) in terms of high R&D costs, a similar insight applies when they are productive in research (e.g., F^j is concentrated on $\underline{v}$).

[29]See, respectively, http://www.sbir.gov/ and https://sbri.innovateuk.org.

[30]See Bedin, Decarolis and Iossa (2015).

5 Related Literature

On prizes versus property rights to motivate innovation. The issue of prize vs. contract is reminiscent of the well-known debate on the effectiveness of the patent system as a source of incentive for innovation– see Maurer and Scotchmer (2004) and Cabral *et al.* (2006) for reviews. Just as in our model, the patent system involves *ex post* distortion (in terms of both too little quantity and foreclosure of competing firms), making prizes apparently preferable – see, e.g., Kremer, 1998. Yet the literature has shown that, as in this paper, *ex post* distortion can be an optimal way to motivate *ex ante* innovation. The difference lies in the motivation for the *ex post* distortion. In the case of Weyl and Tirole (2012), for instance, the supplier has private information at the *ex ante* (innovation) stage; he obtains property rights to facilitate information revelation. In our case, private information in the *ex post* implementation stage, coupled with limited liability, forces the buyer to leave rents to the winning supplier. These rents can be harnessed as incentives for innovation, but only when the allocation of the contract rights is shifted in favor the innovator. That is, the distortion in the allocation of contracts rights arises as a way to incentivize innovation.

On bundling sequential tasks. Our analysis is related to the literature on whether two tasks should be allocated to the same agent ("bundling") or to two different agents ("unbundling"). The existing literature finds that this choice can be driven by problems of adverse selection (see, e.g., Armendariz and Gollier, 1998; Ghatak, 2000), monitoring (Besley and Coate, 1995; Armendariz, 1999; Rai and Sjöström, 2004), moral hazard (Stiglitz, 1990; Varian 1990; Holmstrom and Milgrom, 1991; Itoh, 1993), or agents' limited liability (Laffont and Rey, 2003). A second strand of literature has focused specifically on sequential tasks. Our paper is particularly related to Riordan and Sappington (1989), who highlighted how sole sourcing (bundling) can serve as commitment devise to incentivize R&D effort, by raising the prospect of a lucrative follow-on contract. In their context, the buyer suffers from limited commitment power and the value of the project is non-verifiable. Like them, we show that contract rights can provide incentive for R&D effort, but we consider verifiable project values and full commitment, thus extending the buyer's options to the possibility that his choice depends on the realized project value, or that he commits ex ante to a given bias.

Other more recent papers have studied the role of externalities across tasks (Bennett and Iossa, 2006), budget constraints (Schmitz, 2013), information on the ex post value of the second task (Tamada and Tsai, 2007) or on the future cost of improving the service provision (Hoppe and Schmitz, 2013) or competition among agents (Li and Yu, 2015). Our paper contributes to this literature by showing that the implementation decision should depend on the value of the proposed project(s) as well as on the supplier's characteristics. Full unbundling is therefore typically not optimal, unless innovators and implementors form distinct groups, while pure bundling is optimal only under rather specific conditions – namely,

when the innovator is in a much better position to implement its project.

On discrimination and bidding parity in auctions. Our analysis is also related to the literature on discrimination in auctions, which finds it optimal to distort the allocation to reduce the information rents accruing to the bidders: discrimination against efficient types helps level the playing field and elicit more aggressive bids from otherwise stronger bidders (Myerson, 1981; McAfee and McMillan, 1985). In a similar vein, when bidders can invest in cost reduction, an *ex post* bias in the auction design can help foster bidders' *ex ante* investment incentives (Bag 1997) or prevent the reinforcement of asymmetry among market participants (Arozamena and Cantillon, 2004). Likewise, manipulating the auction rules can help motivate investment in cost reduction by an incumbent firm (Laffont and Tirole, 1988), incentivize monitoring effort by an auditor (Iossa and Legros, 2004) or favor the adoption of an efficient technology by an inefficient firm (Branco, 2002). We contribute to this literature by showing that when investment is "cooperative" (in the sense of Che and Hausch, 1999) and directly benefits the buyer, both favoritism and handicapping are optimal, depending on the value of the proposed project and on the bidders' costs.

On contests. Finally, another large literature studies the provision of incentives through contests or tournaments. Since the seminal contributions by Tullock (1967, 1980) and Krueger (1974) on rent-seeking, and of Becker (1983) on lobbying, the framework has been applied to many other situations, including research contests.[31] This literature typically assumes that agents' efforts affect the probability of winning the contest, but not the associated reward.[32] By contrast, here the principal can reward innovators with contract rights as well as with a monetary prize, which enables her to influence how innovators' efforts affect their information rents. This, in turn, allows us to analyze the optimal composition of a prize.

Dynamic Contracting. The idea of using future "rents" an agent expects to earn to motivate her earlier effort has been developed in the dynamic contracting literature. For example, Board (2011), Andrews and Baron (2016), Calzolari and Spagnolo (2017) and Calzolari *et al.* (2015) consider moral hazard models, showing that the optimal relational contract rewards good behavior with loyalty. However, the relational models do not allow for contingent prizes, as we do, and cannot therefore discuss the relative value of alternative instruments. The idea that the principal can solve short-term moral hazard problem by committing to a long-term and potentially inefficient allocation of business is also present in Garrett and Pavan (2012) who consider repeated adverse selection and examine the distortions in allocation that can occur over time. They consider both the design of retention

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[31]See Corchón (2007), Konrad (2009) and Long (2013) for surveys of the literature on contests. For research contests, see, e.g., Che and Gale (2003) and Piccione and Tan (1996).for an analysis of R&D investment in cost reduction followed by standard procurement auctions.

[32]Some papers allow the reward to depend on agents' efforts. For instance, in "'winner takes all" races, firms' investments in cost reduction may affect not only the probability of winning the market, but also the profit achieved in that case. However, this relation remains exogenously given.

policy (analogous to "contract rights" here) and the use of managemerial compensations (analogous to "prize" here) and analyze how their use evolves over time. An important distinction between the two papers is that the main concern of the principal in their paper is to minimize informational rents, whilst ours is to incentivize innovation.

6 Conclusion

Procuring innovative projects requires incentivizing research efforts from potential suppliers as well as efficiently implementing the selected projects. Our analysis highlights a trade-off between these two objectives when suppliers have private information about their costs. To solve this trade-off, the optimal mechanism relies on contract rights (possibly combined with monetary prizes).

A number of issues are worth exploring further. First, for the most part we have focused on situations in which the value of the proposals can be contracted upon. This is a plausible assumption when, for instance, the proposal involves a prototype or when performance measures – operational or productivity indicators, energy consumption, emissions, etc. – are available and can be used in tender documents; yet another possibility is to rely on evaluation committees. In other situations (e.g., base research), however, the difficulty of describing the project and/or non-verifiability issues may make it impossible to contract *ex ante* on the *ex post* value of the projects. When *ex post* the value is verifiable, the literature suggests that the procurer can replicate the above mechanism (see, e.g., Maskin and Tirole (1999)); a similar remark applies when *ex post* the value of the project is observed by the parties but is non-verifiable by third parties such as courts (see, e.g., Maskin (1999) and Moore and Repullo (1988)). The situation is different when the value of the project is private information (e.g., only the buyer observes it). Yet, the spirit of our insights carries over when for instance the procurer must use the same auction rules whenever she decides to implement the project (see the discussion in Section 4.2). Characterizing the optimal mechanism under private information is beyond the scope of this paper but clearly constitutes an interesting avenue for future research.

Second, we have ignored the costs of participating in procurement tenders. In practice, submitting a tender bid may require tender development costs (e.g., complex estimations and legal advice) that involve significant economic resources, in which case biasing the tender in favor of the innovator may discourage potential suppliers from participating in the tender. It would therefore be worth endogenizing the participation in the tender and exploring how the optimal mechanism should be adjusted to account for these development costs. More generally, accounting for endogenous entry is a promising research avenue.[33]

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[33]For recent work on the role of discrimination in auctions with endogenous entry, see, e.g., Jehiel and

Likewise, we have assumed that the procurer was benevolent. In practice, corruption concerns and, more generally, institution design may matter, which may call for limiting the discretion given to the procuring agency. Balancing this with the provision of innovation incentives constitutes another promising research avenue.[34]

Finally, we have focused on a situation where the innovation is valuable to a single buyer – and thus has no "market" value. An interesting extension would be to consider multiple buyers, so as to allow for the possibility that extra contractual incentives for research effort arise from the commercialization of the innovation. Exploring the role of market forces would also help to shed light on possible anti-competitive effects of alternative mechanisms for the public procurement of innovation.[35]

Lamy (2015).

[34]For recent work on the role of corruption in procurement auctions see, e.g., Burguet (2015).

[35]See the 2014 European State Aid framework for research, development and innovation (EU 2014b).

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Appendix

A   Proof of Proposition 1

To solve $[P - FB]$, we focus on the relaxed problem:

$$[P' - FB] \qquad \max_{x,t} \mathbb{E}_{v,\theta}[v \sum_{i \in N} x_i(v,\theta) - t_i(v,\theta)|e]$$

subject to (LL), (MH) and

$$\mathbb{E}_\theta[t_i(v,\theta) - \theta_i x_i(v,\theta)] \geq 0, \quad \forall v, i. \qquad (IR')$$

This problem is a relaxation of $[P - FB]$ since (IR') requires (IR) to hold only on average. At the same time, whenever a mechanism satisfies (IR'), one can construct at least one mechanism that satisfies (IR), without affecting other constraints. Hence, there is no loss in restricting attention to $[P' - FB]$. To solve $[P' - FB]$, we first observe that for each $i \neq 1$, the constraint (IR') must bind. If not, one can always lower the expected payment to increase the value of the objective without tightening any constraints. Next, define

$$\rho_1(v) := \mathbb{E}_\theta[t_1(v,\theta) - \theta_1 x_1(v,\theta)].$$

Then, we can weaken $[P' - FB]$ further to:

$$[P'' - FB] \qquad \max_{x,t} \mathbb{E}_{v,\theta}[\sum_{i \in N}(v - \theta_i)x_i(v,\theta) - \rho_1(v)|e]$$

subject to

$$\rho_1(v) \geq 0, \quad \forall v, \qquad (IR'')$$

$$\mathbb{E}_\theta[\sum_{i \in N}(v - \theta_i)x_i(v,\theta)] \geq \rho_1(v), \quad \forall v, \qquad (LL'')$$

$$\frac{\partial}{\partial e}\mathbb{E}_v[\rho_1(v)|e] \geq c'(e). \qquad (MH'')$$

Note that the weakening occurs with the moral hazard constraint: (MH'') is a first-order necessary condition of (MH).

