Online Technical Appendix to "Measuring Aggregate Price Indexes with Taste Shocks: Theory and Evidence for CES Preferences" (Not for Publication)*

Stephen J. Redding

Princeton University and NBER†

David E. Weinstein

Columbia University and NBER‡

September, 2019

Table of Contents

*We thank Columbia University and Princeton University for research support. We are especially grateful to the editor, five anonymous referees, Dave Donaldson, Pablo Fajgelbaum, Rob Feenstra, Pete Klenow, and Nick Bloom for helpful comments. We are also grateful to many other colleagues and conference and seminar participants for their helpful comments. We also thank Mark Bambda, Anna Blender, Molly Borden, Mathis Maehlum, Anders Nielsen, Rafael Parente, Dyanne Vaught and Yingjie (Angela) Wu for excellent research assistance. We also want to thank Young and Rubicam for sharing their brand asset value data. Researcher(s) own analyses calculated (or derived) based in part on data from The Nielsen Company (US), LLC and marketing databases provided through the Nielsen Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. The conclusions drawn from the Nielsen data are those of the researcher(s) and do not reflect the views of Nielsen. Nielsen is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein. Responsibility for the results, opinions, and any errors lies with the authors alone.

†Department of Economics and WWS, Julis Romo Rabinowitz Building, Princeton, NJ 08544. Tel: 1 609 258 4016. Fax: 1 609 258 6419. Email: reddings@princeton.edu.

‡Department of Economics, 420 W. 118th Street, MC 3308, New York, NY 10027. Email: dew35@columbia.edu.

A.1 Introduction

This online appendix contains technical derivations, additional information about the data, and supplementary empirical results.

Section A.2 derives the exact CES price index from Section II.C. of the paper and compares it to the Sato-Vartia index. Section A.3 characterizes the taste-shock bias and shows that a positive taste shock for a variety mechanically increases the expenditure-share weight for that variety and reduces the expenditure-share weight for all other varieties. Section A.4 derives the elasticity of substitution implied by the Sato-Vartia index under its assumption of time-invariant tastes for each common variety. Section A.5 considers our robustness test in which we rule out a pure change in consumer tastes by requiring that a generalized mean of order-r of the consumer taste parameters is constant.

Section A.6 develops the extension to non-homothetic CES preferences from Section III.A. of the paper. Section A.7 provides further details for the extension to nested CES preferences from Section III.B. of the paper. Section A.8 develops the generalization to mixed CES with heterogeneous groups of consumers from Section III.C. of the paper. Section A.9 shows that our unified approach to the demand system and the unit expenditure function also can be applied to the closely-related logit and mixed logit preferences, as discussed in Section III.D. of the paper.

Section A.10 shows that our main insight that the demand system can be inverted to construct an exact price index with time-varying taste shocks is not specific to CES, but also holds for the flexible functional forms of translog and almost ideal demand system (AIDS) preferences, as discussed in Section III.E. of the paper. We show that the Törnqvist index for translog preferences exhibits a similar taste-shock bias as the Sato-Vartia index for CES preferences.

Section A.11 provides further details on the Feenstra (1994) estimator used to estimate the elasticity of substitution. Section A.12 develops our joint specification test of the assumption of CES demand and our normalization that tastes have a constant geometric mean across common varieties. Section A.13 contains the data appendix, which reports summary statistics for each of the product groups (sectors) in our data. Section A.14 reports additional empirical results discussed in Sections V.B., V.E. and VII. of the paper.

A.2 Derivation of Exact CES Price Index

In this section of the online appendix, we derive the expression for the exact CES price index in terms of taste-adjusted prices in equation (10) in Section II.D. of the paper. From the common variety expenditure share in equation (5) in the paper, we can express the change in the common variety price index as:

$$\frac{P_t^*}{P_{t-1}^*} = \frac{\left(p_{kt}/\varphi_{kt}\right) / \left(p_{kt-1}/\varphi_{kt-1}\right)}{\left(s_{kt}^*/s_{kt-1}^*\right)^{\frac{1}{1-\sigma}}}. \tag{A.1}$$

Taking logs of both sides, and rearranging, we have:

$$\frac{\ln\left(\frac{P_t^*}{P_{t-1}^*}\right) - \ln\left(\frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}}\right)}{\ln\left(\frac{s_{kt}^*}{s_{kt-1}^*}\right)} = \frac{1}{\sigma - 1}. \tag{A.2}$$

If we now multiply both sides of this equation by $s_{kt}^* - s_{kt-1}^*$ and sum across all common varieties, we obtain:

$$\sum_{k \in \Omega_t^*} \left(s_{kt}^* - s_{kt-1}^*\right) \frac{\ln\left(\frac{P_t^*}{P_{t-1}^*}\right) - \ln\left(\frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}}\right)}{\ln\left(\frac{s_{kt}^*}{s_{kt-1}^*}\right)} = 0 \tag{A.3}$$

or

$$\sum_{k \in \Omega_t^*} \left(\frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*}\right) \ln\left(\frac{P_t^*}{P_{t-1}^*}\right) = \sum_{k \in \Omega_t^*} \left(\frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*}\right) \ln\left(\frac{p_{kt}/\varphi_{kt}}{p_{kt-1}/\varphi_{kt-1}}\right). \tag{A.4}$$

Re-writing this expression, we obtain the log change in our exact CES price index in equation (10) in the paper:

$$\ln\left(\frac{P_t^*}{P_{t-1}^*}\right) = \left[\sum_{k \in \Omega_t^*} \omega_{kt}^* \ln\left(\frac{p_{kt}}{p_{kt-1}}\right)\right] - \left[\sum_{k \in \Omega_t^*} \omega_{kt}^* \ln\left(\frac{\varphi_{kt}}{\varphi_{kt-1}}\right)\right], \tag{A.5}$$

$$\omega_{kt}^* \equiv \frac{\frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*}}{\sum_{\ell \in \Omega_t^*} \frac{s_{\ell t}^* - s_{\ell t-1}^*}{\ln s_{\ell t}^* - \ln s_{\ell t-1}^*}}, \qquad \sum_{k \in \Omega_t^*} \omega_{kt}^* = 1. \tag{A.6}$$

We now show that the exact CES price index in equation (A.5) is equal to the unified price index in equation (8) in the paper. Using our inversion of the demand system from equation (12) in the paper and our normalization that the taste shocks are mean zero across common varieties ($\ln(\tilde{\varphi}_t/\tilde{\varphi}_{t-1}) = 0$), we can substitute for the taste shocks ($\varphi_{kt}/\varphi_{kt-1}$) in equation (A.5) to obtain:

$$\ln\left(\frac{P_t^*}{P_{t-1}^*}\right) = \ln\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right) + \frac{1}{\sigma - 1} \ln\left(\frac{\tilde{s}_t^*}{\tilde{s}_{t-1}^*}\right) - \frac{1}{\sigma - 1} \sum_{k \in \Omega_t^*} \omega_{kt}^* \ln\left(\frac{s_{kt}^*}{s_{kt-1}^*}\right), \tag{A.7}$$

where a tilde above a variable denotes a geometric mean across common varieties such that $\tilde{x}_t = \left(\prod_{k \in \Omega_{t,}^*} x_{kt}\right)^{1/N_t^*}$ for the variable x_{kt}. Using the definition of the Sato-Vartia weights (ω_{kt}^*) from equation (A.6) above, the final term in equation (A.7) is equal to zero, so that equation (A.7) reduces to the CES common variety unified price index:

$$\ln\left(\frac{P_t^*}{P_{t-1}^*}\right) = \ln \Phi_t^{*CCV} = \ln\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right) + \frac{1}{\sigma - 1} \ln\left(\frac{\tilde{s}_t^*}{\tilde{s}_{t-1}^*}\right). \tag{A.8}$$

Finally, using equations (A.5) and (A.8) together with the definition of the Sato-Vartia index (the special case of equation (10) in the paper in which $\varphi_{kt}/\varphi_{kt-1} = 1$ for all $k \in \Omega_t^*$), we can express our common variety exact CES price index as equal to the Sato-Vartia index minus an additional term that we refer to as the taste-shock bias, as in equation (13) in the paper:

$$\ln\left(\frac{P_t^*}{P_{t-1}^*}\right) = \ln \Phi_t^{*CCV} = \ln \Phi_t^{*SV} - \sum_{k \in \Omega_t^*} \omega_{kt}^* \ln\left(\frac{\varphi_{kt}}{\varphi_{kt-1}}\right). \tag{A.9}$$

A.3 Taste-Shock Bias

As discussed in Section II.E. of the paper, the Sato-Vartia index is only unbiased if the taste shocks ($\ln\left(\varphi_{kt}/\varphi_{kt-1}\right)$) are orthogonal to the expenditure-share weights (ω_{kt}^{*}); it is upward-biased if they are positively correlated with these weights; and it is downward-biased if they are negatively correlated with these weights. In principle, either a positive or negative correlation between the taste shocks ($\ln\left(\varphi_{kt}/\varphi_{kt-1}\right)$) and the expenditure-share weights (ω_{kt}^{*}) is possible, depending on the underlying correlation between taste and price shocks. However, there is a mechanical force for a positive correlation, because the expenditure-share weights themselves are functions of the taste shocks. In this section of the online appendix, we show that a positive taste shock for a variety mechanically increases the expenditure-share weight for that variety and reduces the expenditure-share weight for all other varieties.

Note that the Sato-Vartia common variety expenditure share weights (ω_{kt}^{*}) can be written as:

$$\omega_{kt}^{*}=\frac{\xi_{kt}^{*}}{\sum_{\ell\in\Omega_{t}^{*}}\xi_{\ell t}^{*}}, \tag{A.10}$$

$$\xi_{kt}^{*}\equiv\frac{s_{kt}^{*}-s_{kt-1}^{*}}{\ln s_{kt}^{*}-\ln s_{kt-1}^{*}}, \tag{A.11}$$

where

$$s_{kt}^{*}=\frac{\left(p_{kt}/\varphi_{kt}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_{t}^{*}}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}. \tag{A.12}$$

Note also that tastes, prices and expenditure shares at time $t-1$ (φ_{kt-1}, p_{kt-1}, s_{kt-1}) are pre-determined at time t. To evaluate the impact of a positive taste shock for variety k ($\varphi_{kt}/\varphi_{kt-1}>1$), we consider the effect of an increase in tastes at time t for a variety (φ_{kt}) given its tastes at time $t-1$ (φ_{kt-1}). Using the definitions (A.10)-(A.12), we have following two results:

$$\frac{d\omega_{kt}^{*}}{d\xi_{kt}^{*}}\frac{\xi_{kt}^{*}}{\omega_{kt}^{*}}=\left(1-\omega_{kt}^{*}\right)>0, \tag{A.13}$$

$$\frac{d\omega_{\ell t}^{*}}{d\xi_{kt}^{*}}\frac{\xi_{kt}^{*}}{\omega_{\ell t}^{*}}=-\omega_{kt}^{*}<0. \qquad \ell\neq k, \tag{A.14}$$

$$\frac{d\xi_{kt}^{*}}{ds_{kt}^{*}}\frac{s_{kt}^{*}}{\xi_{kt}^{*}}=\frac{1}{\ln\left(s_{kt-1}^{*}/s_{kt}^{*}\right)}-\frac{1}{\left(s_{kt-1}^{*}-s_{kt}^{*}\right)/s_{kt}^{*}}>0, \tag{A.15}$$

where we have used the fact that percentage changes are larger in absolute magnitude than logarithmic changes and hence:

$$\frac{s_{kt-1}^{*}-s_{kt}^{*}}{s_{kt}^{*}}>\ln\left(\frac{s_{kt-1}^{*}}{s_{kt}}\right)>0 \quad \text{for} \quad s_{kt-1}^{*}>s_{kt}^{*},$$

$$\frac{s_{kt-1}^{*}-s_{kt}^{*}}{s_{kt}^{*}}<\ln\left(\frac{s_{kt-1}^{*}}{s_{kt}^{*}}\right)<0 \quad \text{for} \quad s_{kt-1}^{*}<s_{kt}^{*}.$$

We also have the following third result:

$$\frac{ds_{kt}^{*}}{d\varphi_{kt}}\frac{\varphi_{kt}}{s_{kt}^{*}}=(\sigma-1)\left(1-s_{kt}^{*}\right)>0, \qquad \frac{ds_{\ell t}^{*}}{d\varphi_{kt}}\frac{\varphi_{kt}}{s_{\ell t}^{*}}=-\left(\sigma-1\right)s_{kt}^{*}<0. \tag{A.16}$$

Together (A.13), (A.14), (A.15) and (A.16) imply that a positive taste shock for variety k increases the Sato-Vartia expenditure share weight for that variety (ω_{kt}^*):

$$\frac{d\omega_{kt}^*}{d\varphi_{kt}}\frac{\varphi_{kt}}{\omega_{kt}^*} = \left(\frac{d\omega_{kt}^*}{d\xi_{kt}^*}\frac{\xi_{kt}^*}{\omega_{kt}^*}\right)\left(\frac{d\xi_{kt}^*}{ds_{kt}^*}\frac{s_{kt}^*}{\xi_{kt}^*}\right)\left(\frac{ds_{kt}^*}{d\varphi_{kt}}\frac{\varphi_{kt}}{s_{kt}^*}\right) > 0, \tag{A.17}$$

and reduces the Sato-Vartia expenditure share weight for all other varieties $\ell \neq k$ ($\omega_{\ell t}^*$):

$$\frac{d\omega_{\ell t}^*}{d\varphi_{kt}}\frac{\varphi_{kt}}{\omega_{\ell t}^*} = \left(\frac{d\omega_{\ell t}^*}{d\xi_{\ell t}^*}\frac{\xi_{\ell t}^*}{\omega_{\ell t}^*}\right)\left(\frac{d\xi_{\ell t}^*}{ds_{\ell t}^*}\frac{s_{\ell t}^*}{\xi_{\ell t}^*}\right)\left(\frac{ds_{\ell t}^*}{d\varphi_{kt}}\frac{\varphi_{kt}}{s_{\ell t}^*}\right) < 0. \tag{A.18}$$

A.4 Elasticity of Substitution Implied by the Sato-Vartia Index

In this section of the online appendix, we show that the Sato-Vartia index's assumption of time-invariant tastes for each common variety implies that the elasticity of substitution can be recovered from the observed data on prices and expenditure shares with no estimation. We first show that under this assumption there exists an infinite number of approaches to recovering the elasticity of substitution, each of which uses different weights for each common variety. If tastes for *all* common varieties are indeed constant (including no changes in tastes, quality, measurement error or specification error), all of these approaches will recover the same elasticity of substitution. We next show that if consumer tastes for *some* common variety change over time, but a researcher falsely assumes time-invariant tastes for all common varieties, these alternative approaches will return different values for the elasticity of substitution, depending on which weights are used.

Under the Sato-Vartia assumption of constant tastes for each common variety ($\varphi_{kt} = \varphi_{kt-1} = \varphi_k$ for all $k \in \Omega_t^*$ and t), the common variety expenditure share is:

$$s_{kt}^* = \frac{\left(p_{kt}/\varphi_k\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t}/\varphi_\ell\right)^{1-\sigma}}. \tag{A.19}$$

Dividing the expenditure share by its geometric mean across common varieties, we get:

$$\frac{s_{kt}^*}{\tilde{s}_t^*} = \left(\frac{p_{kt}/\varphi_k}{\tilde{p}_t/\tilde{\varphi}}\right)^{1-\sigma}, \tag{A.20}$$

where a tilde above a variable denotes a geometric mean across common varieties. Taking logarithms in (A.20), we obtain:

$$\ln\left(\frac{s_{kt}^*}{\tilde{s}_t^*}\right) = (1-\sigma)\ln\left(\frac{p_{kt}}{\tilde{p}_t}\right) + (\sigma-1)\ln\left(\frac{\varphi_k}{\tilde{\varphi}}\right). \tag{A.21}$$

Taking differences in (A.21), we have:

$$\Delta\ln\left(\frac{s_{kt}^*}{\tilde{s}_t^*}\right) = (1-\sigma)\,\Delta\ln\left(\frac{p_{kt}}{\tilde{p}_t}\right). \tag{A.22}$$

Multiplying both sides of (A.22) by ω_{kt}^* and summing across common varieties, we get:

$$\sum_{k\in\Omega_t^*}\omega_{kt}^*\Delta\ln\left(\frac{s_{kt}^*}{\tilde{s}_t^*}\right) = (1-\sigma)\sum_{k\in\Omega_t^*}\omega_{kt}^*\Delta\ln\left(\frac{p_{kt}}{\tilde{p}_t}\right), \tag{A.23}$$

where ω_{kt}^* are the Sato-Vartia weights:

$$\omega_{kt}^* = \frac{\frac{s_{kt}^* - s_{kt-1}^*}{\ln s_{kt}^* - \ln s_{kt-1}^*}}{\sum_{\ell \in \Omega_t^*} \frac{s_{\ell t}^* - s_{\ell t-1}^*}{\ln s_{\ell t}^* - \ln s_{\ell t-1}^*}}.$$

Equation (A.23) yields the following closed-form solution for σ:

$$\sigma^{SV} = 1 + \frac{\sum_{k \in \Omega_t^*} \omega_{kt}^* \left[\ln\left(\frac{s_{kt}^*}{s_{kt-1}^*}\right) - \ln\left(\frac{\tilde{s}_t^*}{\tilde{s}_{t-1}^*}\right) \right]}{\sum_{k \in \Omega_t^*} \omega_{kt}^* \left[\ln\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right) - \ln\left(\frac{p_{kt}}{p_{kt-1}}\right) \right]}, \tag{A.24}$$

which establishes that the elasticity of substitution (σ) is identified from observed changes in prices and expenditure shares with no estimation under the Sato-Vartia index's assumption of time-invariant tastes for all common varieties ($\varphi_{kt} = \varphi_{kt-1} = \bar{\varphi}_k$ for all $k \in \Omega_t^*$ and t). Note that we could have instead multiplied both sides of (A.22) by any positive finite share that sums to one across common varieties:

$$\sum_{k \in \Omega_t^*} \xi_{kt}^* \Delta \ln\left(\frac{s_{kt}^*}{\tilde{s}_t^*}\right) = (1-\sigma) \sum_{k \in \Omega_t^*} \xi_{kt}^* \Delta \ln\left(\frac{p_{kt}}{\tilde{p}_t}\right), \qquad \sum_{k \in \Omega_t^*} \xi_{kt}^* = 1, \tag{A.25}$$

and obtained another expression for σ given observed prices and expenditure shares:

$$\sigma^{ALT} = 1 + \frac{\sum_{k \in \Omega_t^*} \xi_{kt}^* \left[\ln\left(\frac{s_{kt}^*}{s_{kt-1}^*}\right) - \ln\left(\frac{\tilde{s}_t^*}{\tilde{s}_{t-1}^*}\right) \right]}{\sum_{k \in \Omega_t^*} \xi_{kt}^* \left[\ln\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right) - \ln\left(\frac{p_{kt}}{p_{kt-1}}\right) \right]}. \tag{A.26}$$

Therefore, there exists a continuum of approaches to measuring σ, each of which weights prices and expenditure shares with different non-negative weights that sum to one. Under the Sato-Vartia index's assumption of constant tastes for each variety ($\varphi_{kt} = \varphi_{kt-1} = \varphi_k$ for all $k \in \Omega_t^*$ and t), each of these alternative approaches returns the same value for σ, since all are derived from equation (A.22).