Let $\nu(v)$, $\mu(v)$, and λ denote the multipliers for constraints (IR''), (LL'') and (MH''), respectively. Then, the Lagrangian (more precisely its integrand) is given by:

$$L(v,\theta,e) := [1 + \mu(v)] \left\{ \sum_{i \in N} x_i(v,\theta)(v - \theta_i) \right\} - \rho_1(v)[1 + \mu(v) - \nu(v) - \beta(v)] - \lambda c'(e),$$

where
$$\beta(v) := \lambda \frac{f_e(v|e)}{f(v|e)}.$$

The optimal solution $(e^{FB}, x^{FB}(v,\theta), \rho_1^{FB}(v), \lambda^{FB}, \mu^{FB}(v), \nu^{FB}(v))$ must satisfy the following necessary conditions.

First, since the Lagrangian is linear in x_i's, the optimal solution $x_i^{FB}(v,\theta)$ is as defined in Proposition 1. Next, the Lagrangian L is also linear in $\rho_1(v)$; hence, its coefficient must be equal to zero:
$$1 + \mu^*(v) - \beta^*(v) - \nu^*(v) = 0. \tag{1}$$

Next, the optimal effort e^{FB} must satisfy

$$\left. \frac{\partial}{\partial e} \mathbb{E}_{v,\theta}[L(v,\theta,e)|e] \right|_{e=e^{FB}} = 0. \tag{2}$$

Finally, complementary slackness implies that, for each v,

$$\nu^{FB}(v)\rho_1^{FB}(v) = 0, \tag{3}$$

$$\mu^{FB}(v) \left\{ \mathbb{E}_\theta[\sum_{i \in N} x_i^{FB}(v,\theta)(v-\theta_i) - \rho_1^{FB}(v) \right\} = 0, \tag{4}$$

and

$$\lambda^{FB} \left[\int_v \rho_1^{FB}(v) f_e(v|e^{FB})dv - c'(e^{FB}) \right] = 0. \tag{5}$$

We first prove that $\lambda^{FB} > 0$. Suppose not. Then, $\beta^{FB}(v) = 0$ for all $v \in V$. It then follows from (1) that $\nu^{FB}(v) > 0$ for all $v \in V$. By (3), this means that $\rho_1^{FB}(v) \equiv 0$. As $x_i = x_i^{FB}$, it then follows from (4) that for any $v > \underline{\theta}$, $\mu^{FB}(v) = 0$. Collecting these facts together, we conclude that

$$\mathbb{E}_\theta[L(v,\theta,e)] = \mathbb{E}_\theta[\max\{0, v - \min_i \{\theta_i\}\}],$$

which is increasing in v (and strictly so for a positive measure of v). By (MLRP), this means that
$$\frac{\partial}{\partial e} \mathbb{E}_{v,\theta}[L(v,\theta,e)|e] > 0,$$
a contradiction to (2). We thus conclude that $\lambda^{FB} > 0$.

If $v < \hat{v}^{FB}$, then $\beta^{FB}(v) < 1$, and thus $1 + \mu^{FB}(v) - \beta^{FB}(v) > 0$. Hence, by (1), $\nu^{FB}(v) > 0$ and, by (3), we have $\rho_1^{FB}(v) = 0$. It in turn follows from (4) that $\mu^{FB}(v) = 0$ provided that $v > \underline{\theta}$.

If instead $v > \hat{v}^{FB}$, then $\beta^{FB}(v) > 1$, and thus $1 - \beta^{FB}(v) - \nu^{FB}(v) < 0$. Hence, by (1), $\mu^{FB}(v) > 0$. But then, by (4), we must have

$$\rho_1^{FB}(v) = \mathbb{E}_\theta \left[\sum_{i \in N} x_i^{FB}(v, \theta)(v - \theta_i) \right],$$

as claimed in Proposition 1.

Next, we show that $\underline{v} < \hat{v}^{FB} < \bar{v}$. First, by ($MLRP$), $\beta^{FB}(v)$ is strictly increasing in v, and there exists $\tilde{v} \in (\underline{v}, \bar{v})$ such that $\beta^{FB}(\tilde{v}) = 0 \, (< 1)$; it follows that $\hat{v}^{FB} > \tilde{v} \, (> \underline{v})$. Second, we must have $v > \hat{v}^{FB}$ with positive probability (i.e., λ^{FB} cannot be too small). Suppose to the contrary that $\beta^{FB}(v) < 1$ for all $v \in V$. Then, as argued above $\rho_1^{FB}(v) = \mu^{FB}(v) = 0$ for all $v \in V$. In this case, by the convexity of $c(\cdot)$, we must have $e^{FB} = 0$, or else we obtain a contradiction to (5). But then, we get

$$L(v, \theta, e) = \max\{0, \max_{\theta_i}(v - \theta_i)\} - c'(e).$$

As the first term is increasing in v (and strictly so for a positive measure of v), and $c'(0) = 0$, we thus get a contradiction to (2).

Finally, we prove that $e^{FB} > 0$. Given $\lambda^{FB} > 0$, it follows from (5) that

$$\int_v \rho_1^{FB}(v) f_e(v | e^{FB}) dv = c'(e^{FB}).$$

As $v > \hat{v}^{FB}$ for a positive measure of v, the left side is strictly positive. This implies that $e^{FB} > 0$, or else the right-hand side vanishes as $c'(0) = 0$.

B Proof of Proposition 3

To solve $[P]$, we first reformulate (IC) in terms of interim allocation and payment rules. For each $i \in N$ and for any $v \in V$ and any $\theta_i \in \Theta_i$, let $X_i(v, \theta_i) := \int_{\theta_{-i}} x_i(v, \theta) \, dG_{-i}(\theta_{-i})$ and $T_i(v, \theta_i) := \int_{\theta_{-i}} t_i(v, \theta) \, dG_{-i}(\theta_{-i})$ denote the interim allocation and payment for firm i and

$$U_i(v, \theta_i) := T_i(v, \theta_i) - \theta_i X_i(v, \theta_i) \tag{6}$$

denote firm i's expected profit. For each $i \in N$, (IC) then can be stated as

$$T_i(v, \theta_i) - \theta_i X_i(v, \theta_i) \geq T_i(v, \theta_i') - \theta_i X_i(v, \theta_i'), \forall v, \theta_i, \theta_i'.$$

The associated envelope condition then yields

$$U_i(v, \theta_i) = \rho_i(v) + \int_{\theta_i}^{\bar{\theta}} X_i(v, \theta) d\theta, \tag{7}$$

where

$$\rho_i(v) := U_i(v, \bar{\theta})$$

is the rent enjoyed by firm i when its cost is highest. Using (7), we can express firm i's expected rent as

$$
\begin{aligned}
\int_{\theta_i} U_i(v, \theta_i) dG_i(\theta_i) &= \int_{\theta_i} \left[\rho_i(v) + \int_{\theta_i}^{\bar{\theta}} X_i(v, s) ds \right] dG_i(\theta_i) \\
&= \rho_i(v) + \int_{\theta_i} X_i(v, \theta_i) \frac{G_i(\theta_i)}{g_i(\theta_i)} dG_i(\theta_i).
\end{aligned}
\tag{8}
$$

For each $i \neq 1$, the rent $\rho_i(v)$ does not help to relax any constraint and reduces the surplus for the principal, so it is optimal to set $\rho_i(v) = 0$ for all v.

Using (6) and (8), the total expected transfer to the firms can be expressed as:

$$
\begin{aligned}
\int_\theta \sum_{i \in N} t_i(v, \theta) dG(\theta) &= \sum_{i \in N} \int_{\theta_i} T_i(v, \theta_i) dG_i(\theta_i) \\
&= \sum_{i \in N} \int_{\theta_i} \left[U_i(v, \theta_i) + \theta_i X_i(v, \theta_i) \right] dG_i(\theta_i) \\
&= \sum_{i \in N} \left\{ \rho_i(v) + \int_{\theta_i} X_i(v, \theta_i) J_i(\theta_i) dG_i(\theta_i) \right\} \\
&= \rho_1(v) + \int_\theta \sum_{i \in N} x_i(v, \theta_i) J_i(\theta_i) dG(\theta),
\end{aligned}
\tag{9}
$$

where $J_i(\theta_i) := \theta_i + \frac{G_i(\theta_i)}{g_i(\theta_i)}$ denotes firm i's virtual cost.

Substituting (9) into the principal's objective function, we can rewrite (LL) as follows:

$$\forall v \in V, \quad \int_\theta \left\{ \sum_{i \in N} x_i(v, \theta) \left[v - J_i(\theta_i) \right] \right\} dG(\theta) \geq \rho_1(v). \tag{$\widehat{LL}$}$$

Let $\mu(v) \geq 0$ denote the multiplier associated with this constraint.

The innovating firm's individual rationality simplifies to

$$\forall v \in V, \quad \rho_1(v) \geq 0. \tag{$\widehat{IR}$}$$

Let $\nu(v) \geq 0$ denote the multiplier associated with this constraint.

We next focus on the first-order condition for the effort constraint.

$$\int_v \int_\theta \left[\rho_1(v) + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1(v, \theta) \right] dG(\theta) f_e(v|e) dv \geq c'(e). \tag{$\widehat{MH}$}$$

Note that we formulate the condition as a weak inequality to ensure the nonnegativity of the multiplier. Let $\lambda \geq 0$ be the associated multiplier.

Then, $[P]$ can more succinctly be reformulated as follows:

$$\max_{e,x(v,\theta),\rho_1(v)} \int_v \left\{ \int_\theta \left[\sum_{i \in N} x_i(v,\theta) \left[v - J_i(\theta_i) \right] \right] dG(\theta) - \rho_1(v) \right\} f(v|e) dv$$

$$\text{subject to } (\widehat{LL}), (\widehat{IR}) \text{ and } (\widehat{MH})$$

The integrand of the Lagrangian is given by:

$$L(v,\theta,e) := [1 + \mu(v)] \left\{ \left[v - \theta_1 - \left(1 - \frac{\beta(v)}{1 + \mu(v)} \right) \frac{G_1(\theta_1)}{g_1(\theta_1)} \right] x_1(v,\theta) + \sum_{\substack{j \in N \\ j \neq 1}} [v - J_j(\theta_j)] x_j(v,\theta) \right\}$$

$$- \rho_1(v) \left[1 + \mu(v) - \nu(v) - \beta(v) \right] - \lambda c'(e),$$

where

$$\beta(v) := \lambda \frac{f_e(v|e)}{f(v|e)}.$$

The optimal solution $(e^*, x^*(v,\theta), \rho_1^*(v), \lambda^*, \mu^*(v), \nu^*(v))$ must satisfy the following necessary conditions. First, observe that the Lagrangian L is linear in $\rho_1(v)$; hence, its coefficient must be equal to zero:

$$1 + \mu^*(v) - \beta^*(v) - \nu^*(v) = 0. \tag{10}$$

The Lagrangian is also linear in x_i's, so the optimal allocation must satisfy, for every i, v, θ:

$$x_i^*(v,\theta) = \begin{cases} 1 & \text{if } i \in \arg\min_j \left\{ \tilde{K}_j(v,\theta_j) \right\} \text{ and } \tilde{K}_i(v,\theta_i) \leq v, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\tilde{K}_i(v,\theta_i) := \begin{cases} J_i(\theta_i) - \frac{\beta^*(v)}{1+\mu^*(v)} \frac{G_i(\theta_i)}{g_i(\theta_i)} & \text{if } i = 1, \\ J_i(\theta_i) & \text{if } i \neq 1, \end{cases}$$

where $\beta^*(v) = \lambda^* \frac{f_e(v|e^*)}{f(v|e^*)}$.

Next, the optimal effort e^* must satisfy

$$\frac{\partial}{\partial e} \int_v \int_\theta L(v,\theta,e^*) f(v|e^*) dG(\theta) dv = 0. \tag{11}$$

Finally, complementary slackness implies that, for each v,

$$\nu^*(v)\rho_1^*(v) = 0, \tag{12}$$

$$\mu^*(v) \left\{ \int_\theta \sum_{i \in N} x_i^*(v,\theta) \left[v - J_i(\theta_i) \right] dG(\theta) - \rho_1^*(v) \right\} = 0, \tag{13}$$

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and

$$\lambda^* \left[\int_v \int_\theta \left[\rho_1^*(v) + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1^*(v,\theta) \right] dG(\theta) \, f_e(v|e^*) dv - c'(e^*) \right] = 0. \tag{14}$$

We now provide the characterization. Consider first the case where $v < \underline{\theta}$. From (10), $\tilde{K}_i(v,\theta_i) \geq \theta_i$, and thus $\tilde{K}_i(v,\theta_i)$ and $K_i^*(v,\theta_i)$ both yield $x_i^*(v,\theta) = 0$ for every $i \in N$; furthermore, $(\widehat{LL})$ and $(\widehat{IR})$ together imply

$$\rho_1^*(v) = 0 = \int_\theta \sum_{i \in N} x_i^*(v,\theta) \left[v - J_i(\theta_i) \right] dG(\theta).$$

Hence, the characterization of $x_i^*(v,\theta)$ given in Proposition 3 is correct.

We now focus on the range $v > \underline{\theta}$. Again, there are two cases depending on the value of v. Consider first the case $v < \hat{v}$, where $\beta^*(v) < 1$. Hence, $1 + \mu^*(v) - \beta^*(v) > \mu^*(v) \geq 0$, and (10) thus implies $\nu^*(v) > 0$. The complementary slackness condition (12) then yields $\rho_1^*(v) = 0$. This, together with Lemma 4 (see Online Appendix B) and the complementary slackness condition (13), implies that $\mu^*(v) = 0$. Hence, $\tilde{K}_1(v,\theta_1) = J_1(\theta_1) - \beta^*(v) G_1(\theta_1)/g_1(\theta_1) = K_1^*(v,\theta_1)$.

Let us now turn to the case $v > \hat{v}$, where $\beta^*(v) > 1$. Hence, $1 - \beta^*(v) - \nu^*(v) < 0$, and (10) thus implies that $\mu^*(v) > 0$; from the complementary slackness condition (13), we thus have

$$\rho_1^*(v) = \int_\theta \sum_{i \in N} x_i^*(v,\theta) \left[v - J_i(\theta_i) \right] dG(\theta).$$

Suppose $\nu^*(v) > 0$. Lemma 4 (of Online Appendix B) then implies $\rho_1^*(v) > 0$, contradicting the complementary slackness condition (12). Therefore, $\nu^*(v) = 0$. It follows now from (10) that $1 + \mu^*(v) = \beta^*(v)$. We therefore conclude that $\tilde{K}_1(v,\theta_1) = \theta_1 = K_1^*(v,\theta_1)$.

The expected transfer payment $T_i^*(v,\theta_i)$ follows from (6) and (7), with $\rho_1^*(v)$ as described above and $\rho_j^*(v) = 0$ for all $j \neq 1$. The above characterization is valid only when the optimal allocation is monotonic (another necessary condition from incentive compatibility). This follows the assumption that $\frac{G_i(\theta_i)}{g_i(\theta_i)}$ is nondecreasing in θ_i, which implies that $K_i^*(v,\theta_i) = J_i(\theta_i)$, for $i \neq 1$, and

$$K_1^*(v,\theta_1) = J_1(\theta_1) - \min\{1, \beta(v)\} \frac{G_1(\theta_1)}{g_1(\theta_1)} = \theta_1 + \max\{0, 1 - \beta(v)\} \frac{G_1(\theta_1)}{g_1(\theta_1)},$$

are all nondecreasing in θ_i.

We next prove that $\lambda^* > 0$. Suppose $\lambda^* = 0$. Then, $\beta^*(\cdot) = 0$, so (10) again implies that $\nu^*(\cdot) > 0$ and $\mu^*(\cdot) = \rho_1^*(\cdot) = 0$. Hence,

$$L(v,\theta,e^*) = \max\{0, v - \min_i J_i(\theta_i)\},$$

which increases for a positive measure of v. It follows that

$$\frac{\partial}{\partial e}\int_{\underline{v}}^{\overline{v}}\int_{\theta}L(v,\theta,e)dG(\theta)f(v|e)dv\bigg|_{e=e^*}=\int_{\underline{v}}^{\overline{v}}\int_{\theta}\max\{0,v-\min_{i}J_i(\theta_i)\}dG(\theta)f_e(v|e^*)dv>0,$$

which contradicts (11).

Next, we show that $e^*>0$. It follows from (14) and $\lambda^*>0$ that

$$\int_{v}\int_{\theta}\left[\rho_1(v)+\frac{G_1(\theta)}{g_1(\theta)}x_1^*(v,\theta)\right]g(\theta)d\theta f_e(v|e)dv=c'(e).$$

The left-hand side is strictly positive, which implies that $e^*>0$, or else the right side vanishes since $c'(0)=0$.

Online Appendix
Not for publication

A On the Optimality of Offering a Prize $(\hat{v} < \bar{v})$

As mentioned, whether it is optimal to award a monetary prize (i.e., $\hat{v} < \bar{v}$) depends on how much innovation incentives are required and on how much would already be provided by the standard second-best auction. We show in this Online Appendix that a monetary prize is optimal when: (i) there is either little cost heterogeneity (see Section A.1) or a large number of firms (see Section A.2), as the procurement auction does not generate much information rents, and thus provides little innovation incentives; or (ii) the range of project values is large (see Section A.3), so that innovation incentives then matter a lot.

Throughout this Online Appendix, we start with an environment for which there exists an optimal mechanism with no monetary reward, and then consider variations of this environment for which the optimal mechanism must involve a prize.

The baseline environment, for which there exists an optimal mechanism with no monetary reward, consists of a distribution $F(\cdot|e)$ for the value v and a distribution $G_i(\cdot)$ for the cost of each firm $i \in N$, such that $\rho^*(\cdot) = 0$, which amounts to $\hat{v} > \bar{v}$, or

$$\lambda^* < \bar{\lambda} := \frac{f(\bar{v}|e^*)}{f_e(\bar{v}|e^*)}, \tag{15}$$

and implies that $\mu^*(\cdot) = 0$. The optimal allocation is therefore such that $x_i^*(v, \theta) = 0$ for any $v \le \underline{\theta}$ and, for $v > \underline{\theta}$:

$$x_1^*(v, \theta) = \begin{cases} 1 & \text{if } K_1^*(\theta_1) < \min\{v, J_2(\theta_2), ..., J_n(\theta_n)\}, \\ 0 & \text{otherwise.} \end{cases}$$

For further reference, it is useful to note that the objective of the principal, as a function of e, can be expressed as:

$$\int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^*(v, \theta) [v - J_i(\theta_i)] dG(\theta) dF(v|e)$$

$$+ \quad \lambda \left\{ \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^*(v, \theta_1) G_1(\theta_1) f_e(v|e) d\theta_1 dv - c'(e) \right\},$$

where the innovator's expected probability of obtaining the contract is given by:

$$X_1^*(v, \theta_1) = \int_{\theta_{-1}} x_1^*(v, \theta) dG_{-1}(\theta_{-1}).$$

The first-order condition with respect to e yields:

$$\int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^* (v, \theta) [v - J_i (\theta_i)] \, dG(\theta) f_e(v|e^*) \, dv$$
$$= \lambda \left\{ c''(e^*) - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^*(v, \theta_1) G_1(\theta_1) f_{ee}(v|e^*) d\theta_1 dv \right\}. \tag{16}$$

The optimal effort e^* moreover satisfies the innovator's incentive constraint $c'(e^*) = b(e^*)$, where the innovator's expected benefit is given by:

$$b(e) := \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^*(v, \theta_1) G_1(\theta_1) f_e(v|e) d\theta_1 dv.$$

A.1 Reducing Cost Heterogeneity

Suppose first that costs become increasingly less heterogeneous: the cost of each firm $i \in N$ becomes distributed according to $G_i^m(\theta_i)$ over the range $\Theta_i^m = \left[\underline{\theta}, \bar{\theta}^m = \underline{\theta} + \left(\bar{\theta} - \underline{\theta}\right)/m\right]$. For each $m \in \mathbb{N}^*$, we will denote by e^m, λ^m, $K_1^m(\theta_1)$ and $X_1^m(v, \theta_1)$ the values associated with the optimal mechanism. We now show that, for m large enough, this optimal mechanism must include a monetary prize.

We first note that as m goes to infinity, the innovator's effort tends to the lowest level, $\underline{e}$:

LEMMA 1. e^m tends to $\underline{e}$ as m goes to infinity.

PROOF. The innovator's expected benefit becomes

$$b^m(e) := \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v, \theta_1) G_1^m(\theta_1) f_e(v|e) d\theta_1 dv,$$

and satisfies:

$$|b^m(e)| \le \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\underline{\theta} + \frac{\bar{\theta} - \underline{\theta}}{m}} d\theta_1 |f_e(v|e)| \, dv = \frac{\left(\bar{\theta} - \underline{\theta}\right) \int_{\underline{v}}^{\bar{v}} |f_e(v|e)| \, dv}{m}.$$

Therefore, as m goes to infinity, the expected benefit $b^m(e)$ converges to 0, and the innovator's effort thus converges to the minimal effort, $\underline{e}$. □

Furthermore:

LEMMA 2. As m goes to infinity:

- The left-hand side of (16) tends to

$$B^\infty := \int_{\underline{v}}^{\bar{v}} (v - \underline{\theta}) f_e(v|\underline{e}) \, dv > 0.$$

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- *In the right-hand side of* (16), *the terms within brackets tend to* $c''(\underline{e})$.