Now suppose that some common variety experiences a taste shock ($\varphi_{kt} \neq \varphi_{kt-1}$ for some $k \in \Omega_t^*$ and t), but a researcher falsely assumes that tastes for all common varieties are constant. Dividing the common variety expenditure share by its geometric mean, we get:

$$\frac{s_{kt}^*}{\tilde{s}_t^*} = \left(\frac{p_{kt}/\varphi_{kt}}{\tilde{p}_t/\tilde{\varphi}_t}\right)^{1-\sigma}, \tag{A.27}$$

where a tilde above a variable again denotes a geometric mean across common varieties.

Taking logarithms in (A.27) and taking differences, we obtain:

$$\Delta \ln\left(\frac{s_{kt}^*}{\tilde{s}_t^*}\right) = (1-\sigma)\, \Delta \ln\left(\frac{p_{kt}}{\tilde{p}_t}\right) + (\sigma - 1)\, \Delta \ln \varphi_{kt}, \tag{A.28}$$

where we have used our normalization that the geometric mean of consumer tastes is constant such that $\ln\left(\tilde{\varphi}_t / \tilde{\varphi}_{t-1}\right) = 0$. Multiplying both sides of (A.28) by ω_{kt}^* and summing across common varieties, we get:

$$\sum_{k \in \Omega_t^*} \omega_{kt}^* \Delta \ln\left(\frac{s_{kt}^*}{\tilde{s}_t^*}\right) = (1-\sigma) \sum_{k \in \Omega_t^*} \omega_{kt}^* \Delta \ln\left(\frac{p_{kt}}{\tilde{p}_t}\right) + (\sigma - 1) \sum_{k \in \Omega_t^*} \omega_{kt}^* \Delta \ln \varphi_{kt}. \tag{A.29}$$

Rearranging (A.29), we obtain:

$$\sigma_{\varphi,\omega^*} = 1 + \frac{\sum_{k\in\Omega_t^*} \omega_{kt}^* \left[\ln\left(\frac{s_{kt}^*}{s_{kt-1}^*}\right) - \ln\left(\frac{\tilde{s}_t^*}{\tilde{s}_{t-1}^*}\right)\right]}{\sum_{k\in\Omega_t^*} \omega_{kt}^* \left[\ln\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right) - \ln\left(\frac{p_{kt}}{p_{kt-1}}\right) + \ln\left(\frac{\varphi_{kt}}{\varphi_{kt-1}}\right)\right]}. \tag{A.30}$$

Note that we could have instead multiplied both sides of (A.28) by any positive finite share that sums to one across common varieties:

$$\sum_{k\in\Omega_t^*} \xi_{kt}^* \Delta \ln\left(\frac{s_{kt}^*}{\tilde{s}_t^*}\right) = (1-\sigma) \sum_{k\in\Omega_t^*} \xi_{kt}^* \Delta \ln\left(\frac{p_{kt}}{\tilde{p}_t}\right) + (\sigma-1) \sum_{k\in\Omega_t^*} \xi_{kt}^* \Delta \ln \varphi_{kt}, \tag{A.31}$$

where

$$\sum_{k\in\Omega_t^*} \xi_{kt}^* = 1,$$

and obtained another expression for the elasticity of substitution (σ):

$$\sigma_{\varphi,\xi^*} = 1 + \frac{\sum_{k\in\Omega_t^*} \xi_{kt}^* \left[\ln\left(\frac{s_{kt}^*}{s_{kt-1}^*}\right) - \ln\left(\frac{\tilde{s}_t^*}{\tilde{s}_{t-1}^*}\right)\right]}{\sum_{k\in\Omega_t^*} \xi_{kt}^* \left[\ln\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right) - \ln\left(\frac{p_{kt}}{p_{kt-1}}\right) + \ln\left(\frac{\varphi_{kt}}{\varphi_{kt-1}}\right)\right]}. \tag{A.32}$$

Note that equations (A.30) and (A.32) both return the same value for σ, because both are derived from equation (A.28). However, suppose that a researcher falsely assumes that tastes for all common varieties are constant ($\varphi_{kt} = \varphi_{kt-1} = \varphi_k$ for all $k \in \Omega_t^*$ and t) and uses equations (A.24) and (A.26) to measure σ (instead of equations (A.30) and (A.32)). Under this false assumption, equations (A.24) and (A.26) will return different values for σ, because in general:

$$\sum_{k\in\Omega_t^*} \omega_{kt}^* \ln\left(\varphi_{kt}/\varphi_{kt-1}\right) \neq \sum_{k\in\Omega_t^*} \xi_{kt}^* \ln\left(\varphi_{kt}/\varphi_{kt-1}\right) \qquad \text{for} \qquad \omega_{kt}^* \neq \xi_{kt}^*.$$

Therefore, when tastes for some common variety change over time ($\varphi_{kt} \neq \varphi_{kt-1}$ for some $k \in \Omega_t^*$ and t), but a researcher falsely assumes that tastes for all common varieties are constant ($\varphi_{kt} = \varphi_{kt-1} = \varphi_k$ for all $k \in \Omega_t^*$ and t), the use of different weights for prices and expenditure shares (ω_{kt}^* versus ξ_{kt}^*) in general returns different elasticities of substitution ($\sigma^{SV} \neq \sigma^{ALT}$).

A.5 Robustness to Alternative Normalizations for Consumer Tastes

In this section of the online appendix, we develop our robustness test in which we rule out a pure change in consumer tastes by requiring that a generalized mean of order-r of the consumer taste parameters is constant. From equations (3) and (5) in the paper for expenditure shares in period t, we have:

$$\varphi_{kt} P_t = p_{kt} \left(s_{kt}^*\right)^{\frac{1}{\sigma-1}} \lambda_t^{\frac{1}{\sigma-1}}. \tag{A.33}$$

Taking the mean of order r of equation (A.33) across common varieties, we obtain:

$$\left[\frac{1}{N_t^*} \sum_{k\in\Omega_t^*} \varphi_{kt}^r\right]^{\frac{1}{r}} P_t = \left[\frac{1}{N_t^*} \sum_{k\in\Omega_t^*} p_{kt}^r \left(s_{kt}^*\right)^{\frac{r}{\sigma-1}}\right]^{\frac{1}{r}} \lambda_t^{\frac{1}{\sigma-1}}. \tag{A.34}$$

Similarly, for period $t-1$, we have:

$$\left[\frac{1}{N_t^*}\sum_{k\in\Omega_t^*}\varphi_{kt-1}^r\right]^{\frac{1}{r}} P_{t-1} = \left[\frac{1}{N_t^*}\sum_{k\in\Omega_t^*} p_{kt-1}^r \left(s_{kt-1}^*\right)^{\frac{r}{\sigma-1}}\right]^{\frac{1}{r}} \lambda_{t-1}^{\frac{1}{\sigma-1}}. \tag{A.35}$$

Using the normalization that a generalized mean of order-r of the consumer taste parameters is constant, we have:

$$\left[\frac{1}{N_t^*}\sum_{k\in\Omega_t^*}\varphi_{kt}^r\right]^{\frac{1}{r}} = \left[\frac{1}{N_t^*}\sum_{k\in\Omega_t^*}\varphi_{kt-1}^r\right]^{\frac{1}{r}}. \tag{A.36}$$

Taking the ratio of equations (A.34) and (A.35), and using our normalization (A.36), we obtain:

$$\frac{P_t}{P_{t-1}} = \frac{\left[\frac{1}{N_t^*}\sum_{k\in\Omega_t^*} p_{kt}^r \left(s_{kt}^*\right)^{\frac{r}{\sigma-1}}\right]^{\frac{1}{r}}}{\left[\frac{1}{N_t^*}\sum_{k\in\Omega_t^*} p_{kt-1}^r \left(s_{kt-1}^*\right)^{\frac{r}{\sigma-1}}\right]^{\frac{1}{r}}} \left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{\sigma-1}}, \tag{A.37}$$

which corresponds to equation (16) in the paper.

A.6 Non-Homothetic CES

In this section of the online appendix, we derive the generalization of our common variety unified price index (CUPI) for non-homothetic CES preferences from Section III.A. of the paper.

A.6A. Preferences

In particular, we generalize our analysis to the non-separable class of CES functions in Sato (1975), which satisfy implicit additivity in Hanoch (1975), as recently used in the macroeconomics literature in Comin, Lashkari and Mestieri (2015) and Matsuyama (2019). We suppose that we observe data on households indexed by $h \in \{1,\ldots,H\}$ that differ in income and total expenditure (E_t^h). The non-homothetic CES consumption index for household h (C_t^h) is defined by the following implicit function:

$$\sum_{k\in\Omega_t}\left(\frac{\varphi_{kt}^h c_{kt}^h}{\left(C_t^h\right)^{(\epsilon_k-\sigma)/(1-\sigma)}}\right)^{\frac{\sigma-1}{\sigma}} = 1, \tag{A.38}$$

where c_{kt}^h denotes household h's consumption of variety k at time t; φ_{kt}^h is household h's taste parameter for variety k at time t; σ is the constant elasticity of substitution between varieties; ϵ_k is the constant elasticity of consumption of variety k with respect to the consumption index (C_t^h) that allows preferences to be non-homothetic. Assuming that varieties are substitutes ($\sigma > 1$), we require $\epsilon_k < \sigma$ for the consumption index (A.38) to be globally monotonically increasing and quasi-concave, and hence to correspond to a well-defined utility function. Our baseline homothetic CES specification corresponds to the special case of equation (A.38) in which $\epsilon_k = 1$ for all $k \in \Omega_t$.

A.6B. Expenditure Minimization

The Lagrangian for the utility maximization problem for household h is:

$$\mathcal{L} = C_t^h + \rho^h \left(1 - \sum_{k \in \Omega_t} \left(\frac{\varphi_{kt}^h c_{kt}^h}{\left(C_t^h\right)^{(\epsilon_k - \sigma)/(1-\sigma)}}\right)^{\frac{\sigma-1}{\sigma}}\right) + \lambda^h \left(E_t^h - \sum_{k \in \Omega_t} p_{kt} c_{kt}^h\right), \tag{A.39}$$

where we assume for simplicity that all households face the same prices for a given variety (p_{kt}). The first-order condition with respect to consumption of each variety (c_{kt}^h) can be written as:

$$p_{kt} c_{kt}^h = \frac{\rho^h}{\lambda^h} \left(\frac{1-\sigma}{\sigma}\right) \kappa_{kt}^h, \tag{A.40}$$

where we define κ_{kt}^h as:

$$\kappa_{kt}^h \equiv \left(\frac{\varphi_{kt}^h c_{kt}^h}{\left(C_t^h\right)^{(\epsilon_k - \sigma)/(1-\sigma)}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{A.41}$$

From the first-order condition (A.40) and utility function (A.38), total expenditure by household h is given by:

$$E_t^h = \sum_{k \in \Omega_t} p_{kt} c_{kt}^h = \frac{1-\sigma}{\sigma} \frac{\rho^h}{\lambda^h}. \tag{A.42}$$

Using this result in the first-order condition (A.40), we find that κ_{kt}^h equals the share of variety k in the expenditure of household h at time t:

$$s_{kt}^h = \frac{p_{kt} c_{kt}^h}{E_t^h} = \kappa_{kt}^h = \left(\frac{\varphi_{kt}^h c_{kt}^h}{\left(C_t^h\right)^{(\epsilon_k - \sigma)/(1-\sigma)}}\right)^{\frac{\sigma-1}{\sigma}}. \tag{A.43}$$

Re-arranging this relationship, we obtain the demand function for variety k:

$$c_{kt}^h = \left(\varphi_{kt}^h\right)^{\sigma-1} \left(\frac{p_{kt}}{E_t^h}\right)^{-\sigma} \left(C_t^h\right)^{\epsilon_k - \sigma} = \left(\varphi_{kt}^h\right)^{\sigma-1} \left(\frac{p_{kt}}{P_t^h}\right)^{-\sigma} \left(C_t^h\right)^{\epsilon_k}, \tag{A.44}$$

which highlights that ϵ_k controls the elasticity of demand for variety k with respect to the real consumption index (C_t^h). Using this demand function (A.44), the expenditure share (A.43) can be re-written as:

$$s_{kt}^h = \left(\varphi_{kt}^h\right)^{\sigma-1} \left(\frac{p_{kt}}{P_t^h}\right)^{1-\sigma} \left(C_t^h\right)^{\epsilon_k - 1}. \tag{A.45}$$

Additionally, using the CES demand function (A.44) in utility in equation (A.38), we can solve for the expenditure function for household h:

$$E_t^h = P_t^h C_t^h = \left[\sum_{k \in \Omega_t} \left(\frac{p_{kt}}{\varphi_{kt}^h}\right)^{1-\sigma} \left(C_t^h\right)^{\epsilon_k - \sigma}\right]^{\frac{1}{1-\sigma}}. \tag{A.46}$$

Therefore the price index for household h is given by:

$$P_t^h = \frac{1}{C_t^h} \left[\sum_{k \in \Omega_t} \left(\frac{p_{kt}}{\varphi_{kt}^h}\right)^{1-\sigma} \left(C_t^h\right)^{\epsilon_k - \sigma}\right]^{\frac{1}{1-\sigma}}, \tag{A.47}$$

or equivalently:

$$P_t^h = \left[\sum_{k \in \Omega_t} \left(\frac{p_{kt}}{\varphi_{kt}^h} \right)^{1-\sigma} \left(E_t^h / P_t^h \right)^{\epsilon_k - 1} \right]^{\frac{1}{1-\sigma}}. \tag{A.48}$$

Combining equations (A.45) and (A.48), the share of variety k in expenditure for household h at time t can be written as:

$$s_{kt}^h = \frac{\left(p_{kt} / \varphi_{kt}^h \right)^{1-\sigma} \left(E_t^h / P_t^h \right)^{\epsilon_k - 1}}{\sum_{\ell \in \Omega_t} \left(p_{\ell t} / \varphi_{\ell t}^h \right)^{1-\sigma} \left(E_t^h / P_t^h \right)^{\epsilon_\ell - 1}} = \frac{\left(p_{kt} / \varphi_{kt}^h \right)^{1-\sigma} \left(E_t^h / P_t^h \right)^{\epsilon_k - 1}}{\left(P_t^h \right)^{1-\sigma}}. \tag{A.49}$$

Equations (A.48) and (A.49) correspond to equations (18) and (19) in Section III.A. of the paper respectively.

A.6C. Non-homothetic CES Unified Price Index

We now show that our unified approach to the demand system and the price index can be extended to this case of non-homothetic CES preferences. As for the homothetic CES specification in Section II. of the paper, the price index (A.48) depends on taste-adjusted prices (p_{kt} / φ_{kt}^h) rather than observed prices (p_{kt}). An additional challenge relative to the homothetic CES case is that the overall CES price index (P_t^h) enters the numerator of the expenditure share in equation (A.49). To overcome this additional challenge, we work with the share of each variety in *overall* expenditure (s_{kt}^h) rather than the common variety expenditure share (s_{kt}^{h*} in our earlier notation). In particular, re-arranging the overall expenditure share in equation (A.49) for an individual common variety, we have:

$$P_t^h = \frac{p_{kt}}{\varphi_{kt}} \left(s_{kt}^h \right)^{\frac{1}{\sigma-1}} \left(E_t^h / P_t^h \right)^{\frac{\epsilon_k - 1}{1-\sigma}}. \tag{A.50}$$

Taking logarithms yields:

$$\ln P_t^h = \ln p_{kt} - \ln \varphi_{kt} + \frac{1}{\sigma - 1} \ln s_{kt}^h + \left(\frac{\epsilon_k - 1}{1 - \sigma} \right) \ln \left(E_t^h / P_t^h \right). \tag{A.51}$$

Averaging across the common varieties, we obtain:

$$[1 + \vartheta] \ln P_t^h = \ln \tilde{p}_t + \frac{1}{\sigma - 1} \ln \tilde{s}_t^h + \vartheta \ln \left(E_t^h \right), \tag{A.52}$$

$$\vartheta \equiv \frac{1}{N_t^*} \sum_{k \in \Omega_t^*} \frac{\epsilon_k - 1}{1 - \sigma},$$

where a tilde above a variable denotes an average across common varieties such that $\tilde{p}_t = \left(\prod_{k \in \Omega_t^*} p_{kt} \right)^{1/N_t^*}$, where $N_t^* = |\Omega_t^*|$; we have used our normalization that the average taste shock across common varieties is equal to zero ($\ln \left(\tilde{\varphi}_t / \tilde{\varphi}_{t-1} \right) = 0$); the derived parameter ϑ captures the average across the common varieties of the elasticity of expenditure with respect to the consumption index (ϵ_k) relative to the elasticity of substitution (σ). Rearranging terms in equation (A.52) and exponentiating, we obtain the following closed-form solution for the overall CES unit expenditure function:

$$P_t^h = \left(\tilde{p}_t \right)^{\frac{1}{1+\vartheta}} \left(\tilde{s}_t^h \right)^{\frac{1}{(\sigma-1)(1+\vartheta)}} \left(E_t^h \right)^{\frac{\vartheta}{1+\vartheta}}. \tag{A.53}$$

Taking ratios between the two time periods, we obtain our generalization of our CES unified price index to the non-homothetic case for each household h:

$$\frac{P_t^h}{P_{t-1}^h} = \left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\right)^{\frac{1}{1+\vartheta}} \left(\frac{\tilde{s}_t^h}{\tilde{s}_{t-1}^h}\right)^{\frac{1}{(\sigma-1)(1+\vartheta)}} \left(\frac{E_t^h}{E_{t-1}^h}\right)^{\frac{\vartheta}{1+\vartheta}}, \tag{A.54}$$

which corresponds to equation (21) in the paper. From this expression, the change in the household's cost of living (P_t^h/P_{t-1}^h) now depends directly on the change in income (and hence total expenditure) for parameter values for which preferences are non-homothetic ($\vartheta \neq 0$).

A.7 Nested CES

In our baseline specification in Section II. of the paper, we focus for simplicity on a single CES tier of utility. In this section of the online appendix, we generalize our approach to a nested CES demand system with multiple tiers of utility. For simplicity, we illustrate this generalization for two tiers of utility (an upper tier defined across sectors and a lower tier defined across varieties within sectors), but as discussed in the paper our analysis goes through for any number of tiers of utility.

A.7A. Preferences

We assume that the aggregate unit expenditure function is a constant elasticity function of the unit expenditure function for each sector $g \in \Omega^G$ as follows:

$$P_t = \left[\sum_{g\in\Omega^G} \left(\frac{P_{gt}^G}{\varphi_{gt}^G}\right)^{1-\sigma^G}\right]^{\frac{1}{1-\sigma^G}}, \qquad \sigma^G > 1, \tag{A.55}$$

where σ^G is the elasticity of substitution across sectors; P_{gt}^G is the unit expenditure function for each sector; φ_{gt}^G is the taste parameter for each sector; we assume for simplicity that the set of sectors is constant over time and denote the number of elements in this set by $N^G = \left|\Omega^G\right|$.

The unit expenditure function for each sector is a constant elasticity function of the consumption of varieties $k \in \Omega_{gt}^K$ within that sector as follows:

$$P_{gt}^G = \left[\sum_{k\in\Omega_{gt}^K} \left(\frac{p_{kt}^K}{\varphi_{kt}^K}\right)^{1-\sigma_g^K}\right]^{\frac{1}{1-\sigma_g^K}}, \qquad \sigma_g^K > 1, \tag{A.56}$$

where σ_g^K is the elasticity of substitution across varieties within each sector and can differ across sectors; p_{kt}^K is the price for each variety; φ_{kt}^K is the taste parameter for each variety; we allow the set of varieties within each sector to change over time and denote the number of elements within this set by $N_{gt}^K = \left|\Omega_{gt}^K\right|$; we require that both elasticities of substitution (σ^G and σ_g^K) are greater than one, but do not otherwise restrict their values relative to one another.

A.7B. Aggregate Price Index

Applying Shephard's Lemma to the aggregate unit expenditure function (A.55), the share of aggregate expenditure on each sector (s_{gt}^{G}) is:

$$s_{gt}^{G}=\frac{\left(P_{gt}^{G}/\varphi_{gt}^{G}\right)^{1-\sigma^{G}}}{\sum_{m\in\Omega^{G}}\left(P_{mt}^{G}/\varphi_{mt}^{G}\right)^{1-\sigma^{G}}}=\frac{\left(P_{gt}^{G}/\varphi_{gt}^{G}\right)^{1-\sigma^{G}}}{P_{t}^{1-\sigma^{G}}}. \tag{A.57}$$

Rearranging this expenditure share, and taking logarithms, we obtain the following expression for the aggregate unit expenditure function:

$$\ln P_{t}=\ln P_{gt}^{G}-\ln\varphi_{gt}^{G}+\frac{1}{\sigma^{G}-1}\ln s_{gt}^{G}. \tag{A.58}$$

Differencing over time, and averaging across sectors, the change in the aggregate cost of living can be expressed in the form of our exact CES price index:

$$\Delta\ln P_{t}=\frac{1}{N^{G}}\sum_{g\in\Omega^{G}}\Delta\ln P_{gt}^{G}+\frac{1}{\sigma^{G}-1}\frac{1}{N^{G}}\sum_{g\in\Omega^{G}}\Delta\ln s_{gt}^{G}, \tag{A.59}$$

where we normalize the taste parameters for each sector such that they have a constant geometric mean across sectors, which implies:

$$\frac{1}{N^{G}}\sum_{g\in\Omega^{G}}\ln\left(\frac{\varphi_{gt}^{G}}{\varphi_{gt-1}^{G}}\right)=0. \tag{A.60}$$

A.7C. Sectoral Price Index

We now solve for the change in the unit expenditure function for each sector ($\Delta\ln P_{gt}^{G}$) in equation (A.59) as a function of the characteristics of the varieties within that sector. First, we can decompose the change in the sectoral unit expenditure function between a pair of periods t and $t-1$ into a variety correction term for the entry and exit of varieties ($\left(1/\left(\sigma_{g}^{K}-1\right)\right)\ln\left(\lambda_{gt}^{G}/\lambda_{gt-1}^{G}\right)$) and the change in the price index for common varieties ($P_{gt}^{G*}/P_{gt-1}^{G*}$):

$$\frac{P_{gt}^{G}}{P_{gt-1}^{G}}=\left(\frac{\lambda_{gt}^{G}}{\lambda_{gt-1}^{G}}\right)^{\frac{1}{\sigma_{g}^{K}-1}}\frac{P_{gt}^{G*}}{P_{gt-1}^{G*}}. \tag{A.61}$$

The sectoral unit expenditure function for common varieties (P_{gt}^{G*}) takes the same form as in equation (A.56) but the summation is only over common varieties $k\in\Omega_{gt}^{K*}$:

$$P_{gt}^{G*}=\left[\sum_{k\in\Omega_{gt}^{K*}}\left(\frac{p_{kt}^{K}}{\varphi_{kt}^{K}}\right)^{1-\sigma_{g}^{K}}\right]^{\frac{1}{1-\sigma_{g}^{K}}}, \tag{A.62}$$

and the share of each individual common variety in all expenditure on common varieties (s_{kt}^{K*}) is:

$$s_{kt}^{K*}=\frac{\left(p_{kt}^{K}/\varphi_{kt}^{K}\right)^{1-\sigma_{g}^{K}}}{\sum_{\ell\in\Omega_{gt}^{K*}}\left(p_{\ell t}^{K}/\varphi_{\ell t}^{K}\right)^{1-\sigma_{g}^{K}}}=\frac{\left(p_{kt}^{K}/\varphi_{kt}^{K}\right)^{1-\sigma_{g}^{K}}}{\left(P_{gt}^{G*}\right)^{\sigma_{g}^{K}-1}}. \tag{A.63}$$

Rearranging this common variety expenditure share, and taking logarithms, we obtain the following expression for the sectoral common variety unit expenditure function:

$$\ln P_{gt}^{G*} = \ln p_{kt}^{K} - \ln \varphi_{kt}^{K} + \frac{1}{\sigma_g^K - 1} \ln s_{kt}^{K*}. \tag{A.64}$$

Differencing over time, and averaging across common varieties, the change in the sectoral common varieties unit expenditure function also can be expressed in the form of our exact CES price index:

$$\Delta \ln P_{gt}^{G*} = \frac{1}{N_{gt}^{K*}} \sum_{k \in \Omega_{gt}^{K*}} \Delta \ln p_{kt}^{K} + \frac{1}{\sigma_g^K - 1} \frac{1}{N_{gt}^{K*}} \sum_{k \in \Omega_{gt}^{K*}} \Delta \ln s_{kt}^{K*}, \tag{A.65}$$

where we normalize the taste parameters for each variety such that they have a constant geometric mean across common varieties within each sector, which implies:

$$\frac{1}{N_{gt}^{K*}} \sum_{k \in \Omega_{gt}^{G*}} \ln \left(\frac{\varphi_{kt}^{K}}{\varphi_{kt-1}^{K}} \right) = 0. \tag{A.66}$$

A.7D. Nested CES Unified Price Index

Our CES unified price index (CUPI) for each tier of utility is defined over the mean of the logs of the prices and expenditure shares for that tier of utility. As the mean is a linear operator, we can apply this operator recursively across the tiers of utility to express the change in the aggregate cost of living in terms of means across both sectors and varieties within each sector. In particular, using equations (A.61) and (A.65) for each sector in the aggregate cost of living in equation (A.59), we obtain equation (23) in the paper:

$$\begin{aligned} \ln \left(\frac{P_t}{P_{t-1}} \right) = {} & \frac{1}{N^G} \sum_{g \in \Omega^G} \frac{1}{N_{gt}^{K*}} \sum_{k \in \Omega_{gt}^{K*}} \ln \left(\frac{p_{kt}^{K}}{p_{kt-1}^{K}} \right) + \frac{1}{N^G} \sum_{g \in \Omega^G} \frac{1}{\sigma_g^K - 1} \frac{1}{N_{gt}^{K*}} \sum_{k \in \Omega_{gt}^{K*}} \ln \left(\frac{s_{gkt}^{K*}}{s_{gkt-1}^{K*}} \right) \\ & + \frac{1}{N^G} \sum_{g \in \Omega^G} \frac{1}{\sigma_g^K - 1} \ln \left(\frac{\lambda_{gt}^{K}}{\lambda_{gt-1}^{K}} \right) + \frac{1}{\sigma^G - 1} \frac{1}{N^G} \sum_{g \in \Omega^G} \ln \left(\frac{s_{gt}^{G}}{s_{gt-1}^{G}} \right). \end{aligned} \tag{A.67}$$

This expression decomposes the change in the aggregate cost of living into four terms: (i) the average log change in prices across sectors and common varieties within each sector; (ii) the average log change in common variety expenditure shares across both sectors and common varieties within each sector; (iii) the average variety correction across sectors for the entry and exit of varieties; and (iv) the average log change in expenditure shares across sectors.

Although, for simplicity, we focus on two tiers of utility here, this procedure can be extended for any number of tiers of utility, from the highest to the lowest. In general, we can estimate the elasticity of substitution recursively for each tier of utility. However, conventional measures of the overall cost of living typically aggregate categories using expenditure-share weights. Therefore, we assume that the upper tier of utility across sectors is Cobb-Douglas ($\sigma^G = 1$), and estimate the elasticity of substitution across barcodes within sectors (σ_g^K), separately for each sector.

A.8 Mixed CES

In this section of the online appendix, we show that our results also generalize to a mixed CES specification, in which there are multiple groups of heterogeneous consumers indexed by $h \in \{1, \ldots, H\}$. For simplicity, we return to the case of a single tier of utility, although this mixed CES generalization can be combined with a nesting structure. In the non-homothetic specification in Section A.6 of this appendix, the only source of heterogeneity in expenditure shares across consumers is differences in income. In contrast, in this mixed CES specification, we allow both the elasticity of substitution (σ^h) and the taste parameter for each variety (φ^h_{kt}) to vary across the heterogeneous groups of consumers.

A.8A. Preferences and Expenditure Shares

In particular, the unit expenditure function (P^h_t) and expenditure share (s^h_{kt}) for a household from group h are given by:

$$P^h_t = \left[\sum_{k \in \Omega_t} \left(\frac{p_{kt}}{\varphi^h_{kt}} \right)^{1-\sigma^h} \right]^{\frac{1}{1-\sigma^h}}, \tag{A.68}$$

$$s^h_{kt} = \frac{\left(p_{kt}/\varphi^h_{kt}\right)^{1-\sigma^h}}{\sum_{\ell \in \Omega_t} \left(p_{\ell t}/\varphi^h_{\ell t}\right)^{1-\sigma^h}} = \frac{\left(p_{kt}/\varphi^h_{kt}\right)^{1-\sigma^h}}{\left(P^h_t\right)^{1-\sigma^h}}, \tag{A.69}$$

where s^h_{kt} is a share of variety k in the expenditure of group h at time t; we assume for simplicity that all groups face the same prices (p_{kt}); we also assume that the set of varieties available (Ω_t) is the same for all groups; but we allow for the possibility that some groups do not consume some varieties, which we interpret as corresponding to the limiting case in which the taste parameter converges to zero for that group and variety ($\lim \varphi^h_{kt} \to 0$ for some k and h); these groups of consumers could in principle differ by income and/or other demographic characteristics.

A.8B. Properties of Mixed CES

The presence of heterogeneity across groups relaxes the independence of irrelevant alternatives (IIA) assumption of CES, because the differences in substitution and taste parameters across groups imply that the relative expenditure shares of two varieties in two different markets depend on the relative size of the groups in those markets. In particular, the expenditure share of variety k at time t can be written as:

$$s_{kt} = \frac{x_{kt}}{x_t} = \sum_{h=1}^{H} \frac{x^h_{kt}}{x_t} = \sum_{h=1}^{H} \frac{x^h_t}{x_t} \frac{x^h_{kt}}{x^h_t} = \sum_{h=1}^{H} f^h_t s^h_{kt}, \tag{A.70}$$

where x^h_{kt} is expenditure by group h on variety k at time t; x_{kt} is expenditure on variety k at time t; x^h_t is overall expenditure by group h at time t; and x_t is total expenditure at time t; s_{kt} is the share of variety k in overall expenditure at time t; s^h_{kt} is the share of variety k in group h's expenditure at time t; and f^h_t is the share of group h in overall expenditure at time t. From equation (A.70), the expenditure shares of each variety k (s_{kt}) depend not only on their expenditure shares for each group h (s^h_{kt}), but also on the relative importance of the different groups (f^h_t) in total expenditure, because of the different preferences of these groups.

Similarly, this heterogeneity across groups relaxes the symmetric cross-substitution properties of CES, because the elasticity of expenditure on one variety with respect to a change in the price of another variety in two different markets also depends on group composition. To demonstrate this role for group composition, we begin by writing total expenditure on variety k as the sum across groups h of their expenditure on that variety:

$$x_{kt} = \sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}.$$

Differentiating expenditure on variety k with respect to the price of another variety ℓ, we obtain:

$$\frac{\partial x_{kt}}{\partial p_{\ell t}} = \sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1} \left(\sigma^{h}-1\right) \frac{\partial P_{t}^{h}}{\partial p_{\ell t}} \frac{1}{P_{t}^{h}}.$$

Rearranging this equation, we obtain the following elasticity of expenditure on variety k with respect to the price of another variety ℓ:

$$\frac{\partial x_{kt}}{\partial p_{\ell t}} \frac{p_{\ell t}}{x_{kt}} = \frac{\sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1} \left(\sigma^{h}-1\right) \frac{\partial P_{t}^{h}}{\partial p_{\ell t}} \frac{p_{\ell t}}{P_{t}^{h}}}{\sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}},$$

which can be re-written as follows:

$$\begin{aligned}
\frac{\partial x_{kt}}{\partial p_{\ell t}} \frac{p_{\ell t}}{x_{kt}} &= \sum_{h=1}^{H} \frac{\left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}}{\sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}} \left(\sigma^{h}-1\right) \frac{\partial P_{t}^{h}}{\partial p_{\ell t}} \frac{p_{\ell t}}{P_{t}^{h}}, \\
&= \sum_{h=1}^{H} \frac{\left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}}{\sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}} \left(\sigma^{h}-1\right) s_{\ell t}^{h}, \\
&= \sum_{h=1}^{H} \frac{\sum_{\ell \in \Omega_t} \left(p_{\ell t}/\varphi_{\ell t}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}}{\sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}} \frac{\left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}}{\sum_{\ell \in \Omega_t} \left(p_{\ell t}/\varphi_{\ell t}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}} \left(\sigma^{h}-1\right) s_{\ell t}^{h}, \\
&= \sum_{h=1}^{H} \frac{\sum_{\ell \in \Omega_t} \left(p_{\ell t}/\varphi_{\ell t}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}}{\sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}} \left(\sigma^{h}-1\right) s_{kt}^{h} s_{\ell t}^{h}, \\
&= \sum_{h=1}^{H} \frac{\sum_{\ell \in \Omega_t} \left(p_{\ell t}/\varphi_{\ell t}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}}{\sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}} \frac{\sum_{h=1}^{H} \sum_{\ell \in \Omega_t} \left(p_{\ell t}/\varphi_{\ell t}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}}{\sum_{\ell \in \Omega_t} \left(p_{\ell t}/\varphi_{\ell t}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}} f_{t}^{h} \left(\sigma^{h}-1\right) s_{kt}^{h} s_{\ell t}^{h}, \\
&= \sum_{h=1}^{H} \frac{\sum_{h=1}^{H} \sum_{\ell \in \Omega_t} \left(p_{\ell t}/\varphi_{\ell t}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}}{\sum_{h=1}^{H} \left(p_{kt}/\varphi_{kt}^{h}\right)^{1-\sigma^{h}} x_{t}^{h} \left(P_{t}^{h}\right)^{\sigma^{h}-1}} f_{t}^{h} \left(\sigma^{h}-1\right) s_{kt}^{h} s_{\ell t}^{h}, \\
&= \sum_{h=1}^{H} \frac{1}{s_{kt}} f_{t}^{h} \left(\sigma^{h}-1\right) s_{kt}^{h} s_{\ell t}^{h}.
\end{aligned}$$