PROOF. The left-hand side of (16) is of the form $\int_{\underline{v}}^{\bar{v}} h_1^m(v)\, dv$, where

$$h_1^m(v) := f_e(v|e^m) \int_{\underline{\theta}}^{\bar{\theta}^m} \sum_{i \in N} x_i^m(v, \theta)\, [v - J_i(\theta_i)]\, dG^m(\theta).$$

Furthermore, for any $v > \underline{\theta}$, $\hat{J}(\theta) := \min_{i \in N} \{J_i(\theta_i)\} < v$ for m is large enough (namely, for m such that $\bar{\theta}^m < v$ or $m > (\bar{\theta} - \underline{\theta})/(v - \underline{\theta})$), and so

$$h_1^m(v) = f_e(v|e^m) \int_{\underline{\theta}}^{\bar{\theta}^m} \left[v - \hat{J}(\theta)\right] dG^m(\theta),$$

which is bounded:

$$|h_1^m(v)| < \left| \max_e f_e(v|e) \right| \max\{v - \underline{\theta}, 0\},$$

and converges to

$$\lim_{m \to \infty} h_1^m(v) = (v - \underline{\theta})\, f_e(v|\underline{e}).$$

Using Lebesgue's dominated convergence theorem, we then have:

$$\lim_{m \to \infty} \int_{\underline{v}}^{\bar{v}} h_1^m(v)\, dv = \int_{\underline{v}}^{\bar{v}} \lim_{m \to \infty} h_1^m(v)\, dv = B^\infty.$$

We now turn to the right-hand side (16). The terms within brackets are

$$c''(e^m) - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v, \theta_1)\, G_1^m(\theta_1)\, f_{ee}(v|e^m)\, d\theta_1 dv,$$

where the first term tends to $c''(\underline{e})$ and the second term is of the form $\int_{\underline{v}}^{\bar{v}} h_2^m(v)\, dv$, where

$$h_2^m(v) = f_{ee}(v|e^m) \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v, \theta_1)\, G_1^m(\theta_1)\, d\theta_1$$

satisfies:

$$|h_2^m(v)| < \max_e |f_{ee}(v|e)| \int_{\underline{\theta}}^{\bar{\theta}^m} d\theta_1 = \frac{(\bar{\theta} - \underline{\theta})\max_e |f_{ee}(v|e)|}{m}$$

and thus tends to 0 as m goes to infinity. $\square$

To conclude the argument, suppose that the optimal mechanism never involves a prize. Condition (16) should thus hold for any m, and in addition, the Lagrangian multiplier λ^m should satisfy the boundary condition (15). We should thus have:

$$\int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} \sum_{i \in N} x_i^m(v, \theta)\, [v - J_i(\theta_i)]\, dG(\theta)\, f_e(v|e^m)\, dv$$

$$< \frac{f(\bar{v}|e^m)}{f_e(\bar{v}|e^m)} \left\{ c''(e^m) - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v, \theta_1)\, G_1(\theta_1)\, f_{ee}(v|e^m)\, d\theta_1 dv \right\}.$$

Taking the limit as m goes to infinity, this implies:

$$B^\infty = \int_{\underline{v}}^{\bar{v}} (v - \underline{\theta}) f_e(v|\underline{e}) dv < \frac{f(\bar{v}|\underline{e})}{f_e(\bar{v}|\underline{e})} c''(\underline{e}),$$

which is obviously violated when the return on effort is sufficiently high (e.g., $c''(\underline{e})$ is low enough).

A.2 Increasing the Number of Firms

Let us now keep the cost distributions fixed, and suppose instead that m additional firms are introduced in the environment with the same cost distribution as the innovator: $G_k(\theta_k) = G_1(\theta_k)$ for $k = n+1, ..., n+m$. Letting again denote by e^m, λ^m, $K_1^m(\theta_1)$ and $X_1^m(v, \theta_1)$ the values associated with the optimal mechanism, we now show that the optimal mechanism must involve a prize for m large enough.

By construction, $K_1^m(\theta_1)(> \theta_1) > \underline{\theta}$ for any $\theta_1 > \underline{\theta}$, whereas the lowest $J_j(\theta_j)$ becomes arbitrarily close to $J_1(\underline{\theta}) = \underline{\theta}$ as m increases; it follows that the probability of selecting the innovator, $X_1^m(v, \theta_1)$, tends to 0 as m goes to infinity:

LEMMA 3. $X_1^m(v, \theta_1)$ tends to 0 as m goes to infinity.

PROOF. The probability of selecting the innovator satisfies:

$$
\begin{aligned}
X_1^m(v, \theta_1) &\leq \Pr\left[K_1^m(\theta_1) \leq \min_{j=n+1,...,n+m}\{J_1(\theta_j)\}\right] \\
&\leq \Pr\left[\theta_1 \leq \min_{j=n+1,...,n+m}\{J_1(\theta_j)\}\right] \\
&= \left[1 - G_1\left(J_1^{-1}(\theta_1)\right)\right]^m,
\end{aligned}
\tag{17}
$$

where the second inequality stems from $K_1^m(\theta_1) \geq \theta_1$, and the last expression tends to 0 when m goes to infinity. $\square$

It follows that Lemma 1 still holds, that is, the innovator's effort tends to the lowest level, $\underline{e}$, as m goes to infinity. To see this, it suffices to note that the innovator's expected benefit, now equal to

$$b^m(e) = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v, \theta_1) G_1(\theta_1) f_e(v|e) d\theta_1 dv,$$

satisfies:

$$|b^m(e)| \leq \int_{\underline{v}}^{\bar{v}} h(v) dv,$$

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where

$$h(v) := |f_e(v|e)| \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v, \theta_1) \, d\theta_1$$

is bounded (by $(\bar{\theta} - \underline{\theta}) \max_{v,e} \{|f_e(v|e)|\}$) and, from the previous Lemma, tends to 0 as m goes to infinity. Hence, as m goes to infinity, the expected benefit $b^m(e)$ converges to 0, and the innovator's effort thus tends to $\underline{e}$.

Likewise, Lemma 2 also holds; that is,

- The left-hand side of (16) tends to B^∞. To see this, it suffices to follow the same steps as before, noting that $h_1^m(v)$, now given by

$$h_1^m(v) = \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^m(v, \theta) [v - J_i(\theta_i)] \, dG(\theta) \, f_e(v|e^m),$$

  is still bounded:

$$|h_1^m(v)| < \max\{v - \underline{\theta}, 0\} \left|\max_e f_e(v|e)\right|,$$

  and tends to $(v - \underline{\theta}) f_e(v|\underline{e})$ for any $v > \underline{\theta}$:

  - $\hat{J}(\theta) = \min_{i \in N} \{J_i(\theta_i)\}$ is almost always lower than v when m is large enough. Indeed, for any $\varepsilon > 0$, we have:

$$\begin{aligned}
\Pr\left[\hat{J}(\theta) \le \underline{\theta} + \varepsilon\right] &\ge \Pr\left[\min_{i=n+1,\ldots,n+m} \{J_i(\theta_i)\} \le \underline{\theta} + \varepsilon\right] \\
&= \Pr\left[\min_{i=n+1,\ldots,n+m} \{\theta_i\} \le J_1^{-1}(\underline{\theta} + \varepsilon)\right] \\
&= 1 - \left[1 - G_1\left(J_1^{-1}(\underline{\theta} + \varepsilon)\right)\right]^m,
\end{aligned}$$

    where the last expression converges to 1 as m goes to infinity. Therefore, for any $\varepsilon > 0$, there exists $\hat{m}_1(\varepsilon)$ such that for any $m \ge \hat{m}_1(\varepsilon)$,

$$\Pr\left[\hat{J}(\theta) \le \underline{\theta} + \varepsilon\right] \ge 1 - \varepsilon.$$

  - Hence, for $m \ge \hat{m}_1(\varepsilon)$:

$$v - \underline{\theta} \ge \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^m(v, \theta) [v - J_i(\theta_i)] \, dG(\theta) \ge (1 - \varepsilon)(v - \underline{\theta} - \varepsilon),$$

    where the right-hand side converges to $v - \underline{\theta}$ as ε tends to 0.

The conclusion then follows again from Lebesgue's dominated convergence theorem.

- In the right-hand side of (16), the terms within brackets tend to $c''(\underline{e})$. To see this, it suffices to note that $h_2^m(v)$, now given by

$$h_2^m(v) = f_{ee}(v|e^m) \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v, \theta_1) G_1(\theta_1) d\theta_1$$

  – is still bounded:

$$
\begin{aligned}
|h_2^m(v)| &< \max_e |f_{ee}(v|e)| \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v, \theta_1) d\theta_1 \\
&\leq \max_e |f_{ee}(v|e)| \int_{\underline{\theta}}^{\bar{\theta}} \left[1 - G_1\left(J_1^{-1}(\theta_1)\right)\right]^m d\theta_1.
\end{aligned}
$$

  – and converges to 0: Indeed, for any $\varepsilon > 0$,

$$
\begin{aligned}
|h_2^m(v)| &< \max_e |f_{ee}(v|e)| \left\{ \int_{\underline{\theta}}^{\underline{\theta}+\frac{\varepsilon}{2}} d\theta_1 + \int_{\underline{\theta}+\frac{\varepsilon}{2}}^{\bar{\theta}} \left[1 - G_1\left(J_1^{-1}(\underline{\theta}+\varepsilon)\right)\right]^m d\theta_1 \right\} \\
&< \max_e |f_{ee}(v|e)| \left\{ \frac{\varepsilon}{2} + (\bar{\theta} - \underline{\theta}) \left[1 - G_1\left(J_1^{-1}(\underline{\theta}+\varepsilon)\right)\right]^m \right\}.
\end{aligned}
$$

  But there exists $\hat{m}_2(\varepsilon)$ such that, for any $m \geq \hat{m}_2(\varepsilon)$:

$$(\bar{\theta} - \underline{\theta}) \left[1 - G_1\left(J_1^{-1}(\theta_1)\right)\right]^m \leq \frac{\varepsilon}{2},$$

  and thus

$$|h_2^m(v)| < \max_e |f_{ee}(v|e)| \, \varepsilon.$$

  – It follows that the second term converges again to 0:

$$\lim_{m \to \infty} \int_{\underline{\theta}}^{\bar{v}} h_2^m(v) \, dv = \int_{\underline{\theta}}^{\bar{v}} \lim_{m \to \infty} h_2^m(v) \, dv = 0.$$

The conclusion follows, using the same reasoning as in Section A.1.