Rearranging the final line, we obtain equation (26) in the paper:

$$\frac{\partial x_{kt}}{\partial p_{\ell t}} \frac{p_{\ell t}}{x_{kt}} = \frac{1}{s_{kt}} \sum_{h=1}^{H} f_{t}^{h} \left(\sigma^{h}-1\right) s_{kt}^{h} s_{\ell t}^{h}. \tag{A.71}$$

A.8C. Entry and Exit

We now show that our results for entry and exit and the change in the cost of living hold for each group of consumers separately. Partitioning varieties into entering, exiting and common varieties, the change in the overall cost of living for group h between periods $t-1$ and t can be expressed in terms of the change in the share of expenditure on common varieties ($\lambda_t^h/\lambda_{t-1}^h$) and the change in the cost of living for these common varieties (P_t^{h*}/P_{t-1}^{h*}):

$$\Phi_t^h = \frac{P_t^h}{P_{t-1}^h} = \left(\frac{\lambda_t^h}{\lambda_{t-1}^h}\right)^{\frac{1}{\sigma^h-1}} \frac{P_t^{h*}}{P_{t-1}^{h*}}, \tag{A.72}$$

where (λ_t^h, λ_{t-1}^h) take the same form as in equation (4) in the paper but are defined for each group separately. We again use an asterisk to denote the value of a variable for the common set of varieties, such that P_t^{h*} and P_{t-1}^{h*} are the unit expenditure functions for *common* varieties:

$$P_t^{h*} \equiv \left[\sum_{k\in\Omega_t^*}\left(\frac{p_{kt}}{\varphi_{kt}^h}\right)^{1-\sigma^h}\right]^{\frac{1}{1-\sigma^h}}. \tag{A.73}$$

In addition to the aggregate shares of common varieties in total expenditure (λ_t^h, λ_{t-1}^h), we can also define the share of an individual common variety $k \in \Omega_t^*$ in expenditure on all common varieties (s_{kt}^{h*}) for household h:

$$s_{kt}^{h*} = \frac{\left(p_{kt}/\varphi_{kt}^h\right)^{1-\sigma^h}}{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t}/\varphi_{\ell t}^h\right)^{1-\sigma^h}} = \frac{\left(p_{kt}/\varphi_{kt}^h\right)^{1-\sigma^h}}{\left(P_t^{h*}\right)^{1-\sigma^h}}, \qquad k \in \Omega_t^*. \tag{A.74}$$

A.8D. Exact Price indexes

All our results for the exact CES price index in Section II.D. of the paper also hold for each group of consumers separately. Using equations (A.73) and (A.74), the log change in group h's cost of living for common varieties ($\ln \Phi_t^{h*}$) between periods $t-1$ and t can be expressed in the following form:

$$\ln \Phi_t^{h*} = \sum_{k\in\Omega_t^*} \omega_{kt}^{h*} \ln\left(\frac{p_{kt}}{p_{kt-1}}\right) - \sum_{k\in\Omega_t^*} \omega_{kt}^{h*} \ln\left(\frac{\varphi_{kt}^h}{\varphi_{kt-1}^h}\right), \tag{A.75}$$

where the weights ω_{kt}^{h*} are the logarithmic mean of common variety expenditure shares (s_{kt}^{h*}) in periods t and $t-1$ and sum to one for each group,

$$\omega_{kt}^{h*} \equiv \frac{\frac{s_{kt}^{h*}-s_{kt-1}^{h*}}{\ln s_{kt}^{h*}-\ln s_{kt-1}^{h*}}}{\sum\limits_{\ell\in\Omega_t^*}\frac{s_{\ell t}^{h*}-s_{\ell t-1}^{h*}}{\ln s_{\ell t}^{h*}-\ln s_{\ell t-1}^{h*}}}, \tag{A.76}$$

where the derivation is the same as that for a single group in Section A.2 of this online appendix.

We use the invertibility of the CES demand system for each group to express the unobserved time-varying taste parameter for that group (φ_{kt}^h) in terms of observed prices (p_{kt}) and common variety expenditure shares (s_{kt}^{h*}). In particular, taking logarithms in the common variety expenditure share (5), differencing over time, and then differencing from the mean across common varieties within each time period for each group separately, we obtain the following closed-form expression for the log change in tastes that is analogous to the expression for a single group in equation (12) in the paper:

$$\ln\left(\frac{\varphi_{kt}^{h}}{\varphi_{kt-1}^{h}}\right)=\ln\left(\frac{p_{kt}/\tilde{p}_{t}}{p_{kt-1}/\tilde{p}_{t-1}}\right)+\frac{1}{\sigma^{h}-1}\ln\left(\frac{s_{kt}^{h*}/\tilde{s}_{t}^{h*}}{s_{kt-1}^{h*}/\tilde{s}_{t-1}^{h*}}\right), \tag{A.77}$$

where a tilde denotes a geometric mean across the set of common varieties, such that $\tilde{x}_t=\left(\prod_{k\in\Omega_t^*}x_{kt}\right)^{1/N_t^*}$ for the variable x_{kt}; and we normalize the tastes for each variety to have a constant geometric mean across common varieties for each group of consumers: $\tilde{\varphi}_t^h/\tilde{\varphi}_{t-1}^h=1$.

Using equation (A.77) to substitute for the taste shocks in equation (A.75), we obtain an exact CES common variety price index (CCV) for each group separately:

$$\ln\Phi_t^{h*}=\frac{1}{N_t^*}\sum_{k\in\Omega_t^*}\ln\left(\frac{p_{kt}}{p_{kt-1}}\right)+\frac{1}{\sigma^h-1}\frac{1}{N_t^*}\sum_{k\in\Omega_t^*}\ln\left(\frac{s_{kt}^{h*}}{s_{kt-1}^{h*}}\right). \tag{A.78}$$

Substituting this common variety price index into our earlier expression for the overall price index (A.72), we have our exact CES unified price index (CUPI) for each group separately as in equation (27) in the paper:

$$\ln\Phi_t^{h}=\frac{1}{\sigma^h-1}\ln\left(\frac{\lambda_t^h}{\lambda_{t-1}^h}\right)+\frac{1}{N_t^*}\sum_{k\in\Omega_t^*}\ln\left(\frac{p_{kt}}{p_{kt-1}}\right)+\frac{1}{\sigma^h-1}\frac{1}{N_t^*}\sum_{k\in\Omega_t^*}\ln\left(\frac{s_{kt}^{h*}}{s_{kt-1}^{h*}}\right). \tag{A.79}$$

A.9 Logit Specification

In the discrete choice literature, a well-known result is that CES preferences can be derived as the aggregation of the choices of individual consumers with extreme-value-distributed idiosyncratic preferences, as shown in Anderson de Palma and Thisse (1992) and Train (2009). In this section of the online appendix, we use this result to show that our CES unified price index (CUPI) holds for logit preferences, as widely used in applied microeconometric research.

Following McFadden (1974), we suppose that the utility of an individual consumer i who consumes c_{ik} units of variety k at time t is given by:

$$U_{it}=u_{kt}+z_{ikt},\qquad u_{kt}\equiv\ln\varphi_{kt}+\ln c_{ikt} \tag{A.80}$$

where φ_{kt} captures common consumer tastes for each variety; z_{ikt} captures idiosyncratic consumer tastes for each variety that are drawn from an independent Type-I Extreme Value distribution:

$$G(z)=e^{-e^{(-z/\nu+\kappa)}}, \tag{A.81}$$

where ν is the scale parameter of the extreme value distribution and $\kappa\approx 0.577$ is the Euler-Mascheroni constant.

Each consumer has the same expenditure E_t and chooses their preferred variety given the observed realizations for idiosyncratic tastes. Therefore the consumer's budget constraint implies:

$$c_{ikt}=\frac{E_t}{p_{ikt}}. \tag{A.82}$$

The probability that individual i chooses variety k at time t is:

$$
\begin{aligned}
x_{ikt} &= \text{Prob}\left(u_{ikt} + z_{ikt} > u_{i\ell t} + z_{i\ell t}, \forall \ell \neq k\right), \\
&= \text{Prob}\left(z_{i\ell t} < z_{ikt} + v_{ikt} - v_{i\ell t}, \forall \ell \neq k\right).
\end{aligned}
$$

Therefore, using the distribution of idiosyncratic tastes (A.81), we have:

$$
x_{ikt}|z_{ikt} = \prod_{\ell \neq k} e^{-e^{-\left(z_{ikt}+u_{ikt}-u_{i\ell}\right)/\nu+\kappa}}.
$$

Integrating across the probability density function for z_{ikt}, we have:

$$
x_{ikt} = \int_{-\infty}^{\infty} \left(\prod_{\ell \neq k} e^{-e^{-\left(y+u_{ikt}-u_{i\ell t}\right)/\nu+\kappa}}\right) \frac{1}{\mu} e^{-y/\nu+\kappa} e^{-e^{-(-y\nu+\kappa)}} dy.
$$

Noting that $u_{ikt} - u_{ikt} = 0$, this expression can be re-written as:

$$
x_{ikt} = \int_{-\infty}^{\infty} \left(\prod_{\ell \in \Omega_t} e^{-e^{-\left(y+u_{ikt}-u_{i\ell t}\right)/\nu+\kappa}}\right) \frac{1}{\mu} e^{-y/\nu+\kappa} dy,
$$

which can be in turn re-written as:

$$
x_{ikt} = \int_{-\infty}^{\infty} \exp\left(-\sum_{\ell \in \Omega_t} e^{-\left(y+u_{ikt}-u_{i\ell t}\right)/\nu+\kappa}\right) \frac{1}{\mu} e^{-y/\nu+\kappa} dy,
$$

and hence:

$$
x_{ikt} = \int_{-\infty}^{\infty} \exp\left(-e^{-y/\nu+\kappa} \sum_{\ell \in \Omega_t} e^{-\left(u_{ikt}-u_{i\ell t}\right)/\nu}\right) \frac{1}{\mu} e^{-y/\nu+\kappa} dy.
$$

Now define the following change of variable:

$$
h = \exp\left(-y/\nu + \kappa\right),
$$

where

$$
-\frac{1}{\nu} \exp\left(-y/\nu + \kappa\right) dy = dh.
$$

As $y \to \infty$, we have $h \to 0$. As $y \to -\infty$, we have $h \to \infty$. Using this change of variable, we have:

$$
x_{ikt} = \int_{\infty}^{0} \exp\left(-h \sum_{\ell \in \Omega_t} e^{-\left(u_{ikt}-u_{i\ell t}\right)/\nu}\right) - dh,
$$

or equivalently:

$$
x_{ikt} = \int_{0}^{\infty} \exp\left(-h \sum_{\ell \in \Omega_t} e^{-\left(u_{ikt}-u_{i\ell t}\right)/\nu}\right) dh,
$$

which yields:

$$
x_{ikt} = \left[\frac{\exp\left(-h \sum_{\ell \in \Omega_t} e^{-\left(u_{ikt}-u_{i\ell t}\right)/\nu}\right)}{-\sum_{\ell \in \Omega_t} e^{-\left(u_{ikt}-u_{i\ell t}\right)/\nu}}\right]_0^{\infty},
$$

and hence:

$$
x_{ikt} = \frac{1}{\sum_{\ell \in \Omega_t} e^{-\left(u_{ikt}-u_{i\ell t}\right)/\nu}}.
$$

The probability that individual i chooses variety k at time t is therefore:

$$x_{ikt} = \frac{e^{u_{ikt}/\nu}}{\sum_{\ell\in\Omega_t} e^{u_{i\ell t}/\nu}},$$

which from the definition of u_{ikt} in (A.80) and the consumer's budget constraint in (A.82) becomes:

$$s_{ikt} = s_{kt} = \frac{\left(p_{kt}/\varphi_{kt}\right)^{-1/\nu}}{\sum_{\ell\in\Omega_t}\left(p_{\ell t}/\varphi_{\ell t}\right)^{-1/\nu}}, \tag{A.83}$$

which makes clear that our consumer taste shocks $(\varphi_{kt}/\varphi_{kt-1})$ correspond to shifts in the common component of tastes for each variety for all consumers (φ_{kt}). As shown in Anderson, De Palma and Thisse (1992), the expected utility of consumer i at time t is:

$$\mathbb{E}\left[U_{it}\right] = \mathbb{E}\left[\max\left\{u_{i1t}+z_{i1t},\ldots,u_{iNt}+z_{iNt}\right\}\right] = \nu\ln\left[\sum_{\ell\in\Omega_t}\exp\left(\frac{u_{ikt}}{\nu}\right)\right]. \tag{A.84}$$

Using the definition of u_{ikt} in (A.80) and the consumers budget constraint in (A.82), expected utility can be written as:

$$\mathbb{E}\left[U_{it}\right] = \frac{E_t}{P_t}, \tag{A.85}$$

where P_t is the unit expenditure function:

$$P_t = \left[\sum_{k\in\Omega_t}\left(p_{kt}/\varphi_{kt}\right)^{-1/\nu}\right]^{-\nu}. \tag{A.86}$$

Total expenditure on variety k across all consumers i at time t is:

$$E_{kt} = \sum_i E_{ikt} = \sum_i s_{kt}E_{it} = s_{kt}E_t, \tag{A.87}$$

where we have used the fact that each consumer has the same expenditure E_t. Combining equations (A.83) and (A.87), total expenditure on variety k at time t can be written as:

$$E_{kt} = \left(p_{kt}/\varphi_{kt}\right)^{-1/\nu} P_t^{1/\nu} E_t, \tag{A.88}$$

where P_t is again the unit expenditure function (A.86).

Note that equations (A.85), (A.86) and (A.88) take the same form as in our baseline CES specification in Section II. of the paper, where $1/\nu = \sigma - 1$. Therefore, our unified price index (CUPI) can be applied for the closely-related logit model. Additionally, in the same way that our baseline CES specification can be generalized to accommodate mixed CES (as in Section A.8 of this online appendix), this baseline logit model can be generalized to accommodate a mixed logit specification, as in McFadden and Train (2000).

A.10 Flexible Functional Forms

In this section of the online appendix, we show that our approach also holds for the flexible functional forms of homothetic translog preferences and the non-homothetic almost ideal demand system (AIDS). We first present results for the homothetic translog case, before turning to the non-homothetic AIDS specification.

A.10A. Homothetic Translog Preferences

Homothetic translog preferences provide an arbitrarily close local approximation to any continuous and twice-differentiable homothetic expenditure function. Following a similar approach to that used for CES preferences in the paper, we show that the translog demand system can be inverted to solve for unobserved time-varying tastes in terms of observed prices and expenditure shares. We use this result to derive an exact price index for translog preferences in terms of only prices and expenditure shares. We compare this exact price index to the conventional Törnqvist index, which is exact for translog preferences under the assumption of time-invariant tastes for each variety. We show that this conventional Törnqvist index for translog is subject to a similar taste-shock bias as the Sato-Vartia index for CES in the presence of time-varying taste shocks.

We consider the following translog unit expenditure function defined over the price (p_{kt}) and taste parameter (φ_{kt}) for a constant set of varieties $k \in \Omega$ with number of elements $N = |\Omega|$:

$$\ln P_t = \ln P\left(p_t, \varphi_t, \sigma\right) = \ln \alpha_0 + \sum_{k \in \Omega} \alpha_k \ln \left(\frac{p_{kt}}{\varphi_{kt}}\right) + \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln \left(\frac{p_{kt}}{\varphi_{kt}}\right) \ln \left(\frac{p_{\ell t}}{\varphi_{\ell t}}\right), \tag{A.89}$$

where the parameters β_{kl} control substitution patterns between varieties; symmetry between varieties requires $\beta_{k\ell} = \beta_{\ell k}$; and symmetry and homotheticity together imply $\sum_{k \in \Omega} \alpha_k = 1$ and $\sum_{k \in \Omega} \beta_{k\ell} = \sum_{\ell \in \Omega} \beta_{\ell k} = 0$.

We begin by deriving the demand system from the unit expenditure function (A.89), which can be re-written as:

$$\begin{aligned}
\ln P_t = \ln \alpha_0 &+ \sum_{k \in \Omega} \alpha_k \ln p_{kt} - \sum_{k \in \Omega} \alpha_k \ln \varphi_{kt} \\
&+ \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln p_{kt} \ln p_{\ell t} - \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln p_{kt} \ln \varphi_{\ell t} \\
&- \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln \varphi_{kt} \ln p_{\ell t} + \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln \varphi_{kt} \ln \varphi_{\ell t}.
\end{aligned}$$

Differentiating with respect to p_{mt}, we have:

$$\begin{aligned}
\frac{\partial P_t / \partial p_{mt}}{P_t} = \frac{\alpha_m}{p_{mt}} &+ \frac{1}{2} \sum_{\ell \in \Omega} \frac{\beta_{m\ell}}{p_{mt}} \ln p_{\ell t} - \frac{1}{2} \sum_{\ell \in \Omega} \frac{\beta_{m\ell}}{p_{mt}} \ln \varphi_{\ell t} \\
&+ \frac{1}{2} \sum_{k \in \Omega} \frac{\beta_{km}}{p_{mt}} \ln p_{kt} - \frac{1}{2} \sum_{k \in \Omega} \frac{\beta_{km}}{p_{mt}} \ln \varphi_{kt}.
\end{aligned}$$

Assuming symmetry ($\beta_{m\ell} = \beta_{km}$), this simplifies to:

$$\frac{\partial P_t / \partial p_{mt}}{P_t} = \frac{\alpha_m}{p_{mt}} + \sum_{\ell \in \Omega} \frac{\beta_{m\ell}}{p_{mt}} \ln p_{\ell t} - \sum_{\ell \in \Omega} \frac{\beta_{m\ell}}{p_{mt}} \ln \varphi_{\ell t},$$

which implies:

$$\frac{\partial P_t}{\partial p_{mt}} \frac{p_{mt}}{P_t} = \alpha_m + \sum_{\ell \in \Omega} \beta_{m\ell} \ln \left(\frac{p_{\ell t}}{\varphi_{\ell t}}\right),$$

and hence:

$$s_{kt} = \alpha_k + \sum_{\ell \in \Omega} \beta_{k\ell} \ln \left(\frac{p_{\ell t}}{\varphi_{\ell t}}\right). \tag{A.90}$$

We assume that a variety's expenditure share is decreasing in its own taste-adjusted price ($\beta_{kk} < 0$), and increasing in the taste-adjusted price of other varieties ($\beta_{k\ell} > 0$ for $\ell \neq k$). This assumption ensures that the demand system satisfies the "connected substitutes" conditions from Berry, Gandhi and Haile (2013), which rule out the possibility that some varieties are substitutes while others are complements.