A.3 Increasing the Value of the Innovation

Let us now keep the supply side (number of firms and their cost distributions) fixed and suppose instead that:

- v is initially distributed over $V = [\underline{v}, \bar{v}]$; for the sake of exposition, we assume $\underline{v} \gg \bar{\theta}$,[36] so that the innovation is always implemented.

---

[36] Namely, $\underline{v} > \min_{i \in N} \{K_i(\underline{v}, \bar{\theta})\}$.

- For every $m \in \mathbb{N}^*$, the value v^m becomes distributed over $V^m = [\underline{v}, \bar{v}^m = \underline{v} + m(\bar{v} - \underline{v})]$, according to the c.d.f. $F^m(v^m|e) = F(\underline{v} + (v^m - \underline{v})/m|e)$.

As before, letting e^m, λ^m, $K_1^m(\theta_1)$, and $X_1^m(v, \theta_1)$ denote the values associated with the optimal mechanism, we now show that this optimal mechanism must involve a prize for m large enough.

We first note that the virtual costs remain invariant here: $K_i^m(v^m, \theta_i) = K_i(v, \theta_i) = J_i(\theta_i)$ for $i > 1$ and, as

$$\beta^m(v^m) = \lambda \frac{f_e^m(v^m|e)}{f^m(v^m|e)} = \lambda \frac{f_e(v|e)}{f(v|e)},$$

we also have

$$
\begin{aligned}
K_1^m(v^m, \theta_1) &= J_1(\theta_1) - \min\{\beta^m(v^m), 1\} \frac{G_1(\theta_1)}{g_1(\theta_1)} \\
&= J_1(\theta_1) - \min\{\beta(v), 1\} \frac{G_1(\theta_1)}{g_1(\theta_1)} \\
&= K_1(v, \theta_1).
\end{aligned}
$$

As by assumption, the innovation is always implemented in this variant, the probability of obtaining the contract only depends on these virtual costs and thus also remains invariant: $x_i^m(v^m, \theta) = x_i^*(v, \theta)$ for any $i \in N$. It follows that, in the right-hand side of (16), the terms within brackets also remained unchanged: using $X_1^m(v^m, \theta_1) = X_1^*(v, \theta_1)$ and $f_{ee}^m(v^m|e) dv^m = f_{ee}(v|e) dv$, we have:

$$c''(e) - \int_{\underline{v}}^{\bar{v}^m} \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v^m, \theta_1) G_1(\theta_1) f_{ee}^m(v^m|e) d\theta_1 dv^m = \Gamma^*(e),$$

where

$$\Gamma^*(e) := c''(e) - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^*(v, \theta_1) G_1(\theta_1) f_{ee}(v|e) d\theta_1 dv.$$

By contrast, the left-hand side of (16) is unbounded as m goes to infinity: using $\sum_{i \in N} x_i^*(v, \theta) = 1$ (as by assumption, the innovation is always implemented here), $f_e^m(v|e) dv^m = f_e(v|e) dv$ and $\int_{\underline{v}}^{\bar{v}} f_e(v|e) dv = 0$, we have:

$$
\begin{aligned}
\int_{\underline{v}}^{\bar{v}^m} \int_{\underline{\theta}}^{\bar{\theta}} & \sum_{i \in N} x_i^m(v^m, \theta)[v^m - J_i(\theta_i)] dG(\theta) f_e^m(v^m|e) dv^m \\
&= \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^*(v, \theta)[\underline{v} + m(v - \underline{v}) - J_i(\theta_i)] dG(\theta) f_e(v|e) dv \\
&= mB^*(e) - C^*(e),
\end{aligned}
$$

where:

$$B^* (e) = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^* (v, \theta) \, v dG (\theta) \, f_e (v|e) \, dv = \int_{\underline{v}}^{\bar{v}} v f_e (v|e) \, dv,$$

$$C^* (e) = \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^* (v, \theta) \, J_i (\theta_i) \, dG (\theta) \, f_e (v|e) \, dv.$$

To conclude the argument, suppose that the optimal mechanism never involves a prize. Condition (16) should thus hold for any m, and in addition, the Lagrangian multiplier λ^m should satisfy the boundary condition (15). We should thus have:

$$mB^* (e) < C^* (e) + \frac{f (\bar{v}|e)}{f_e (\bar{v}|e)} \Gamma^* (e),$$

which is obviously violated for a large enough m.

B   Proof of Proposition 4

As earlier, the incentive compatibility constraint can be replaced by the envelope condition:

$$U_i(\mathbf{v}, \theta_i) = \rho_i(\mathbf{v}) + \int_{\theta_i}^{\bar{\theta}} X_i(\mathbf{v}, s) ds, \quad \forall (\mathbf{v}, \theta_i) \in V^N \times \Theta, \forall i \in N, \tag{18}$$

where

$$X_i(\mathbf{v}, \theta_i) = \mathbb{E}_{\theta_{-i}} \left[\sum_{k \in N} x_i^k(\mathbf{v}, \theta_i, \theta_{-i}) \right].$$

Using (18), firm i's expected rent can be expressed as

$$\int_{\theta_i} U_i(\mathbf{v}, \theta_i) dG_i(\theta_i) = \rho_i(\mathbf{v}) + \int_{\theta_i} X_i(v, \theta_i) \frac{G_i(\theta_i)}{g_i(\theta_i)} dG_i(\theta_i), \tag{19}$$

Using this condition, we can rewrite the limited liability constraint as:

$$\mathbb{E}_{\theta} \left[\sum_{k,i \in N} x_i^k(\mathbf{v}, \theta) \left\{ v^k - \psi_i^k - J_i(\theta_i) \right\} \right] \geq \sum_{i \in N} \rho_i(\mathbf{v}), \quad \forall \mathbf{v} \in V^n. \tag{LL}$$

Let $\mu(\mathbf{v}) \geq 0$ denote the multiplier associated with this constraint.

Also, from (18), individual rationality boils down to

$$\rho_i(\mathbf{v}) \geq 0, \quad \forall \mathbf{v} \in V^n, \forall i \in N. \tag{IR}$$

Let $\nu_i(\mathbf{v}) \geq 0$ denote the multiplier associated with this constraint.

The moral hazard constraint can be replaced by the associated first-order condition, which, using (19), can be expressed as:[37]

$$\int_v \int_\theta \left(\rho_i(\mathbf{v}) + \frac{G_i(\theta_i)}{g_i(\theta_i)} \sum_{k \in N} x_i^k(\mathbf{v}, \theta) \right) dG(\theta) f_{e^i}^i \left(v^i | e^i \right) f^{-i} \left(v^{-i} | e^{-i} \right) dv \geq c'(e^i), \quad \forall i \in N. \tag{MH}$$

We formulate again these conditions as weak inequalities to ensure the nonnegativity of the associated multipliers, which we will denote by $\lambda = (\lambda^1, ..., \lambda^n)$.

The principal's problem can then be more succinctly reformulated as follows:

$$[P] \qquad \max_{\mathbf{x}, (\rho_i), \mathbf{e}} \mathbb{E}_{\mathbf{v}, \theta} \left[\sum_{k, i \in N} x_i^k(\mathbf{v}, \theta) \left(v^k - J_i(\theta_i) - \psi_i^k \right) - \sum_{i \in N} \rho_i(\mathbf{v}) \middle| \mathbf{e} \right]$$
$$\text{subject to } (LL), (IR), \text{ and } (MH).$$

The analysis of this problem follows the same steps as for the case of a single innovator, and we only sketch them here. The integrand of the Lagrangian is now given by:

$$L(\mathbf{v}, \theta, e) := [1 + \mu(\mathbf{v})] \left\{ \sum_{k, i \in N} \left[v^k - \theta_i - \left(1 - \frac{\beta^i(v^i)}{1 + \mu(\mathbf{v})} \right) \frac{G_i(\theta_i)}{g_i(\theta_i)} - \psi_i^k \right] x_i^k(\mathbf{v}, \theta) \right\}$$
$$- \sum_{i \in N} \rho_i(\mathbf{v}) \left[1 + \mu(\mathbf{v}) - \nu_i(\mathbf{v}) - \beta^i(v^i) \right] - \sum_{i \in N} \lambda^i c'(e^i),$$

where

$$\beta^i(v^i) := \lambda^i \frac{f_e^i(v^i | e)}{f(v^i | e)}.$$

The first-order conditions for the monetary prize $\rho_i(\mathbf{v})$ and for the probability $x_i^k(\mathbf{v}, \theta)$ yield, respectively:

$$1 + \mu^*(\mathbf{v}) - \nu_i^*(\mathbf{v}) - \beta^{i*}(v^i) = 0, \quad \forall \mathbf{v} \in V^n, \forall i \in N, \tag{20}$$

and

$$x_i^{k*}(\mathbf{v}, \theta) = \begin{cases} 1 & \text{if } v^k - \tilde{K}_i(\mathbf{v}, \theta_i) - \psi_i^k \geq \max \left\{ \max_{(l,j) \neq (k,i)} v^l - \tilde{K}_j(\mathbf{v}, \theta_j) - \psi_j^l, 0 \right\}, \\ 0 & \text{otherwise,} \end{cases} \tag{21}$$

where

$$\tilde{K}_i(\mathbf{v}, \theta_i) := J_i(\theta_i) - \frac{\beta^{i*}(v^i)}{1 + \mu^*(\mathbf{v})} \frac{G_i(\theta_i)}{g_i(\theta_i)}.$$

Note that $\tilde{K}_i(\mathbf{v}, \theta_i)$ can be expressed as

$$\theta_i + \left[1 - \frac{\beta^{i*}(v^i)}{1 + \mu^*(\mathbf{v})} \right] \frac{G_i(\theta_i)}{g_i(\theta_i)},$$

---

[37]For simplicity, we normalize the firms' efforts in such a way that firms face the same cost $c(e)$; any asymmetry can, however, be accommodated through the distributions $F^k(v^k | e)$.

where (20) and $\nu_i^*(\mathbf{v}) \geq 0$ together imply that the term within brackets is non-negative. It follows that

$$\tilde{K}_i(\mathbf{v}, \theta_i) \geq \theta_i \tag{22}$$

and that $\tilde{K}_i(\mathbf{v}, \theta_i)$ increases with θ_i.