We next use the unit expenditure function (A.89) to derive the exact price index for translog preferences. Consider any quadratic function of the following form:

$$F(\mathbf{z_t}) = a_0 + \sum_{k\in\Omega} a_k z_{kt} + \sum_{k\in\Omega}\sum_{\ell\in\Omega} a_{k\ell} z_{kt} z_{\ell t}, \tag{A.91}$$

where bold font is used to denote a matrix or vector. Under the assumption that the parameters of this quadratic function $\{a_0, a_k, a_{k\ell}\}$ are constant, the following result holds exactly:

$$F(\mathbf{z_t}) - F(\mathbf{z_{t-1}}) = \frac{1}{2}\sum_{k\in\Omega}\left[\frac{\partial F(z_{kt})}{\partial z_{kt}} + \frac{\partial F(z_{kt-1})}{\partial z_{kt-1}}\right](z_{kt} - z_{kt-1}). \tag{A.92}$$

Now note that the homothetic translog unit expenditure function (A.89) corresponds to such a quadratic function where:

$$F(\mathbf{z_t}) = \ln P_t, \qquad z_{kt} = \ln p_{kt}, \qquad \frac{\partial F(\mathbf{z_t})}{\partial z_{kt}} = \frac{\partial \ln P_t}{\partial \ln p_{kt}} = \frac{\partial P_t}{\partial p_{kt}}\frac{p_{kt}}{P_t}.$$

Applying the result (A.92) for this homothetic translog unit expenditure function, we obtain:

$$\ln P_t - \ln P_{t-1} = \frac{1}{2}\sum_{k\in\Omega}\left(\frac{\partial P_t}{\partial p_{kt}}\frac{p_{kt}}{P_t} + \frac{\partial P_{t-1}}{\partial p_{kt-1}}\frac{p_{kt-1}}{P_{t-1}}\right)\left(\ln\left(\frac{p_{kt}}{\varphi_{kt}}\right) - \ln\left(\frac{p_{kt-1}}{\varphi_{kt-1}}\right)\right), \tag{A.93}$$

which using the properties of the unit expenditure function can be re-written as:

$$\ln P_t - \ln P_{t-1} = \sum_{k\in\Omega}\frac{1}{2}(s_{kt} + s_{kt-1})\left(\ln\left(\frac{p_{kt}}{\varphi_{kt}}\right) - \ln\left(\frac{p_{kt-1}}{\varphi_{kt-1}}\right)\right), \tag{A.94}$$

which corresponds to the exact price index for translog ($\ln \Phi_t^{TR}$) in equation (30) in the paper:

$$\ln \Phi_t^{TR} = \ln\left(\frac{P_t}{P_{t-1}}\right) = \sum_{k\in\Omega}\frac{1}{2}(s_{kt} + s_{kt-1})\ln\left(\frac{p_{kt}}{p_{kt-1}}\right) - \sum_{k\in\Omega}\frac{1}{2}(s_{kt} + s_{kt-1})\ln\left(\frac{\varphi_{kt}}{\varphi_{kt-1}}\right), \tag{A.95}$$

where the weights are the arithmetic mean of expenditure shares in the two time periods ($1/2\,(s_{kt} + s_{kt-1})$) and hence necessarily sum to one.

In the same way that our CES unified price index (CUPI) is a generalization of the Sato-Vartia price index to allow for taste shocks for each variety, so the translog exact price index in equation (A.95) is a generalization of the Törnqvist index ($\ln \Phi_t^{TO}$), which corresponds to the special case in which tastes are assumed to be constant for all varieties ($(\varphi_{kt}/\varphi_{kt-1}) = 1$ for all $k \in \Omega$):

$$\ln \Phi_t^{TO} = \ln\left(\frac{P_t}{P_{t-1}}\right) = \sum_{k\in\Omega}\frac{1}{2}(s_{kt} + s_{kt-1})\ln\left(\frac{p_{kt}}{p_{kt-1}}\right). \tag{A.96}$$

From equations (A.95) and (A.96), the exact translog price index with time-varying taste shocks differs from the conventional Törnqvist index that assumes time-invariant tastes by a correction term that we term the taste-shock bias:

$$\ln \Phi_t^{TR} = \ln \Phi_t^{TO} - \underbrace{\left[\sum_{k \in \Omega} \frac{1}{2} \left(s_{kt} + s_{kt-1} \right) \ln \left(\frac{\varphi_{kt}}{\varphi_{kt-1}} \right) \right]}_{\text{taste-shock bias}}. \tag{A.97}$$

Comparing equation (A.97) for translog with equation (13) in the paper for CES, this taste-shock bias takes a similar form as for CES, except that the taste shock for each variety is weighted by the arithmetic mean of expenditure shares in the two time periods rather than the logarithmic mean of these expenditure shares. The Törnqvist index index is unbiased if the demand shocks ($\ln \left(\varphi_{kt} / \varphi_{kt-1} \right)$) are orthogonal to the expenditure-share weights ($\frac{1}{2} \left(s_{kt} + s_{kt-1} \right)$), upward-biased if they are positively correlated with these weights, and downward-biased if they are negatively correlated with these weights. In principle, either a positive or negative correlation between the taste shocks ($\ln \left(\varphi_{kt} / \varphi_{kt-1} \right)$) and the expenditure-share weights ($\frac{1}{2} \left(s_{kt} + s_{kt-1} \right)$) is possible, depending on the underlying correlation between taste shocks ($\ln \left(\varphi_{kt} / \varphi_{kt-1} \right)$) and price shocks ($\ln \left(p_{kt} / p_{kt-1} \right)$) in the two time periods. However, there is a mechanical force for a positive correlation, because the expenditure-share weights themselves are endogenous to the taste shocks, as for our baseline CES specification in Section II.E. of the paper. In particular, a positive taste shock for a variety mechanically increases the expenditure-share weight for that variety and reduces the expenditure-share weight for all other varieties in the demand system (A.90). We therefore obtain the following result for translog preferences, which is analogous to our result in equation (14) in Section II.E. of the paper for CES preferences.

Proposition. *A positive taste shock for a variety k (i.e., $\ln \left(\varphi_{kt} / \varphi_{kt-1} \right) > 0$ for some $k \in \Omega$) increases the expenditure share for that variety k at time t (s_{kt}) and reduces the expenditure share for all other varieties $\ell \neq k$ at time t ($s_{\ell t}$).*

Proof. Note that tastes, prices and expenditure shares at time $t-1$ (φ_{kt-1}, p_{kt-1}, s_{kt-1}) are pre-determined at time t. To evaluate the impact of a positive taste shock for variety k ($\ln \left(\varphi_{kt} / \varphi_{kt-1} \right) > 0$), we consider the effect of an increase in tastes at time t for that variety (φ_{kt}) given its taste parameter at time $t-1$ (φ_{kt-1}). From the expenditure share (A.90), and using our assumption that the parameters ($\boldsymbol{\beta}$) satisfy "connected substitutes," we have:

$$\begin{aligned}
\frac{ds_{kt}}{d\varphi_{kt}} \frac{\varphi_{kt}}{s_{kt}} &= -\frac{\beta_{kk}}{s_{kt}} > 0, \qquad \text{since} \qquad \beta_{kk} < 0, \\
\frac{ds_{\ell t}}{d\varphi_{kt}} \frac{\varphi_{kt}}{s_{\ell t}} &= -\frac{\beta_{\ell k}}{s_{\ell t}} < 0, \qquad \text{since} \qquad \beta_{\ell k} > 0, \qquad \ell \neq k.
\end{aligned}$$

□

For both translog and CES preferences, the source for the taste-shock bias is the failure to take into account that an increase in taste for a variety is analogous to a fall in its price. This failure induces a systematic overstatement of the increase in the cost of living, because consumers substitute towards varieties that become more desirable. Therefore, other things equal, varieties experiencing an increase in tastes (for which the change in observed prices is greater than the true change in taste-adjusted prices) receive a higher expenditure-share weight than varieties experiencing a decrease in tastes (for which the change in observed prices is smaller than the true change in taste-adjusted prices).

As for our CES specification in Section II. of the paper, the challenge in implementing the exact price index (A.95) empirically is that taste-adjusted prices (p_{kt}/φ_{kt}) are not directly observed in the data. Again we overcome this challenge by inverting the demand system to solve for the taste parameters (φ_{kt}) as a function of the observed prices and expenditure shares (p_{kt}, s_{kt}). Differencing over time in the demand system (A.90), we obtain the following expression for the change in the expenditure share for each variety, which corresponds to equation (31) in the paper,

$$\Delta s_{kt} = \sum_{\ell \in \Omega} \beta_{k\ell} \left[\Delta \ln \left(p_{\ell t} \right) - \Delta \ln \varphi_{\ell t} \right]. \tag{A.98}$$

We solve for the unobserved taste shocks ($\Delta \ln \left(\varphi_{\ell t} \right)$) by inverting the demand system in equation (A.98). This demand system (A.98) consists of a system of equations for the change in the expenditure shares (Δs_{kt}) of the N varieties that is linear in the change in the log price ($\Delta \ln p_{kt}$) and log tastes parameter ($\Delta \ln \varphi_{kt}$) for each variety. This demand system can be written in the following matrix form:

$$\Delta \mathbf{s_t} = \boldsymbol{\beta} \Delta \ln \mathbf{p_t} - \boldsymbol{\beta} \Delta \ln \boldsymbol{\varphi}_{\mathbf{t}}, \tag{A.99}$$

where we use bold math font to denote a vector or matrix.

In this demand system (A.99), the changes in expenditure shares ($\Delta \mathbf{s_t}$) must sum to zero across varieties. Furthermore, under our assumptions of symmetry and homotheticity, the rows and columns of the symmetric matrix $\boldsymbol{\beta}$ must each sum to zero. Therefore, without loss of generality, we omit the equation for the first variety. We nevertheless recover the taste shock for all varieties (including the omitted one) using our result that the taste shocks are mean zero across varieties ($(1/N) \sum_{k \in \Omega} \Delta \ln \left(\varphi_{\ell t} \right) = 0$). In particular, we define the following augmented variables:

$$\Delta \tilde{\mathbf{s}}_{\mathbf{t}} \equiv \begin{pmatrix} 0 \\ \Delta \mathbf{s}_{\mathbf{t}}^{-} \end{pmatrix}, \qquad \tilde{\boldsymbol{\beta}} \equiv \begin{pmatrix} 0, \ldots, 0 \\ \boldsymbol{\beta}^{-} \end{pmatrix}, \qquad \tilde{\gamma} \equiv \begin{pmatrix} 1, \ldots, 1 \\ \boldsymbol{\beta}^{-} \end{pmatrix}, \tag{A.100}$$

where $\Delta \mathbf{s}_{\mathbf{t}}^{-}$ denotes the vector of changes in expenditure shares omitting the first variety; and $\boldsymbol{\beta}^{-}$ denotes the symmetric matrix of substitution parameters omitting the first row. Using this notation, the demand system (A.99) can be written in the following form:

$$\Delta \tilde{\mathbf{s}}_{\mathbf{t}} = \tilde{\boldsymbol{\beta}} \Delta \ln p_t - \tilde{\gamma} \Delta \ln \boldsymbol{\varphi}_t, \tag{A.101}$$

which can be inverted to solve for the vector of taste shocks ($\Delta \ln \boldsymbol{\varphi}_t$). We thus obtain the unobserved taste shock for each variety in terms of observed prices and expenditure shares:

$$\Delta \ln \varphi_{kt} = S_{kt}^{-1} \left(\Delta \mathbf{s_t}, \Delta \ln \mathbf{p_t}, \boldsymbol{\beta} \right). \tag{A.102}$$

Substituting for the unobserved taste shock in equation (A.95), we obtain the following exact price index for translog preferences with time-varying taste parameters:

$$\ln \Phi_t^{TCG} = \sum_{k \in \Omega} \frac{1}{2} \left(s_{kt} + s_{kt-1} \right) \ln \left(\frac{p_{kt}}{p_{kt-1}} \right) - \sum_{k \in \Omega} \frac{1}{2} \left(s_{kt} + s_{kt-1} \right) S_{kt}^{-1} \left(\Delta \mathbf{s_t}, \Delta \ln \mathbf{p_t}, \boldsymbol{\beta} \right). \tag{A.103}$$

This exact translog common variety price index (Φ_t^{TCG}) is the analog of our exact CES common variety price index ($\ln \Phi_t^{CCG}$) in equation (9) in the paper.

Therefore, our main insight that the demand system can be unified with the unit expenditure function to construct an exact price index that allows for time-varying taste shocks for individual varieties is not specific to CES, but also holds for flexible functional forms. Furthermore, the taste-shock bias is again present, because a conventional price index that assumes time-variant tastes interprets all movements in expenditure shares as reflecting changes in prices, and hence does not take into account that these movements in expenditure shares are also influenced by the time-varying demand residual.

A.10B. Non-Homothetic Almost Ideal Demand System (AIDS)

The non-homothetic almost ideal demand system (AIDS) provides an arbitrary first-order linear approximation to the demand system and is based on translog functions. In particular, the AIDS expenditure function is defined over the price (p_{kt}) and taste parameter (φ_{kt}) for a constant set of varieties $k \in \Omega$ with number of elements $N = |\Omega|$:

$$\ln E\left(p_t, \varphi_t, u_t, \sigma\right) = \ln \alpha_0 + \sum_{k \in \Omega} \alpha_k \ln\left(\frac{p_{kt}}{\varphi_{kt}}\right) + \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \beta_{k\ell} \ln\left(\frac{p_{kt}}{\varphi_{kt}}\right) \ln\left(\frac{p_{\ell t}}{\varphi_{\ell t}}\right) + u_t \gamma_0 \prod_{k \in \Omega} \left(\frac{p_{kt}}{\varphi_{kt}}\right)^{\gamma_k}, \tag{A.104}$$

where u denotes the level of utility and linear homogeneity in p_{kt} requires $\sum_{k \in \Omega} \alpha_k = 1$, $\sum_{k \in \Omega} \beta_{k\ell} = \sum_{\ell \in \Omega} \beta_{\ell k} = 0$, and $\sum_{k \in \Omega} \gamma_k = 0$. Using Shephard's Lemma, we differentiate with respect to p_{kt} in equation (A.104) to obtain the following expression for the expenditure share for each variety k:

$$s_{kt} = \alpha_k + \sum_{\ell \in \Omega} \beta_{k\ell} \ln\left(\frac{p_{\ell t}}{\varphi_{\ell t}}\right) + \gamma_k \ln\left(\frac{E_t}{P_t}\right), \tag{A.105}$$

where P_t is a price index defined by:

$$\ln P_t = \ln \alpha_0 + \sum_{k \in \Omega} \alpha_k \ln\left(\frac{p_{kt}}{\varphi_{kt}}\right) + \frac{1}{2} \sum_{k \in \Omega} \sum_{\ell \in \Omega} \bar{\beta}_{k\ell} \ln\left(\frac{p_{kt}}{\varphi_{kt}}\right) \ln\left(\frac{p_{\ell t}}{\varphi_{\ell t}}\right), \tag{A.106}$$

where

$$\bar{\beta}_{k\ell} = \frac{1}{2}\left(\beta_{k\ell} + \beta_{\ell k}\right) = \bar{\beta}_{\ell k}. \tag{A.107}$$

Taking the derivative of the expenditure share (s_{kt}) of a variety k in equation (A.105) with respect to the price of any variety j, we have:

$$\frac{ds_{kt}}{dp_{jt}} \frac{p_{jt}}{s_{kt}} = \frac{\beta_{kj}}{s_{kt}} - \frac{\gamma_j \alpha_j}{s_{kt}} - \frac{\gamma_j}{s_{kt}} \sum_{\ell \in \Omega} \bar{\beta}_{j\ell} \ln\left(\frac{p_{\ell t}}{\varphi_{\ell t}}\right), \tag{A.108}$$

where connected substitutes requires $\frac{ds_{kt}}{dp_{kt}} \frac{p_{kt}}{s_{kt}} < 0$ for all k and $\frac{ds_{kt}}{dp_{jt}} \frac{p_{jt}}{s_{kt}} > 0$ for all $j \neq k$.