The complementary slackness associated with (LL) implies that for every $\mathbf{v} \in V^n$,

$$\mu^*(\mathbf{v}) \left\{ \mathbb{E}_\theta \left[\sum_{k,i \in N} x_i^{k*}(\mathbf{v}, \theta) \left\{ v^k - \psi_i^k - J_i(\theta_i) \right\} \right] - \sum_{i \in N} \rho_i^*(\mathbf{v}) \right\} = 0, \tag{23}$$

whereas the complementary slackness associated with (IR) implies that for every $i \in N$ and every $\mathbf{v} \in V^n$,

$$\nu_i^*(\mathbf{v})\rho_i^*(\mathbf{v}) = 0. \tag{24}$$

We now prove the following result:

LEMMA 4. *Fix any* $\mathbf{v}$ *such that* $\max_{k,i} \left\{ v^k - \psi_i^k \right\} > \underline{\theta}$. *We have*

$$\mathbb{E}_\theta \left[\sum_{k,i \in N} x_i^{k*}(\mathbf{v}, \theta) \left[v^k - \psi_i^k - J_i(\theta_i) \right] \right] > 0, \tag{25}$$

if either (i) $n \geq 2$ *or (ii)* $n = 1$ *and either* $v^1 - \psi_1^1 > \overline{\theta}$ *or* $\nu_1(v^1) > 0$.

PROOF. We first focus on the case in which $n \geq 2$. Fix any $\mathbf{v}$ such that $v^l - \psi_j^l - \underline{\theta} > 0$ for some l, j. Further, fix any k such that $\sum_i x_i^k(\mathbf{v}, \theta) > 0$ for a positive measure of θs (a project that does not satisfy this property is never adopted with positive probability and can be ignored).

Consider first the particular case in which project k is always implemented and allocated to the same firm i: $x_i^k(\mathbf{v}, .) = 1$ (this can, for instance, happen when v^k is large and ψ_j^k is prohibitively high for $j \neq i$). In that case:

$$\mathbb{E}_\theta \left[\sum_{i \in N} x_i^k(\mathbf{v}, \theta) \left[v^k - \psi_i^k - J_i(\theta_i) \right] \right]$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i)$$
$$= v^k - \psi_i^k - \overline{\theta}$$
$$> 0,$$

where the inequality stems from (21), applied to $\theta_i = \overline{\theta}$,[38] and (22).

---
[38] Generically, this condition implies $v^k - \psi_i^k > \tilde{K}_i(\mathbf{v}, \overline{\theta})$; we ignore here the non-generic case $v^k - \psi_i^k = \tilde{K}_i(\mathbf{v}, \overline{\theta})$.

Let us now turn to the case in which no firm is selected with probability 1 to implement project k (because project k is not always implemented and/or different firms are selected to implement it). By (21), the optimal allocation rule is then such that

$$X_i^k(\mathbf{v}, \theta_i) := \mathbb{E}_{\theta_{-i}} \left[x_i^k(\mathbf{v}, \theta_i, \theta_{-i}) \right]$$

is nonincreasing in θ_i for all $\theta_i \leq v^k - \psi_i^k$ and equals zero for any $\theta_i > v^k - \psi_i^k$. Further, it is strictly decreasing in θ_i for a positive measure of θ_i if $X_i^k(\mathbf{v}, \theta_i) > 0$, and by the choice of k, there is at least one such firm.

Now, for every i define

$$\bar{X}_i^k(\mathbf{v}, \theta_i) = \begin{cases} \bar{z}_i^k & \text{if } \theta_i \leq v^k - \psi_i^k \\ 0 & \text{if } \theta_i > v^k - \psi_i^k, \end{cases}$$

where $\bar{z}_i^k$ is a constant in $(0, 1)$ chosen so that

$$\int_{\underline{\theta}}^{\overline{\theta}} \bar{X}_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i) = \bar{z}_i^k G_i(v^k - \psi_i^k) = \int_{\underline{\theta}}^{\overline{\theta}} X_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i).$$

Clearly, $\bar{z}_i^k$, and hence $\bar{X}_i^k(\mathbf{v}, \cdot)$, is well defined.

We have:

$$\mathbb{E}_\theta \left[\sum_{i \in N} x_i^k(\mathbf{v}, \theta) \left[v^k - \psi_i^k - J_i(\theta_i) \right] \right]$$

$$= \sum_i \int_{\underline{\theta}}^{\overline{\theta}} X_i^k(\mathbf{v}, \theta_i) \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i)$$

$$= \sum_i \int_{\underline{\theta}}^{\min\{\overline{\theta}, v^k - \psi_i^k\}} X_i^k(\mathbf{v}, \theta_i) \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i)$$

$$> \sum_i \int_{\underline{\theta}}^{\min\{\overline{\theta}, v^k - \psi_i^k\}} \bar{X}_i^k(\mathbf{v}, \theta_i) \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i)$$

$$= \sum_i \bar{z}_i^k \int_{\underline{\theta}}^{\min\{\overline{\theta}, v^k - \psi_i^k\}} \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i)$$

$$= \sum_i \bar{z}_i^k \left(\max\{v^k - \psi_i^k - \overline{\theta}, 0\} \right)$$

$$\geq 0.$$

The second equality stems from the fact that $X_i^k(\mathbf{v}, \theta_i) = 0$ for $\theta_i > v^k - \psi_i^k$, and the strict inequality follows from the fact that: (i) $v^k - \psi_i^k - J_i(\theta_i)$ is strictly decreasing in θ_i; (ii) in the relevant range $\left[\underline{\theta}, \min\{\overline{\theta}, v^k - \psi_i^k\} \right]$, $X_i^k(\mathbf{v}, \theta_i)$ is nonincreasing in θ_i and, for some i, it is

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moreover strictly decreasing in θ_i for a positive measure of θ_i; (iii) $\bar{X}_i^k(\mathbf{v}, \cdot)$ is constant; and (iv)

$$\int_{\underline{\theta}}^{\min\{\bar{\theta}, v^k - \psi_i^k\}} \bar{X}_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i) = \int_{\underline{\theta}}^{\min\{\bar{\theta}, v^k - \psi_i^k\}} X_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i).$$

Summing the above string of inequalities over all k, we obtain the desired result.

Next consider the case in which $n = 1$. In this case, $X_1^1(\mathbf{v}, \theta_1) = x_1^1(\mathbf{v}, \theta_1) = 1$ for $\tilde{K}_1(v^1, \theta_1) \le v^1$ and zero otherwise. Because $X_i^k(\mathbf{v}, \theta_i)$ is constant when it is strictly positive, the strict inequality above does not follow from the above argument. But the strict inequality does still hold if $v^1 - \psi_1^1 > \bar{\theta}$ or if $\nu_1(v^1) > 0$.

In the former case, the last inequality above becomes strict, thus yielding the desired result. To consider the latter case, assume without loss $v^1 - \psi_1^1 \le \bar{\theta}$. Because $\nu_1(v^1) > 0$, we have $\beta^1(v^1) < 1 + \mu(v^1)$, so $\tilde{K}_1(v^1, \theta_1) > \theta_1$, which implies that there exists $\tilde{\theta} < v^1 - \psi_1^1$ such that $x_1^1(\mathbf{v}, \theta_1) = 1$ for $\theta_1 < \tilde{\theta}$ and $x_1^1(\mathbf{v}, \theta_1) = 0$ for $\theta_1 > \tilde{\theta}$. Let $\check{\theta} := \sup\{\theta \le \bar{\theta} | v^1 - \psi_1^1 - J_1(\theta) \ge 0\}$. If $\tilde{\theta} \le \check{\theta}$, then

$$\mathbb{E}_\theta\left[x_1^1(\mathbf{v}, \theta)\left[v^1 - \psi_1^1 - J_1(\theta_1)\right]\right] = \int_{\underline{\theta}}^{\tilde{\theta}}\left[v^1 - \psi_1^1 - J_1(\theta_1)\right] dG(\theta_1) > 0.$$

If $\tilde{\theta} > \check{\theta}$, the same result holds because

$$\mathbb{E}_\theta\left[x_1^1(\mathbf{v}, \theta)\left[v^1 - \psi_1^1 - J_1(\theta_1)\right]\right]$$
$$= \int_{\underline{\theta}}^{\tilde{\theta}}\left[v^1 - \psi_1^1 - J_1(\theta_1)\right] dG(\theta_1)$$
$$> \int_{\underline{\theta}}^{\tilde{\theta}}\left[v^1 - \psi_1^1 - J_1(\theta_1)\right] dG_1(\theta_1) + \int_{\tilde{\theta}}^{v^1 - \psi_1^1}\left[v^1 - \psi_1^1 - J_1(\theta_1)\right] dG_1(\theta_1)$$
$$= \int_{\underline{\theta}}^{v^1 - \psi_1^1}\left[v^1 - \psi_1^1 - J_1(\theta_1)\right] dG_1(\theta_1)$$
$$= 0,$$

where the strict inequality holds because $v^1 - \psi_1^1 - J_1(\theta_1) < 0$ for $\theta_1 \in (\tilde{\theta}, v^1 - \psi_1^1)$ (which in turn holds because $\check{\theta} < \tilde{\theta} < v^1 - \psi_1^1$), and the last equality follows from integration by parts. $\square$

Without loss of generality, assume $n \ge 2$ (otherwise, there would be a single innovator, a case studied earlier). There are two cases. Consider first the case in which $\beta^i(v^i) < 1$ for every $i \in N$. By (20), we must then have

$$\nu_i^*(\mathbf{v}) = 1 + \mu^*(\mathbf{v}) - \beta^{i*}(v^i) > 0,$$

and the complementary slackness condition (24) thus yields $\rho_i^*(\mathbf{v}) = 0$ for every firm $i \in N$. This, together with (25) and the complementary slackness condition (23), implies that $\mu^*(\mathbf{v}) = 0$, and thus

$$\tilde{K}_i(\mathbf{v}, \theta_1) = J_i(\theta_i) - \beta^{i*}(v^i) \frac{G_i(\theta_i)}{g_i(\theta_i)} := K_i^*(v, \theta_1).$$

Consider next the case in which $\max_{i \in N} \{\beta^{i*}(v^i)\} > 1$. Let $\hat{I} = \arg\max_{i \in N} \{\beta^{i*}(v^i)\}$ for the firms that have the highest $\beta^{i*}(v^i)$. Applying (20) to $i \in \hat{I}$ then yields

$$\mu^*(\mathbf{v}) = \nu_i^*(\mathbf{v}) + \beta^{i*}(v^i) - 1 > \nu_i^*(\mathbf{v}) \geq 0, \tag{26}$$

whereas applying (20) to firm $j \notin \hat{I}$ yields

$$1 + \mu^*(\mathbf{v}) - \nu_i^*(\mathbf{v}) = \beta^{i*}(v^i) > \beta^{j*}(v^j) = 1 + \mu^*(\mathbf{v}) - \nu_j^*(\mathbf{v}).$$

It follows that $\nu_j^*(\mathbf{v}) > \nu_i^*(\mathbf{v}) \geq 0$ for $i \in \hat{I}, j \notin \hat{I}$. Therefore, by complementary slackness (24), $\rho_j^*(\mathbf{v}) = 0$, so that only firms $i \in \hat{I}$ can receive a positive monetary prize: $\rho_j^*(\mathbf{v}) = 0$ for $j \notin \hat{I}$. Finally, the complementary slackness condition (23) yields

$$\sum_{i \in \hat{I}} \rho_i^*(\mathbf{v}) = \sum_{i \in N} \rho_i^*(\mathbf{v}) = \mathbb{E}_\theta \left[\sum_{k, i \in N} x_i^{k*}(\mathbf{v}, \theta) \left\{ v^k - \psi_i^k - J_i(\theta_i) \right\} \right].$$

By Lemma 4, the total prize must be strictly positive for all $\mathbf{v}$ such that $v^k > \psi_i^k + \underline{\theta}$ for some k, i. Given the atomlessness of $F_i(\cdot|e)$ for all e, $\hat{I}$ is a singleton with probability one. Hence, for any $\mathbf{v}$ such that $v^k > \psi_i^k + \underline{\theta}$ for some k, i, and $\max_i\{\beta^{i*}(v^i)\} > 1$, with probability one only one firm receives the monetary prize.

Last, we derive the characterization of the optimal allocation rule and transfers. By the above argument, there exists at least one firm $i \in \hat{I}$ such that $\rho_i^*(\mathbf{v}) > 0$, and for that firm, (24) yields $\nu_i^*(\mathbf{v}) = 0$. However, then (20) applied to all $j \in \hat{I}$ along with the fact that $\beta^{i*}(v^i) = \beta^{j*}(v^j)$ for $i, j \in \hat{I}$ means that $\nu_i^*(\mathbf{v}) = 0$ for all $i \in \hat{I}$. It then follows that

$$1 + \mu^*(\mathbf{v}) = \max_i\{\beta^{i*}(v^i)\}.$$

We thus conclude that

$$\tilde{K}_i(\mathbf{v}, \theta_1) = J_i(\theta_i) - \left(\frac{\beta^{i*}(v^i)}{\max_k \beta^{k*}(v^k)} \right) \left(\frac{G_i(\theta_i)}{g_i(\theta_i)} \right) := K_i^*(v, \theta_1).$$

Finally, the expected transfers to firm i, $T_i(v, \theta_i)$, can be derived from (19) using the allocation described above and $U_i(\mathbf{v}, \theta_i) = T_i(v, \theta_i) - \mathbb{E}_{\theta-i} \left[\sum_k \left(\psi_i^k + \theta_i \right) x_i^{k*}(\mathbf{v}, \theta) \right]$.

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C Forbidding Handicaps

We explore here how the optimal mechanism is modified when handicaps are ruled out. Specifically, we suppose that the innovator cannot be handicapped compared to the standard second best allocation. That is, for every v and θ:

$$x_1(v,\theta) \geq x_1^{SB}(v,\theta), \tag{NH}$$

where:

$$x_1^{SB}(v,\theta) := \begin{cases} 1 & \text{if } J_1(\theta_1) \leq \min\{v, \min_{j\neq 1} J_j(\theta_j)\}, \\ 0 & \text{otherwise.} \end{cases}$$

Letting $\alpha(v,\theta) \geq 0$ be the multiplier of the no-handicap constraint (NH), the Lagrangian becomes

$$L(v,e) := [1+\mu(v)] \left\{ \left[v - J_1(\theta_1) + \frac{\beta(v)}{1+\mu(v)} \frac{G_1(\theta_1)}{g_1(\theta_1)} + \frac{\alpha(v,\theta)}{1+\mu(v)} \right] x_1(v,\theta) + \sum_{\substack{j\in N \\ j\neq 1}} [v - J_j(\theta_j)] x_j(v,\theta) \right\}$$

$$- \rho_1(v) [1+\mu(v)-\nu(v)-\beta(v)] - \lambda c'(e) + \alpha(v,\theta)[x_1(v,\theta) - x_1^{SB}(v,\theta)]$$

and the additional complementary slackness is

$$\alpha(v,\theta) \left[x_1(v,\theta) - x_1^{SB}(v,\theta) \right] = 0. \tag{27}$$

The Lagrangian is still linear in x_i's, so the optimal allocation must satisfy, for every i, v, θ:

$$\bar{x}_i(v,\theta) = \begin{cases} 1 & \text{if } i \in \arg\min_j \{\bar{K}_j(v,\theta_j)\} \text{ and } \bar{K}_i(v,\theta_i) \leq v, \\ 0 & \text{otherwise,} \end{cases}$$

where the shadow cost is now given by:

$$\bar{K}_i(v,\theta_i) := \begin{cases} J_i(\theta_i) - \frac{\beta(v)}{1+\mu(v)} \frac{G_i(\theta_i)}{g_i(\theta_i)} - \frac{\alpha(v,\theta)}{1+\mu(v)} & \text{if } i = 1, \\ J_i(\theta_i) & \text{if } i \neq 1, \end{cases} \quad \text{with } \beta(v) := \lambda \frac{f_e(v|e)}{f(v|e)}.$$

When $v > \tilde{v}$, $\bar{K}_1(v,\theta_1) < J_1(\theta_1)$, and we can thus ignore the constraint (NH); hence $\alpha(v,\theta) = 0$, implying $\bar{K}_1(v,\theta_1) = K_1(v,\theta_1)$ and $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$. Let us now consider the case $v < \tilde{v}$. If $\alpha(v,\theta) = 0$, the above characterization yields again $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$, and $v < \tilde{v}$ then implies $\bar{K}_1(v,\theta_1) > J_1(\theta_1)$ and thus $\bar{x}_1(v,\theta) < x_1^{SB}(v,\theta)$ for at least some θs, contradicting (NH); therefore, we must have $\alpha(v,\theta) > 0$, and the complementary slackness condition (27) thus implies $\bar{x}_1(v,\theta) = x_1^{SB}(v,\theta)$, and thus $\bar{K}_1(v,\theta_1) = J_1(\theta_1)$.

The other constraints are unaffected; thus the optimal effort e must satisfy

$$\frac{\partial}{\partial e} \int_v \int_\theta L(v,\theta,e) f(v|e) dv dG(\theta) = 0,$$

and complementary slackness implies that, for each v,

$$\nu(v)\rho_1(v) = 0,$$

$$\mu(v)\left\{\int_\theta \sum_{i\in N} \bar{x}_i(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta) - \rho_1(v)\right\} = 0,$$

and

$$e\left[\int_v\int_\theta \left[\rho_1(v) + \frac{G_1(\theta)}{g_1(\theta)}\bar{x}_1(v,\theta)\right] g(\theta)d\theta f_e(v|e)dv - c'(e)\right] = 0.$$

Going through the same steps as before and summing up, we have:

- Ruling out handicaps implies that contract rights are allocated according to the standard second-best for low-value projects: For $v < \tilde{v}$, $\alpha(v,\theta) = -\beta(v) G_1(\theta_1)/g_1(\theta_1)$ (> 0) and $\bar{K}_i(v,\theta_i) = J_i(\theta_i)$ for all i (and thus, $\bar{x}_i(v,\theta) = x^{SB}(\theta)$ for all i as well).

- Ruling out handicaps has instead no impact on optimal contract rights for high-value projects: For $v > \tilde{v}$, $\alpha(v,\theta) = 0$ and $\bar{x}_i(v,\theta) = x_i^*(v,\theta)$ for all i.

In addition, forbidding handicaps does not affect the size of the monetary prize when such a prize is given:

- For $v < \hat{v}$, $\nu(v) = 1 - \beta(v) > 0$ and thus $\rho_1(v) = 0$ and $\mu(v) = 0$.

- For $v > \hat{v}$, $\nu(v) = 0$ and $\beta(v) = 1 + \mu(v)$, and thus $\bar{K}_1(v,\theta_1) = \theta_1$ and thus $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$, based on $K_1(v,\theta_1) = \theta_1$ and $K_i(v,\theta_i) = J_i(\theta_i)$ for $i \neq 1$; it follows that

$$\rho_1(v) = \int_\theta \sum_{i\in N} x_i^*(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta) = \rho_1^*(v).$$

Note however that ruling out handicaps can affect the conditions under which a prize is given: banning handicaps alters the multiplier λ, which in turn affects the threshold $\hat{v}$, which is determined by the condition $\lambda f_e(v|e)/f(v|e) = 1$.

D Fixed allocation

We show here that our main insight carries over when the procurer is required to use the same tender rules whenever she decides to implement the project. The optimal mechanism relies on contract rights (possibly combined with monetary prizes) to induce the innovator to exert effort. Indeed, as long as the project is not always implemented, it is optimal to

bias the implementation auction in favor of the innovator (handicaps instead should never be used).

Specifically, we consider a setup where, should the procurer wish to implement the project, the mechanism $(x, t) \in \Delta^n \times \mathbb{R}^n$ cannot depend on v. We can then simply denote by $x_i(\theta)$ the probability that firm i implements the project and by $t_i(\theta)$ the transfer payment that it receives.

The timing of the game is now as follows:

1. The buyer offers a mechanism specifying the allocation x and a payment t_i to each firm i.

2. The innovator chooses e; the value v is then realized.

3. The buyer observes v and decides whether to implement the project, in which case firms observe their costs and decide whether to participate.

4. Participating firms report their costs, the project is allocated (or not), and transfers are made according to the mechanism (x, t).

If the procurer decides to implement the project, firm i's expected profit no longer depends on the project value v, and can thus be written as

$$U_i(\theta_i) := T_i(\theta_i) - \theta_i X_i(\theta_i)$$

where $X_i(\theta_i) := \int_{\theta_{-i}} x_i(\theta) \, dG_{-i}(\theta_{-i})$ and $T_i(\theta_i) := \int_{\theta_{-i}} t_i(\theta) \, dG_{-i}(\theta_{-i})$. Using incentive compatibility, this expected profit can be expressed as

$$U_i(\theta_i) = \rho_i + \int_{\theta_i}^{\bar{\theta}} X_i(\theta) d\theta,$$

where

$$\rho_i := U_i(\bar{\theta}).$$

is the rent enjoyed by firm i when its cost is highest. As before, it is optimal to set $\rho_i = 0$ for $i \neq 1$, and thus the total expected transfer to the firms is given by

$$\int_\theta \sum_{i \in N} t_i(\theta) \, dG(\theta) = \rho_1 + \int_\theta \sum_{i \in N} x_i(\theta) \, J_i(\theta_i) dG(\theta),$$

where $J_i(\theta_i) := \theta_i + \frac{G_i(\theta_i)}{g_i(\theta_i)}$ denotes firm i's virtual cost. It follows that the procurer chooses to implement the project when:

$$\int_\theta \left\{ \sum_{i \in N} x_i(\theta) \left[v - J_i(\theta_i) \right] \right\} dG(\theta) \geq \rho_1.$$

As the left-hand side strictly increases with v, there exists a unique $\check{v} \in [\underline{v}, \bar{v}]$ such that this constraint is strictly satisfied if $v > \check{v}$, and violated if $v < \check{v}$.