Assuming that connected substitutes is satisfied, we can again invert the demand system (A.105) to solve for unique values for tastes (φ_{kt}) up to a normalization for the geometric mean for consumer tastes ($\frac{1}{N} \sum_{k \in \Omega} \ln \varphi_{kt} = 0$):

$$\varphi_{kt} = S_{kt}^{-1}\left(\mathbf{s_t}, \mathbf{p_t}\right). \tag{A.109}$$

Using these solutions for consumer tastes ($\varphi_{kt} = S_{kt}^{-1}(\mathbf{s_t}, \mathbf{p_t})$) in the expenditure function (A.104), we obtain:

$$\ln E_t = \ln \alpha_0 + \sum_{k\in\Omega} \alpha_k \ln \left(\frac{p_{kt}}{S_{kt}^{-1}(\mathbf{s_t}, \mathbf{p_t})} \right) + \frac{1}{2} \sum_{k\in\Omega} \sum_{\ell\in\Omega} \beta_{k\ell} \ln \left(\frac{p_{kt}}{S_{kt}^{-1}(\mathbf{s_t}, \mathbf{p_t})} \right) \ln \left(\frac{p_{\ell t}}{S_{kt}^{-1}(\mathbf{s_t}, \mathbf{p_t})} \right) + u_t \gamma_0 \prod_{k\in\Omega} \left(\frac{p_{kt}}{S_{kt}^{-1}(\mathbf{s_t}, \mathbf{p_t})} \right)^{\gamma_k}, \quad \text{(A.110)}$$

which can be used to compute the change welfare between any pair of time periods.

A.11 Feenstra (1994) Estimator

In this section of the online appendix, we provide further details on the Feenstra (1994) estimator used to estimate the elasticity of substitution. We estimate a separate elasticity of substitution for each sector, but suppress the subscript on parameters for sectors to simplify notation. We start by taking logarithms and double-differences in the CES demand system for common varieties in equation (5) in the paper:

$$\Delta \ln \bar{s}_{kt}^{*} = \beta_0 + \beta_1 \Delta \ln \bar{p}_{kt} + u_{kt}, \quad \text{(A.111)}$$

where the first difference is over time and the second difference is from the geometric mean across common varieties; Δ denotes the time-difference operator such that $\Delta \ln \bar{p}_{kt} = \ln(\bar{p}_{kt}/\bar{p}_{kt-1})$; a bar above a variable indicates that it is normalized by its geometric mean across common varieties, such that $\ln(\bar{p}_{kt}) = \ln(p_{kt}/\tilde{p}_t)$; the regression error ($u_{kt}$) includes the time-varying taste shock ($\Delta \ln \bar{\varphi}_{kt}$); and any time-invariant component of tastes is differenced out between the two time periods.

We combine this relationship from the CES demand system in equation (A.111) above with an analogous supply-side relationship:

$$\Delta \ln \bar{s}_{kt}^{*} = \delta_0 + \delta_1 \Delta \ln \bar{p}_{kt} + w_{kt}. \quad \text{(A.112)}$$

The identifying assumption of the Feenstra (1994) estimator is that the double-differenced demand and supply shocks (u_{kt}, w_{kt}) are orthogonal and heteroskedastic. The orthogonality assumption defines a rectangular hyperbola for each variety in the space of the demand and supply elasticities. The heteroskedasticity assumption implies that these rectangular hyperbolas for different varieties do not lie on top of one another. With two varieties, the intersection of these rectangular hyperbolas exactly identifies the elasticity of substitution. With more than two varieties, the model is overidentified.

In particular, following Broda and Weinstein (2006), the orthogonality of the double-differenced demand and supply shocks defines a set of moment conditions (one for each variety within a sector):

$$G(\varsigma) = \mathbb{E}_T [\xi_{kt}(\varsigma)] = 0, \quad \text{(A.113)}$$

where $\varsigma = \begin{pmatrix} \beta_1 \\ \delta_1 \end{pmatrix}$; $\xi_{kt} = u_{kt} w_{kt}$; and $\mathbb{E}_T$ is the expectations operator over time. We stack the moment conditions for all varieties within a sector to form the GMM objective function and obtain:

$$\hat{\varsigma} = \arg\min \left\{ G^S(\varsigma)' W G^S(\varsigma) \right\}, \quad \text{(A.114)}$$

where $G^S(\varsigma)$ is the sample analog of $G(\varsigma)$ stacked over all varieties within a given sector and W is a positive definite weighting matrix. As in Broda and Weinstein (2010), we weight the data for each variety by the number of raw buyers for that variety to ensure that our objective function is more sensitive to varieties purchased by larger numbers of consumers.

A.12 Specification Check Using a Subset of Common Varieties

In this section of the online appendix, we discuss the joint specification test of our assumptions of CES demand and a constant geometric mean of consumer tastes from Section V.E. of the paper. We use the independence of irrelevant alternatives (IIA) property of CES, which implies that the change in the cost of living can be computed either (i) using all common varieties and an entry/exit term or (ii) choosing a subset of common varieties and adjusting the entry/exit term for the omitted common varieties. If preferences are CES and taste shocks average out across varieties such that the geometric mean of tastes is constant for both definitions of common varieties, we should obtain the same change in the cost of living from these two different specifications. We start with the following expression for the change in the cost of living under CES preferences:

$$\frac{P_t}{P_{t-1}} = \left[\frac{\sum_{\ell\in\Omega_t}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_{t-1}}\left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}. \tag{A.115}$$

Our first approach to analyzing the change in the cost of living uses the full set of common varieties and rewrites equation (A.115) as:

$$\begin{aligned}\frac{P_t}{P_{t-1}} &= \left[\frac{\sum_{\ell\in\Omega_t}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}\frac{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_{t-1}}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}\frac{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}, \\ &= \left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{\sigma-1}}\frac{P_t^*}{P_{t-1}^*},\end{aligned} \tag{A.116}$$

where

$$\Omega_t^* = \Omega_t \cap \Omega_{t-1} \tag{A.117}$$

is the set of common varieties and

$$\lambda_t = \frac{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}, \qquad \lambda_{t-1} = \frac{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_{t-1}}\left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}, \tag{A.118}$$

are the shares of expenditure on common varieties in total expenditure in the two time periods, as in Feenstra (1994). We can also define the share of an individual common variety in all expenditure on common varieties:

$$s_{kt}^* = \frac{\left(p_{kt}/\varphi_{kt}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t^*}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}} = \frac{\left(p_{kt}/\varphi_{kt}\right)^{1-\sigma}}{\left(P_t^*\right)^{1-\sigma}}, \tag{A.119}$$

where

$$P_t^* = \left[\sum_{\ell\in\Omega_t^*}\left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{A.120}$$

is the unit expenditure function for common varieties and we use an asterisk to denote the value of a variable for common varieties $k \in \Omega_t^*$. Rearranging equation (A.119), taking logarithms, then taking means across common varieties, and exponentiating, we obtain the following equivalent expression for the unit expenditure function for common varieties:

$$P_t^* = \frac{\tilde{p}_t}{\tilde{\varphi}_t} \left(\tilde{s}_t^*\right)^{\frac{1}{\sigma-1}}, \tag{A.121}$$

where a tilde denotes a geometric mean across common varieties. Using the final line of equation (A.116) and this equivalent expression for the unit expenditure function for common varieties (A.121), we obtain the following expression for the change in the cost living:

$$\frac{P_t}{P_{t-1}} = \left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{p}_t / \tilde{\varphi}_t}{\tilde{p}_{t-1} / \tilde{\varphi}_{t-1}} \left(\frac{\tilde{s}_t^*}{\tilde{s}_{t-1}^*}\right)^{\frac{1}{\sigma-1}}. \tag{A.122}$$

Our second approach to measuring the change in the cost of living uses a subset of common varieties and rewrites equation (A.115) as follows:

$$\begin{aligned} \frac{P_t}{P_{t-1}} &= \left[\frac{\sum_{\ell\in\Omega_t} \left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t^{**}} \left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}} \frac{\sum_{\ell\in\Omega_t^{**}} \left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_{t-1}} \left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}} \frac{\sum_{\ell\in\Omega_t^{**}} \left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t^{**}} \left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}, \\ &= \left(\frac{\mu_t}{\mu_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{P_t^{**}}{P_{t-1}^{**}}, \end{aligned} \tag{A.123}$$

where

$$\Omega_t^{**} \subset \Omega_t^* = \Omega_t \cap \Omega_{t-1}, \tag{A.124}$$

is a subset of common varieties and

$$\mu_t = \frac{\sum_{\ell\in\Omega_t^{**}} \left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t} \left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}}, \qquad \mu_{t-1} = \frac{\sum_{\ell\in\Omega_t^{**}} \left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_{t-1}} \left(p_{\ell t-1}/\varphi_{\ell t-1}\right)^{1-\sigma}}, \tag{A.125}$$

are the shares of expenditure on this subset of common varieties in total expenditure in the two time periods. We can also define the share of an individual variety from this subset in all expenditure on this subset:

$$s_{kt}^{**} = \frac{\left(p_{kt}/\varphi_{kt}\right)^{1-\sigma}}{\sum_{\ell\in\Omega_t^{**}} \left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}} = \frac{\left(p_{kt}/\varphi_{kt}\right)^{1-\sigma}}{\left(P_t^{**}\right)^{1-\sigma}}, \tag{A.126}$$

where

$$P_t^{**} = \left[\sum_{\ell\in\Omega_t^{**}} \left(p_{\ell t}/\varphi_{\ell t}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{A.127}$$

is the unit expenditure function for this subset of common varieties and we use a double asterisk to denote the value of a variable for this subset of common varieties. Rearranging equation (A.123), taking logarithms, then taking means across the subset of common varieties, and exponentiating, we obtain the following equivalent expression for the unit expenditure function for this subset of common varieties:

$$P_t^{**} = \frac{\tilde{\tilde{p}}_t}{\tilde{\tilde{\varphi}}_t} \left(\tilde{\tilde{s}}_t^{**}\right)^{\frac{1}{\sigma-1}}, \tag{A.128}$$

where a double tilde denotes a geometric mean across this subset of common varieties. Using the final line of equation (A.123) and this equivalent expression for the unit expenditure function for common varieties (A.128), we obtain the following alternative expression for the change in the cost of living:

$$\frac{P_t}{P_{t-1}} = \left(\frac{\mu_t}{\mu_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{\tilde{p}}_t / \tilde{\tilde{\varphi}}_t}{\tilde{\tilde{p}}_{t-1} / \tilde{\tilde{\varphi}}_{t-1}} \left(\frac{\tilde{\tilde{s}}^*_t}{\tilde{\tilde{s}}^*_{t-1}}\right)^{\frac{1}{\sigma-1}}. \tag{A.129}$$

Our two approaches to measuring the change in the cost of living in equations (A.122) and (A.129) provide the basis for a joint specification test of the sensitivity of our results to our assumptions of CES demand and a constant geometric mean of consumer tastes. If preferences are CES and the geometric mean of consumer tastes is constant across both (i) all common varieties ($\tilde{\varphi}_t = \tilde{\varphi}_{t-1}$) and (ii) this subset of common varieties ($\tilde{\tilde{\varphi}}_t = \tilde{\tilde{\varphi}}_{t-1}$), we should obtain the same change in the cost of living whether we use all common varieties in equation (A.122) or this subset of common varieties in equation (A.129):

$$\frac{P_t}{P_{t-1}} = \left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{p}_t}{\tilde{p}_{t-1}} \left(\frac{\tilde{s}^*_t}{\tilde{s}^*_{t-1}}\right)^{\frac{1}{\sigma-1}} = \left(\frac{\mu_t}{\mu_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{\tilde{p}}_t}{\tilde{\tilde{p}}_{t-1}} \left(\frac{\tilde{\tilde{s}}^{**}_t}{\tilde{\tilde{s}}^{**}_{t-1}}\right)^{\frac{1}{\sigma-1}}, \tag{A.130}$$

which corresponds to equation (35) in the paper.

In contrast, if preferences are not CES, the two expressions in equation (A.130) differ in general from one another. Similarly, if preferences are CES but the geometric means of consumer tastes are not constant for both definitions of common varieties, these two expressions also differ in general. In this case, from equations (A.122) and (A.129), we have:

$$\left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{p}_t}{\tilde{p}_{t-1}} \left(\frac{\tilde{s}^*_t}{\tilde{s}^*_{t-1}}\right)^{\frac{1}{\sigma-1}} = \left(\frac{\mu_t}{\mu_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{\tilde{p}}_t}{\tilde{\tilde{p}}_{t-1}} \left(\frac{\tilde{\tilde{s}}^{**}_t}{\tilde{\tilde{s}}^{**}_{t-1}}\right)^{\frac{1}{\sigma-1}} \frac{\tilde{\varphi}_t / \tilde{\varphi}_{t-1}}{\tilde{\tilde{\varphi}}_t / \tilde{\tilde{\varphi}}_{t-1}}, \tag{A.131}$$

which implies that the two sides of equation (A.130) differ from one another by the ratio of the changes in the geometric means for these two definitions of common varieties.

A.13 Data Appendix

In this data appendix, we report our full list of product groups (sectors) and summary statistics for each sector in Table A.1, as a supplement to Table I in the paper. Consistent with our discussion for the full sample in Section IV. of the paper, we find pervasive entry and exit for all sectors, combined with substantial variation across these sectors in the share of barcoded goods (varieties) that enter and exit and the share of common varieties in expenditure in period t relative to period $t-1$.

A.14 Additional Empirical Results

In this section of the online appendix, we report additional empirical results for the specifications discussed in Sections V.B., V.E. and VII. of the paper. First, we examine the relationship between our estimated demand residuals and separate measures of Brand Asset Values (BAVs) from consumer surveys. Second, we examine the sensitivity of our measured changes in the cost of living to the Feenstra (1994) estimated elasticities. Third, we illustrate the relevance of our results for official measures of the consumer price index (CPI). Fourth, we demonstrate the robustness of our results to the treatment of varieties with smaller expenditure shares for which measurement error could be relatively more important. Fifth, we show that we find the same pattern of results using the Kilts-Nielsen Retail Scanner Dataset as an alternative source of scanner data to the Kilts-Nielsen Homescan data used in the paper.

A.14A. Relationship Between Estimated Tastes and Measured Brand Asset Values

To provide additional empirical evidence on the extent to which our estimates capture consumer tastes rather than specification or measurement error, we obtained survey data on consumer evaluations of brands from Young and Rubicam (the U.S. subsidiary of the world's largest marketing firm WPP). In particular, Young and Rubicam (Y&R) conducts an annual survey of approximately 17,000 U.S. consumers in which they try to ascertain "brand asset values" (BAVs) based on how surveyed consumers respond to a large number of questions about brands. We then matched the Y&R brands with the Nielsen brands. One difficulty we faced is that the definition of "brand" is not standardized across sources. For example, while both Nielsen and Young and Rubicam agree that Diet Coke and Coke Zero are different brands, Breakstone's butter, cottage cheese, and yogurt count as three brands in Nielsen data but only one brand ("Breakstone's") in Young and Rubicam's survey data. Young and Rubicam have fewer brands, so in the end we used a many-to-one match of 12,541 Nielsen brands to 1,370 Young and Rubicam brands.

Young and Rubicam aggregate consumer responses about brand perceptions into four basic factors, each of which is designed to capture a brand attribute that causes consumers to purchase that brand instead of another brand. They identify four factors as important in assessing a brand's value. "Energized differentiation" or "differentiation" is a measure of perceptions of the uniqueness or innovativeness of a product. Consumers rate brands with high levels of differentiation when they feel loyalty to those products and are likely to choose them despite price premia over other products with similar physical characteristics. In Young & Rubicam data, brands with high levels of differentiation are iPhones, Bose headphones, Trader Joe's products, Mountain Dew, Listerine, and Ben&Jerry's ice cream. In contrast, brands like DonQ rum, Mazola corn oil, and Cheer detergent have low levels of energized differentiation, reflecting the fact that consumers state that they are not willing to choose these products when their price is relatively high.

A second important brand characteristic is "relevance," which is a measure of whether consumers feel that the brand is relevant for them. For example, tobacco brands tend to have low relevance because many consumers do not smoke and so are unlikely to purchase tobacco products regardless of price. Other low-relevance brands are Botox, NuvaRing, and Nicorettes. On the other hand, high-relevance brands contain products that many consumers think are "necessary." Examples of high-relevance brands are Band-Aids, Heinz ketchup, and Kleenex.

"Esteem" is a third characteristic that captures the perceived prestige of the brand. For example, Lucky Strike cigarettes, Schlitz Malt Liquor, and Method dish soap are brands that consumers hold in low esteem. Consumers are unlikely to purchase these products in order to impress people. By contrast, Duracell batteries, Band-Aid brand bandaids, and Ziploc bags are high esteem brands because consumers think of them as the best in their classes.

Finally, and most relevant for many economic models of advertising, "knowledge" measures how familiar a consumer is with the brand. If consumers feel that they have a good understanding of the characteristics of a brand, then the brand obtains a high knowledge score. Whether they like the products or not, consumers report having a excellent understanding of exactly what they are buying when they purchase Coca-Cola, M&Ms, and Hershey's bars.

Each of these brand characteristics are imperfectly correlated with one another. For example, many brands have "energized differentiation," in the sense that consumers who know about the brands really like them, but are largely unknown (e.g., Pat LaFrieda meats and Kagome drinks). Other brands are held in esteem even if their products are not well known (e.g., Corning and DeWalt).

One issue with these variables is that it is difficult to interpret the raw scores of each variable since they are based on a variety of scaled survey questions (e.g., self-reported familiarity with a brand on 7 point scale). Since it is not obvious that a movement from 1 to 2 in a raw score means the same thing as a movement from 2 to 3, we expressed each variable as a percentile. Thus, a one unit movement in knowledge corresponds to moving one percentile in the distribution of consumer assessment of familiarity.

Since marketing data is reported at the level of the brand rather than the barcode, we estimate the nested CES specification from Section III.B. of the paper, with product groups (sectors) and Nielsen brands as our nests. In this specification, consumer taste for a barcode depends on both consumer taste for that barcode relative to other barcodes within the Nielsen brand (φ_{kt}^{K}) and consumer taste for the Nielsen brand itself (φ_{bt}^{B}). We normalize consumer tastes such that the geometric mean of barcode tastes is constant across common barcodes within each Nielsen brand and the geometric mean of brand tastes is constant across common Nielsen brands within each sector.