Obviously, if $\check{v} = \bar{v}$, then the project is never implemented, and thus the innovator has no incentive to provide any effort. The assumption that $\bar{v} > \underline{\theta}$ guarantees that this is not optimal. Conversely, if $\check{v} = \underline{v}$ then the project is always implemented. This could be optimal if even low-value projects were still sufficiently desirable, but implies again the innovator has no incentive to provide any effort, as it obtains for sure the same information rents, regardless of the realized value of the project. From now on, we will focus on the case where the optimal threshold is interior, i.e., $\check{v} \in (\underline{v}, \bar{v})$. Let $\mu \geq 0$ denote the multiplier associated with the above constraint for $v = \check{v}$:

$$\int_{\theta} \left\{ \sum_{i \in N} x_i(\theta) \left[\check{v} - J_i(\theta_i)\right] \right\} dG(\theta) \geq \rho_1. \tag{LL}$$

The innovating firm's individual rationality boils down here to

$$\rho_1 \geq 0. \tag{IR}$$

Let $\nu \geq 0$ denote the multiplier associated with this constraint.

Finally, the first-order condition for the effort constraint becomes:

$$\int_{v \geq \check{v}} \int_{\theta} \left[\rho_1 + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1(\theta) \right] dG(\theta) f_e(v|e) dv \geq c'(e). \tag{MH}$$

Let $\lambda \geq 0$ be the associated multiplier.

The buyer's problem can then be formulated as follows:

$$\max_{e, \check{v}, x(\theta), \rho_1} \quad \int_{v \geq \check{v}} \left\{ \int_{\theta} \left[\sum_{i \in N} x_i(\theta) \left[v - J_i(\theta_i)\right] \right] dG(\theta) - \rho_1 \right\} f(v|e) dv$$

$$\text{subject to } (\widehat{LL}), (\widehat{IR}) \text{ and } (\widehat{MH})$$

The Lagrangian is given by:

$$\begin{aligned}
L = & \int_{v \geq \check{v}} \left\{ \int_{\theta} \left[\sum_{i \in N} x_i(\theta) \left[v - J_i(\theta_i)\right] \right] dG(\theta) - \rho_1 \right\} f(v|e) dv \\
& + \mu \left[\int_{\theta} \left\{ \sum_{i \in N} x_i(\theta) \left[\check{v} - J_i(\theta_i)\right] \right\} dG(\theta) - \rho_1 \right] \\
& + \nu \rho_1 + \lambda \left[\int_{v \geq \check{v}} \int_{\theta} \left[\rho_1 + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1(\theta) \right] dG(\theta) f_e(v|e) dv - c'(e) \right].
\end{aligned}$$

Re-arranging terms, it can be expressed as $L = \int_\theta L(\theta, e, \check{v}) dG(\theta)$, where

$$L(\theta, e, \check{v}) := [1 - F(\check{v}|e) + \mu] \left\{ \left[\check{v}^e - J_1(\theta_1) + \frac{\beta^e}{1 + \check{\mu}} \frac{G_1(\theta_1)}{g_1(\theta_1)} \right] x_1(\theta) + \sum_{\substack{i \in N \\ i \neq 1}} [\check{v}^e - J_i(\theta_i)] x_i(\theta) \right\}$$

$$- [1 - F(\check{v}|e)] \rho_1 (1 - \beta^e + \check{\mu} - \check{\nu}) - \lambda c'(e).$$

where:
$$\check{\mu} := \frac{\mu}{1 - F(\check{v}|e)} \text{ and } \check{\nu} := \frac{\nu}{1 - F(\check{v}|e)}$$

denote the weighted value of the Lagrangian multipliers μ and ν (weighted by the probability of implementing the project), and:

$$\beta^e := \lambda \int_{v \geq \check{v}} \frac{f_e(v|e)}{1 - F(\check{v}|e)} dv \text{ and } \check{v}^e := \frac{v^e + \check{\mu}\check{v}}{1 + \check{\mu}},$$

where
$$v^e := \int_{v \geq \check{v}} v \frac{f(v|e)}{1 - F(\check{v}|e)} dv.$$

The optimal solution $(e^*, \check{v}^*, x^*(\theta), \rho_1^*, \lambda^*, \mu^*, \nu^*)$ must satisfy the following necessary conditions. First, observe that the Lagrangian L is linear in $\rho_1(v)$; hence, its coefficient must be equal to zero:
$$1 - \beta^{e*} + \check{\mu}^* - \check{\nu}^* = 0, \tag{28}$$

where $\check{\mu}^*$ and $\check{\nu}^*$ denote the optimal values of the weighted multipliers, and

$$\beta^{e*} := \frac{\lambda^*}{1 - F(\check{v}|e^*)} \int_{v \geq \check{v}} f_e(v|e^*) dv.$$

The Lagrangian is also linear in x_i's, so the optimal allocation must satisfy, for every i, v, θ:

$$x_i^*(\theta) = \begin{cases} 1 & \text{if } i \in \arg\min_j \left\{ \tilde{K}_j(\theta_j) \right\} \text{ and } \tilde{K}_i(\theta_i) \leq \frac{v^e + \check{\mu}^* \check{v}}{1 + \check{\mu}^*}, \\ 0 & \text{otherwise}, \end{cases}$$

where
$$\tilde{K}_i(\theta_i) := \begin{cases} J_i(\theta_i) - \frac{\beta^{e*}}{1 + \check{\mu}^*} \frac{G_i(\theta_i)}{g_i(\theta_i)} & \text{if } i = 1, \\ J_i(\theta_i) & \text{if } i \neq 1. \end{cases} \tag{29}$$

We next prove that $\lambda^* > 0$. Suppose $\lambda^* = 0$, which implies $\beta^{e*} = 0$. Together with (29) and (28), this yields

$$L(\theta, e, \check{v}^*) = [1 - F(\check{v}^*|e) + \mu^*] \int_\theta \max \left\{ 0, \check{v}^{e*} - \min_i J_i(\theta_i) \right\} dG(\theta),$$

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and thus

$$\frac{\partial}{\partial e} \int_\theta L(\theta, e, \check{v}^*)dG(\theta)\bigg|_{e=e^*} = -F_e(\check{v}^*|e^*) \int_\theta \max\left\{0, \check{v}^{e*} - \min_i J_i(\theta_i)\right\} dG(\theta), \qquad (30)$$

where, in the right-hand side:

- The first term, $-F_e(\check{v}^*|e^*)$, as

$$- F_e(\check{v}|e) > 0. \qquad (31)$$

  for any $\check{v}^* \in (\underline{v}, \bar{v})$. To see this, note that

$$-F_e(\check{v}|e) = \frac{\partial}{\partial e}\left[1 - F(\check{v}|e)\right] = \int_{v \geq \check{v}} f_e(v|e)dv,$$

  where from $(MLRP)$, $f_e(v|e) > 0$ for $v > \tilde{v}$ and $f_e(v|e) < 0$ for $v < \tilde{v}$. Therefore, if $\check{v} \geq \tilde{v}$, then $\int_{v \geq \check{v}} f_e(v|e)dv > 0$. If instead $\check{v} < \tilde{v}$, then $\int_{v < \check{v}} f_e(v|e)dv < 0$; but by construction,

$$\int_{v \geq \check{v}} f_e(v|e)dv + \int_{v < \check{v}} f_e(v|e)dv = \frac{\partial}{\partial e}\int f(v|e)dv = 0,$$

  implying again that $\int_{v \geq \check{v}} f_e(v|e)dv > 0$.

$$\int_\theta \sum_{i \in N} x_i(\theta)\left[\check{v} - J_i(\theta_i)\right]dG(\theta) \geq \rho_1$$

- The second term is also positive. Indeed, we have:

$$\int_\theta \max\left\{0, \check{v}^{e*} - \min_i J_i(\theta_i)\right\}dG(\theta) \;>\; \int_\theta \max\left\{0, \check{v} - \min_i J_i(\theta_i)\right\}dG(\theta)$$
$$\geq \;\int_\theta \sum_{i \in N} x_i^*(\theta)\left[\check{v} - J_i(\theta_i)\right]dG(\theta)$$
$$\geq \; 0,$$

  where the first inequality stems $\check{v}^e > \check{v}$ for any $\check{v}^* < \bar{v}$, and the last one follows from (LL) and (IR).

It follows that the right-hand side of (30) is positive, and thus

$$\frac{\partial}{\partial e}\int_\theta L(\theta, e, \check{v}^*)dG(\theta)\bigg|_{e=e^*} > 0,$$

which violates the optimality of e^*. We thus conclude that $\lambda^* > 0$.

Given $\lambda^* > 0$, (31) implies $\beta^{e*} > 0$. It then follows from (29) that *the innovator benefits from a favorable bias in the allocation of the contract rights.*

Finally, complementary slackness implies that, for each v,

$$\check{\nu}^* \rho_1^* = 0, \tag{32}$$

$$\check{\mu}^* \left\{ \int_\theta \sum_{i \in N} x_i^*(\theta) \left[v - J_i(\theta_i) \right] dG(\theta) - \rho_1^* \right\} = 0, \tag{33}$$

When $(0 <) \beta^{e*} < 1$, we have:

$$1 - \beta^{e*} + \check{\mu}^* > \check{\mu}^* \geq 0,$$

and (28) thus implies $\check{\nu}^* > 0$. The complementary slackness condition (32) then yields $\rho_1^* = 0$.

When instead $\beta^{e*} > 1$ we have:

$$1 - \beta^{e*} - \check{\nu}^* < 0,$$

and (28) thus implies that $\check{\mu}^*(v) > 0$; from the complementary slackness condition (33), we thus have

$$\rho_1^*(v) = \int_\theta \sum_{i \in N} x_i^*(\theta) \left[\tilde{v}^* - J_i(\theta_i) \right] dG(\theta).$$