We can examine the relationship between our estimates of consumer tastes and the Y&R measured Brand Asset Values (BAVs) in a regression framework by estimating the following specification:

$$\ln \varphi_{bt}^{B} = \alpha_b + \mu_{gt} + \xi \mathrm{BAV}_{rt}^{i} + \varepsilon_{bt}, \tag{A.132}$$

where α_b is a Nielsen brand fixed effect; μ_{gt} is a sector-time fixed effect corresponding the sector containing brand b; BAV_{rt}^{i} is the Y&R BAV component i ($i \in$ {differentiation, relevance, esteem, knowledge}) matched to Nielsen brand b; and $\boldsymbol{\xi}$ are the associated coefficients on these variables. We cluster the standard errors by Y&R-brand-time to take account of the fact that the BAV measures take the same value across Nielsen brands within each Y&R-brand-time-period.

We can see the relationship between our estimates of consumer tastes and the Y&R BAV components through binned scatter plots. In Figure A.1 we present a binned scatter plot of the residuals from regressing $\ln \varphi_{bt}^{B}$ on sector-time fixed effects (μ_{gt}) against the residuals from regressing each of the BAVs (BAV_{rt}^{i}) on sector-time fixed effects. This plot gives us a sense of whether brands that have high estimated consumer tastes also correspond to brands that have characteristics associated high brand asset values. We see that this is generally true for each of the BAV measures, each of which is strongly and positively associated with our estimates of consumer tastes. In Figure A.2, we repeat this exercise, this time plotting the residuals from regressing $\ln \varphi^{B}_{bt}$ on Nielsen brand (α_b) and sector-time fixed effects (μ_{gt}) against the residuals from regressing BAVs (BAV^{i}_{rt}) on Nielsen brand and sector-time fixed effects, so the residuals can now be interpreted as how estimated consumer tastes or BAVs shift over time. This specification enables us to see if our estimated consumer taste shifts correspond to shifts in preferences or knowledge as measured in consumer surveys. The binned scatters clearly indicate a strong association. There appears to be an almost linear relationship between the two measures of taste shifts.

In Table A.2, we estimate equation (A.132) without Nielsen brand fixed effects (α_b) to see whether brands with high estimated taste parameters also score highly on BAV components. We find a positive and statistically significant correlation between our estimates of brand tastes and each of these BAV measures. Inevitably, the BAV measures based on consumer surveys are imperfect proxies for the whole host of characteristics that influence the appeal of a brand to consumers (including physical characteristics, quality, fashion, lifestyle etc). Furthermore, the Nielsen brands (b) are measured at a more disaggregated level than the Young and Rubicam brands (r). Thus, knowing how consumers perceive Breakstone's varieties in general tells us about *average* Nielsen measured brand appeal (as we saw in Figures A.1 and A.2), but is not informative about the differential perceptions of sub-brands like Breakstone's butter or Breakstone's yogurt. For both these and other reasons, there remains substantial idiosyncratic variation in estimated consumer tastes that is not captured by the BAV measures. Nevertheless, these results confirm that our estimates of consumer tastes for brands are systematically related to separate measures of the extent to which these brands appeal to consumers from consumer surveys.

In Table A.3 we augment the regression specification with Nielsen brand fixed effects, which implies that the estimated coefficients are now identified from the relationship between changes in estimated consumer tastes and changes in BAVs. We find that changes in each of the four BAVs are positively correlated with changes in our estimated consumers tastes, and the coefficients on relevance, esteem, and knowledge are statistically significant at conventional critical values. We also find an important role for the brand fixed effects, which is consistent with both our measures of consumer tastes and the BAV measures capturing persistent characteristics of brands.

Taken together, in both levels and changes, our estimated demand residuals are systematically related to separate measures of brand asset values, consistent with them capturing consumer tastes.

A.14B. Grid Search over the Elasticity of Substitution

We now examine the robustness of our findings of a substantial taste-shock bias to the estimated elasticity of substitution. In particular, we undertake a grid search over the range of plausible values for the elasticity of substitution. We consider a grid of thirty-eight evenly spaced values for this elasticity ranging from 1.5 to 20. For each value on the grid, we compute our CCV and CUPI for each sector and year, and then aggregate across sectors using expenditure-share weights. In Figure A.3, we compare these changes in the cost of living to the Fisher index. A smaller elasticity of substitution implies that varieties are more differentiated, which increases the absolute magnitude of the variety correction term for entering varieties being more desirable than exiting varieties $((1/(\sigma-1))\ln(\lambda_t/\lambda_{t-1})<0)$. As a result, we find that the CCV and CUPI fall further below the Fisher index as the elasticity of substitution becomes small. Nevertheless, across the entire range of plausible values for this elasticity, we find a quantitatively relevant taste-shock bias.

A.14C. Comparison with Official CPI Categories

We next illustrate the relevance of our estimated changes in the cost of living using the Nielsen data for official measures of the consumer price index (CPI). In particular, we map 89 of our 104 sectors to official CPI categories. For each of these 89 sectors, we compute conventional Laspeyres and Paasche indexes, and aggregate across sectors using expenditure share weights to compute the change in the aggregate cost of living over time. As shown in Figure A.4, we find that conventional price indexes computed using the Nielsen data are remarkably successful in replicating properties of official price indexes, with a positive and statistically significant correlation of 0.99 between the Laspeyres (based on Nielsen data) and the CPI. Moreover, the average changes in the cost of living as measured by the Laspeyres index and the CPI are almost identical: 2.65 versus 2.35 percent respectively. The Paasche index (based on Nielsen data) has the same correlation with the CPI, but has an average change that is only 1.9 percent per year. In other words, annual movements in changes in the cost of living as measured by the BLS for this set of goods can be closely approximated by using a Laspeyres index and the Nielsen data, and the difference between the Laspeyres and the Paasche indexes in the Nielsen data is less than one percentage point per year (consistent with the findings of the Boskin Commission in Boskin et al. 1996). In contrast, we find a substantial bias from abstracting from entry/exit and taste shocks, with our CUPI more than one percentage point below the CPI.

A.14D. Measurement Error in Small Expenditure Shares

We next examine the sensitivity of our results to measurement error in expenditure shares for varieties that account for small shares of expenditure. In particular, we use the property that the change in the cost of living can be computed either (i) using all common varieties and an entry/exit term or (ii) choosing a subset of common varieties and adjusting the entry/exit term for the omitted common varieties, as discussed in Section V.E. of the paper. Using this property, we recompute the CUPI using the subset of our baseline sample of common varieties with above-median expenditure shares. This specification is less sensitive to measurement error for varieties that account for small shares of expenditure, because expenditures on varieties with below-median expenditure shares only enter the change in the cost of living through the aggregate share of expenditure on varieties with above-median expenditure shares. In Figure A.5, we compare the resulting measures of the CCV and CUPI to those in our baseline specification that does not distinguish between common varieties with above-median versus below-median expenditure shares. As apparent from the figure, we find a similar change in the aggregate cost of living as in our baseline specification in the paper. This pattern of results suggests that our results are not sensitive to measurement error in expenditure shares for varieties that account for small shares of expenditure.

A.14E. Price Changes with Retail Scanner Data

Finally, we examine the robustness of our results to using the Kilts-Nielsen Retail Scanner Dataset as an alternative source of scanner data to the Kilts-Nielsen Homescan data used in the paper. Our choice of the Kilts-Nielsen Homescan data is motivated by the fact that it is designed to provide a representative sample of household expenditures on barcodes. In contrast, point of sale data, like that in the Retail Scanner Dataset, typically are drawn from a non-representative sample of stores and barcodes.

We begin by providing some further information about the advantages and disadvantages associated with working with retail scanner data. The Retail Scanner Dataset is comprised of typically 90 chains and their 35,000 stores. Since all scans from these stores are included in the database, an attractive feature of the data is that there are vastly more purchases recorded in retail scanner data relative to Homescan data. Both datasets contain purchase information for approximately five million barcodes (5.2 and 5.4 million barcodes respectively) and span approximately the same set of product groups. However, the non-representativeness of the retail sample appears to have an important impact on the market shares of many varieties. For example, Kilts estimates that only 2 percent of liquor purchases, 1 percent of convenience store purchases, and no online purchases or purchases from non-chain stores are in the Retail Scanner Dataset. In contrast, about half of all purchases from food and drug stores and about a third of all purchases from mass-merchandisers are in the retail sample. [§]

The skewness in the sampling in the Retail Scanner Dataset appears to produce a corresponding skewness in the market shares of barcodes. While both Homescan and retail scanner data tend to report similar market share numbers for products with large market shares, retail scanner data appears to severely underreport the market shares for varieties not sold intensively in the 90 chains. For example, while only 1 percent of barcodes had sales shares of less than 2.0×10^{-7} in Homescan data, more than a quarter of all market shares fell below this cutoff in the retail dataset. As a result of this skewing, the median market share in Homescan data is 8.5 times larger than in retail scanner data despite the fact that the each dataset has approximately the same number of barcodes. In order to deal with the apparently severe mismeasurement of the market shares of barcodes not sold intensively in the sampled stores, we exclude barcodes with a market share of less than 10^{-6} from the set of common varieties and instead include them in the set of entering and exiting varieties.

In Figure A.6, we compare changes in the cost of living using the Homescan and Retail Scanner data. The solid black lines show the CCV and CUPI in our baseline specification in the paper using the Homescan data. The dashed lines recompute the CCV and CUPI using the Homescan data, but excluding varieties with expenditure shares of less than 10^{-6} from the set of common varieties and instead include them in the set of entering and exiting varieties. As one can see from the figure, reclassifying varieties with trivial market shares has almost no impact on either index when using the Homescan data. Finally, the grey lines report the same results for the retail scanner data, in which we again exclude varieties with market shares of less than 10^{-6} from the set of common varieties and include them in the set of entering and exiting varieties. The CCV and CUPI's using retail scanner data display the same year-to-year pattern in measured changes in the cost of living, but the average change in the retail scanner data is about a percentage point lower than in the Homescan data. This

[§] Kilts Center for Marketing (2013) "Retail Scanner Dataset Manual," University of Chicago Booth School of Business, mimeo.

difference likely reflects the fact that the retail scanner data is not a random sample of barcodes or stores, so that expenditure shares and prices differ systematically between the two datasets.

References

Anderson, Simon, André de Palma and Jacques-François Thisse, *Discrete Choice Theory of Product Differentiation*, (Cambridge MA: MIT Press, 1992).

Berry, Steven, Amit Gandhi, and Philip Hale, "Connected Substitutes and Invertibility of Demand," Econometrica, 81 (2013), 2087-2111.

Boskin, Michael J., Ellen R. Dulberger, Robert J. Gordon, Zvi Griliches, and Dale W. Jorgenson, Toward a More Accurate Measure of the Cost of Living, Final Report to the Senate Finance Committee, (Washington, DC: Senate Finance Committee, 1996).

Broda, Christian and David E. Weinstein, "Globalization and the Gains from Variety," Quarterly Journal of Economics, 121 (2006), 541-86.

Broda, Christian and David E. Weinstein, "Product Creation and Destruction: Evidence and Price Implications," *American Economic Review*, 100 (2010), 691-723.

Comin, Diego A., Danial Lashkari, and Martí Mestieri, "Structural Change with Long-run Income and Price Effects," *NBER Working Paper*, 21595, 2015.

Feenstra, Robert C., "New Product Varieties and the Measurement of International Prices," American Economic Review, 84 (1994), 157-177.

Hanoch, Giora, "Production and Demand Models with Direct or Indirect Implicit Additivity," *Econometrica*, 43 (1975), 395-419.

Kilts Center for Marketing, Retail Scanner Dataset Manual, Chicago Booth, mimeograph (2013).

Matsuyama, Kiminori, "Engel's Law in the Global Economy: Demand-Induced Patterns of Structural Change, Innovation, and Trade," Econometrica, 87 (2019), 497-528.

McFadden, Daniel, "Conditional Logit Analysis of Qualitative Choice Behavior," in Frontiers in Econometrics, Paul Zaremba, ed. (New York: Academic Press, 1974).

McFadden, Daniel and Kenneth Train, "Mixed MNL Models of Discrete Response," Journal of Applied Econometrics, 15 (2000), 447-470.

Sato, R., "The Most General Class of CES Functions," Econometrica, 43 (1975), 999-1003.

Train, Kenneth E., Discrete Choice Methods with Simulation, (Cambridge: Cambridge University Press, 2009).

TABLE A.1: Descriptive Statistics by Sector

Code	Description	Number of UPCs	Mean $\frac{\lambda_t}{\lambda_{t-1}}$	Percent of UPCs that Enter in a Year	Percent of UPCs that Exit in a Year
501	BABY FOOD	4180	.86	41.21	38.02
503	CANDY	56780	.86	45.92	45.05
504	FRUIT - CANNED	8090	.95	25.65	26.84
505	GUM	3511	.85	41.11	40.49
506	JAMS, JELLIES, SPREADS	9621	.95	31.78	31.62
507	JUICE, DRINKS - CANNED, BOTTLED	24188	.93	34.36	34.52
508	PET FOOD	32085	.86	35.02	33.15
510	PREPARED FOOD-READY-TO-SERVE	11907	.96	32	31.74
511	PREPARED FOOD-DRY MIXES	11179	.92	29.7	29.04
512	SEAFOOD - CANNED	4159	.95	33.27	33.91
513	SOUP	11502	.94	28.73	27.96
514	VEGETABLES - CANNED	17941	.98	24.73	24.72
1001	BAKING MIXES	5651	.91	32.57	31.87
1002	BAKING SUPPLIES	8636	.95	27.27	26.06
1004	BREAKFAST FOOD	7436	.85	38.04	35.86
1005	CEREAL	13941	.88	30.73	29.69
1006	COFFEE	13536	.88	40.81	37.99
1007	CONDIMENTS, GRAVIES, AND SAUCES	27557	.95	32.61	31.43
1008	DESSERTS, GELATINS, SYRUP	6269	.94	28.54	27.55
1009	FLOUR	1706	.98	25.93	24.99
1010	FRUIT - DRIED	7342	.87	39.41	37.95
1011	NUTS	15802	.91	38.31	37.26
1012	PACKAGED MILK AND MODIFIERS	5272	.93	28.19	27.71
1013	PASTA	11223	.97	29.39	29.6
1014	PICKLES, OLIVES, AND RELISH	10841	.98	29.76	30.51
1015	SALAD DRESSINGS, MAYO, TOPPINGS	8494	.93	30.98	30.84
1016	SHORTENING, OIL	6118	.96	31.32	30.59
1017	SPICES, SEASONING, EXTRACTS	22056	.96	37.5	35.84
1018	SUGAR, SWEETENERS	2647	.96	25.21	23.49
1019	TABLE SYRUPS, MOLASSES	2370	.96	29.37	29.44
1020	TEA	11428	.92	38.13	35.52
1021	VEGETABLES AND GRAINS - DRIED	5687	.99	29.1	28.51
1501	BREAD AND BAKED GOODS	60452	.94	35.66	35.91
1503	CARBONATED BEVERAGES	19409	.97	33.58	33.01
1505	COOKIES	24560	.87	39.67	39.51
1506	CRACKERS	7421	.89	32.7	32.23
1507	SNACKS	49223	.8	40.24	38.36
1508	SOFT DRINKS-NON-CARBONATED	13594	.91	38.43	36.51
2001	BAKED GOODS-FROZEN	4585	.94	30.89	30.34
2002	BREAKFAST FOODS-FROZEN	4175	.87	33.29	29.45
2003	DESSERTS/FRUITS/TOPPINGS-FROZEN	3801	.94	29.78	28.1
2005	ICE CREAM, NOVELTIES	25770	.92	35.71	35.7
2006	JUICES, DRINKS-FROZEN	1417	.98	22.09	25.46
2007	PIZZA/SNACKS/HORS DOEURVES-FRZN	9799	.88	37.69	35.17
2008	PREPARED FOODS-FROZEN	23503	.89	38.16	36.52
2009	UNPREP MEAT/POULTRY/SEAFOOD-FRZN	9773	.95	37.78	37.18
2010	VEGETABLES-FROZEN	13442	.94	24.32	24.45
2501	BUTTER AND MARGARINE	2522	.96	24.24	24.28
2502	CHEESE	25314	.96	28.34	26.81
2503	COT CHEESE, SOUR CREAM, TOPPINGS	5112	.98	20.89	21.69
2504	DOUGH PRODUCTS	2965	.92	27.28	27.06
2505	EGGS	3891	.98	22.41	23.16

Note: Sample pools all households and aggregates to the national level using sampling weights to construct a nationally-representative quarterly database by barcode on the total value sold, total quantity sold, and average price; λ_t and λ_{t-1} are the shares of expenditure on common barcodes in total expenditure in each sector in time t and $t-1$ respectively (four-quarter difference). Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

Code	Description	Number of UPCs	Mean $\frac{\lambda_t}{\lambda_{t-1}}$	Percent of UPCs that Enter in a Year	Percent of UPCs that Exit in a Year
2506	MILK	14652	.98	22.53	23.59
2508	SNACKS, SPREADS, DIPS-DAIRY	6661	.93	36.41	34.64
2510	YOGURT	11258	.85	33.23	30.75
3001	DRESSINGS/SALADS/PREP FOODS-DELI	26637	.91	41.88	39.59
3002	PACKAGED MEATS-DELI	24312	.95	30.84	30.63
3501	FRESH MEAT	2862	.95	32.02	28.28
4001	FRESH PRODUCE	22274	.95	36.87	33.18
4501	DETERGENTS	9249	.81	39.08	37.06
4502	DISPOSABLE DIAPERS	5265	.76	51.44	50.03
4503	FRESHENERS AND DEODORIZERS	15414	.73	51.58	49.3
4504	HOUSEHOLD CLEANERS	9485	.89	36.8	36.14
4505	HOUSEHOLD SUPPLIES	21366	.93	44.83	44.88
4506	LAUNDRY SUPPLIES	11730	.86	39.48	39.14
4507	PAPER PRODUCTS	43683	.73	45.46	44.48
4508	PERSONAL SOAP AND BATH ADDITIVES	19684	.81	48.68	46.88
4509	PET CARE	41877	.86	49.75	47.59
4510	TOBACCO & ACCESSORIES	11476	.99	42.77	42.89
4511	WRAPPING MATERIALS AND BAGS	10912	.92	30.3	29.66
5001	BEER	9307	.98	42.1	38.66
5002	LIQUOR	14253	.96	42.61	40.41
5003	WINE	23813	.97	45.02	42.61
5501	AUTOMOTIVE	6324	.97	37.82	39.3
5502	BATTERIES AND FLASHLIGHTS	8849	.88	40.49	40.13
5503	BOOKS AND MAGAZINES	1118	.86	39.49	42.88
5505	CHARCOAL, LOGS, ACCESSORIES	2260	.9	44.56	41.91
5507	ELECTRONICS, RECORDS, TAPES	56166	.69	61.16	57.19
5508	FLORAL, GARDENING	3005	.97	46.08	49.35
5509	GLASSWARE, TABLEWARE	43016	.74	63.81	63.84
5511	HARDWARE, TOOLS	15584	.87	57.75	56.42
5513	HOUSEWARES, APPLIANCES	20261	.8	46.18	45.36
5514	INSECTICDS/PESTICDS/RODENTICDS	3565	.94	40.65	39.66
5515	KITCHEN GADGETS	45310	.83	54.68	53.21
5516	LIGHT BULBS, ELECTRIC GOODS	18024	.72	42.77	42.6
5519	SEWING NOTIONS	959	.91	53.89	54.15
5520	SHOE CARE	751	.98	52.34	53.02
5522	STATIONERY, SCHOOL SUPPLIES	79576	.84	51.77	52.62
6001	BABY NEEDS	9324	.91	51.75	51.76
6002	COSMETICS	44897	.84	52.15	51.03
6003	COUGH AND COLD REMEDIES	13231	.85	36.71	36.32
6004	DEODORANT	4694	.84	39.42	39.19
6007	FEMININE HYGIENE	1426	.94	37.49	38.52
6008	FIRST AID	10594	.91	38.01	36.59
6009	FRAGRANCES - WOMEN	16031	.69	66.6	65.65
6010	GROOMING AIDS	26596	.86	55.36	54.12
6011	HAIR CARE	25309	.86	44.26	43.47
6012	MEDICATIONS/REMEDIES/HEALTH AIDS	38693	.9	39.19	37.96
6013	MEN'S TOILETRIES	4047	.53	59.6	57.21
6014	ORAL HYGIENE	12336	.86	37.45	37.23
6015	SANITARY PROTECTION	4447	.89	37.17	36.89
6016	SHAVING NEEDS	4987	.85	40.91	40.65
6017	SKIN CARE PREPARATIONS	18297	.81	49.88	46.74
6018	VITAMINS	35327	.89	43.43	42.68

Note: Sample pools all households and aggregates to the national level using sampling weights to construct a nationally-representative quarterly database by barcode on the total value sold, total quantity sold, and average price; λ_t and λ_{t-1} are the shares of expenditure on common barcodes in total expenditure in each sector in time t and $t-1$ respectively (four-quarter difference). Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

TABLE A.2: Regressions of Estimated Brand Consumer Tastes on BAVs Including Sector-Time Fixed Effects

	(1)	(2)	(3)	(4)
	$\ln(\varphi_{bt}^{B})$	$\ln(\varphi_{bt}^{B})$	$\ln(\varphi_{bt}^{B})$	$\ln(\varphi_{bt}^{B})$
Energized Differentiation	0.00870***			
	(0.000440)			
Relevance		0.00179***		
		(0.000492)		
Esteem			0.00340***	
			(0.000504)	
Knowledge				0.00110**
				(0.000449)
Observations	70,276	70,276	70,276	70,276
R^2	0.10	0.08	0.09	0.08
Group-Time FE	Yes	Yes	Yes	Yes
Brand FE	No	No	No	No

Note: $\ln \varphi_{bt}^{B}$ is the log of the estimated brand consumer tastes parameter calculated using the Nielsen definition of brand, where we use the nested CES specification from Section III.B. of the paper to aggregate from barcodes to brands. The right-hand side variables—Energized Differentiation, Relevance, Esteem, and Knowledge—are the components of Brand Asset Value as measured by Young and Rubicam (Y&R). They are expressed in percentiles ranging from the lowest to the highest. Because there are often several Nielsen brands matched to a Y&R brand, standard errors, reported in parentheses are clustered by Y&R-brand-time. Brands with only one observation are dropped. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

TABLE A.3: Regressions of Estimated Brand Consumer Tastes on BAVs Including Nielsen-Brand and Sector-Time Fixed Effects

	(1)	(2)	(3)	(4)
	$\ln(\varphi_{bt}^{B})$	$\ln(\varphi_{bt}^{B})$	$\ln(\varphi_{bt}^{B})$	$\ln(\varphi_{bt}^{B})$
Energized Differentiation	0.000103			
	(0.000313)			
Relevance		0.00246***		
		(0.000654)		
Esteem			0.00236***	
			(0.000672)	
Knowledge				0.00364***
				(0.00100)
Observations	67,598	67,598	67,598	67,598
R^2	0.76	0.76	0.76	0.76
Group-Time FE	Yes	Yes	Yes	Yes
Brand FE	Yes	Yes	Yes	Yes

Note: $\ln \varphi_{bt}^{B}$ is the log of the estimated brand consumer tastes parameter calculated using the Nielsen definition of brand, where we use the nested CES specification from Section III.B. of the paper to aggregate from barcodes to brands. The right-hand side variables—Energized Differentiation, Relevance, Esteem, and Knowledge—are the components of Brand Asset Value as measured by Young and Rubicam (Y&R). They are expressed in percentiles ranging from the lowest to the highest. Because there are often several Nielsen brands matched to a Y&R brand, standard errors, reported in parentheses are clustered by Y&R-brand-time. Brands with only one observation are dropped. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

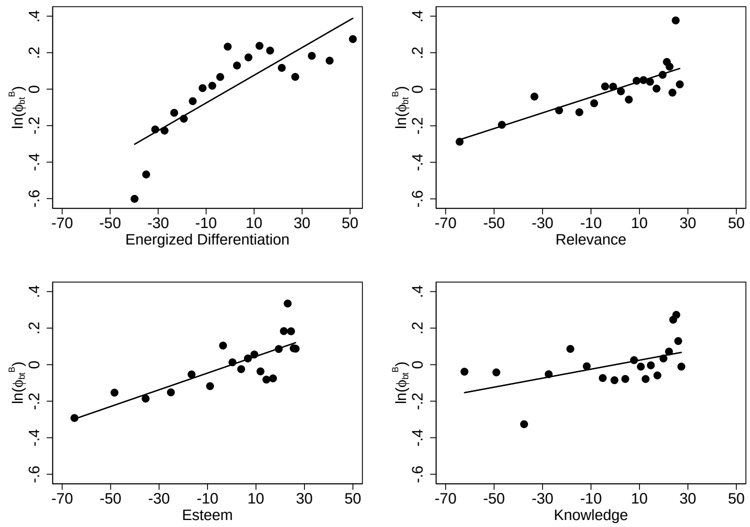

FIGURE A.1: Partial Regression and Binned Scatter Plot of Estimated Brand Consumer Tastes vs. BAVs After Conditioning on Sector-Time Fixed Effects

Note: $\ln \varphi^B_{bt}$ is the log of the estimated Nielsen brand consumer tastes parameter, where we use the nested CES specification from Section III.B. of the paper to aggregate from barcodes to brands. Energized Differentiation, Relevance, Esteem, and Knowledge are the components of Brand Asset Value as measured by Young and Rubicam. They are expressed in percentiles ordered so that brands with high values for a BAV component have high percentiles. The figure portrays binned scatter plots of the residuals from regressing $\ln \varphi^B_{bt}$ on sector-time fixed effects against the residuals from regressing each of our BAVs against sector-time fixed effects. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

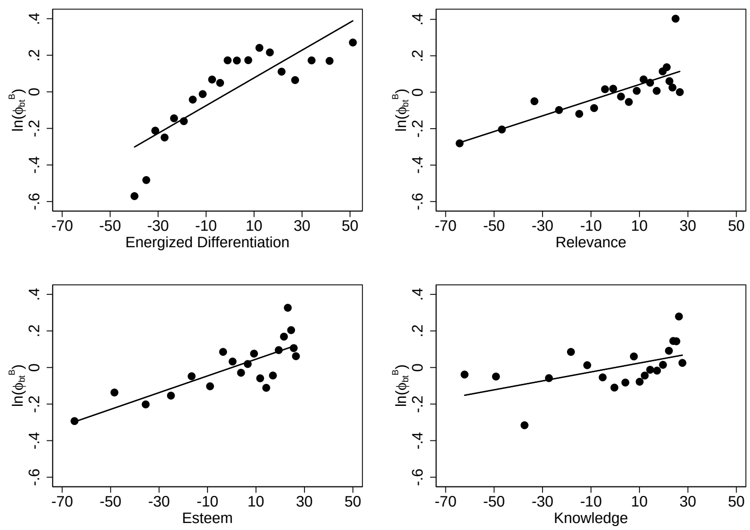

FIGURE A.2: Partial Regression and Binned Scatter Plot of Estimated Brand Consumer Tastes vs. BAVs After Conditioning on Sector-Time and Nielsen Brand Fixed Effects

Note: $\ln \varphi_{bt}^{B}$ is the log of the estimated brand consumer tastes parameter, where we use the nested CES specification from Section III.B. of the paper to aggregate from barcodes to brands. Energized Differentiation, Relevance, Esteem, and Knowledge are the components of Brand Asset Value as measured by Young and Rubicam. They are expressed in percentiles ordered so that brands with high values for a BAV component have high percentiles. The figure portrays binned scatter plots of the residuals from regressing $\ln \varphi_{bt}^{B}$ on sector-time and Nielsen-brand fixed effects against the residuals from regressing each of our BAVs against sector-time and Nielsen-brand fixed effects. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

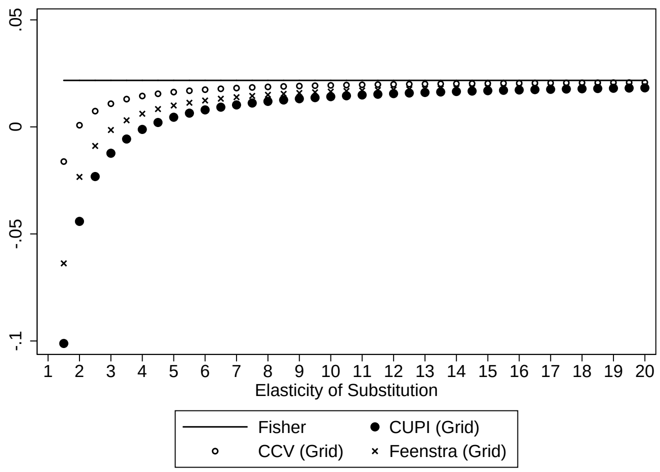

FIGURE A.3: Average of Four-Quarter Proportional Changes in the Aggregate Cost of Living $((P_t - P_{t-1}) / P_{t-1})$ from 2005-2013 for Alternative Elasticities of Substitution

Note: Average of four-quarter proportional changes in the aggregate cost of living from 2005-2013. Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the sectors in our data $\left(\left(P_{gt} - P_{gt-1}\right) / P_{gt-1}\right)$ by their expenditure shares. Figure shows the time-averaged values of (i) the Fisher index; (ii) the Feenstra (1994) index, which combines the variety correction term with the Sato-Vartia price index for common barcodes (the special case of equation (10) in the paper in which $\varphi_{kt} / \varphi_{kt-1} = 1$ for all $k \in \Omega_t^*$); (iii) the CCV (equation (9) in the paper); and (iv) the CUPI (equation (8) in the paper) for thirty-eight evenly-spaced values of the elasticity of substitution ranging from 1.5 to 20. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

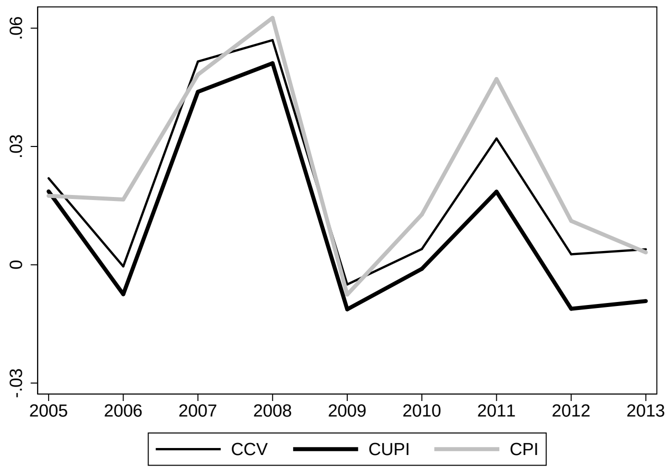

FIGURE A.4: Four-Quarter Proportional Changes in the Aggregate Cost of Living $\left(\left(P_t - P_{t-1}\right)/P_{t-1}\right)$, CPI Matched Sample

Note: This figure shows alternative measures of the four-quarter proportional change in the aggregate cost of living using different price indexes for the 89 out of 104 sectors that we can match to subcategories of the CPI. Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the sectors $\left(\left(P_{gt} - P_{gt-1}\right)/P_{gt-1}\right)$ by their expenditure shares. The thick gray line shows the aggregate price index based on the CPI subcategories. The other lines show alternative price indexes computed using the Nielsen data. CCV and CUPI are our exact common variety price index (equation (9)) and unified price index (equation (8)), respectively, using the Feenstra (1994) estimated elasticities of substitution. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

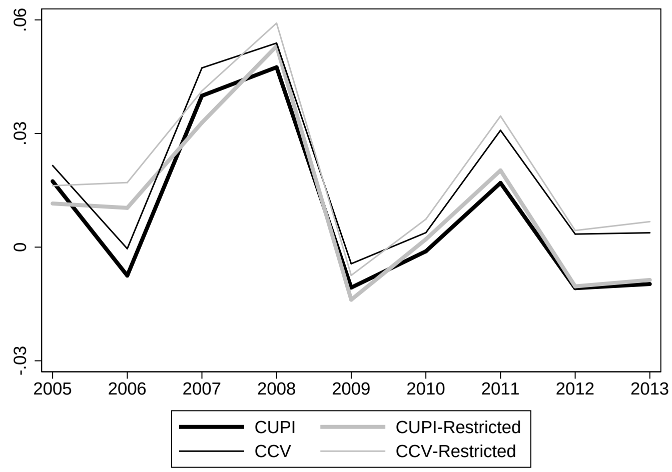

FIGURE A.5: Robustness of Four-Quarter Proportional Changes in the Aggregate Cost of Living $((P_t - P_{t-1})\,/P_{t-1})$ to Measurement Error in Small Expenditure Shares

Note: Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the sectors in our data $((P_{gt} - P_{gt-1})\,/P_{gt-1})$ by their expenditure shares. CUPI is our baseline CES unified price index from equation (8) in the paper using the Feenstra (1994) estimated elasticities. CUPI-Restricted is the robustness check using the subset of common barcodes with above-median expenditure shares. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.

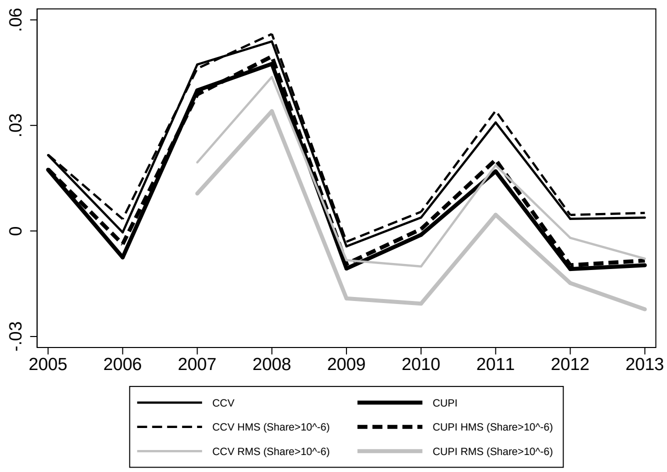

FIGURE A.6: Robustness of Four-Quarter Proportional Changes in the Aggregate Cost of Living $((P_t - P_{t-1})\,/P_{t-1})$ to Using the Retail Scanner Dataset

Note: Change in the aggregate cost of living is computed by weighting the four-quarter proportional change in the cost of living for each of the sectors in our data $((P_{gt} - P_{gt-1})\,/P_{gt-1})$ by their expenditure shares. CUPI and CCV are our baseline indices from the paper using the Homescan data and the Feenstra (1994) estimated elasticities. CUPI HMS and CCV HMS recalculate these indexes using the Homescan data, excluding varieties with expenditure shares of less than 10^{-6} from the set of common varieties, and instead including them in the set of entering and exiting varieties. CUPI RMS and CCV RMS recalculate these indexes using the Retail Scanner Dataset, again excluding varieties with expenditure shares of less than 10^{-6} from the set of common varieties, and including them in the set of entering and exiting varieties. Calculated based on data from The Nielsen Company (US), LLC and provided by the Marketing Data Center at The University of Chicago Booth School of Business.