Application of cut cell methods to simulations of surge barriers in 1D/2D

Judah Ryoo

Columbia University

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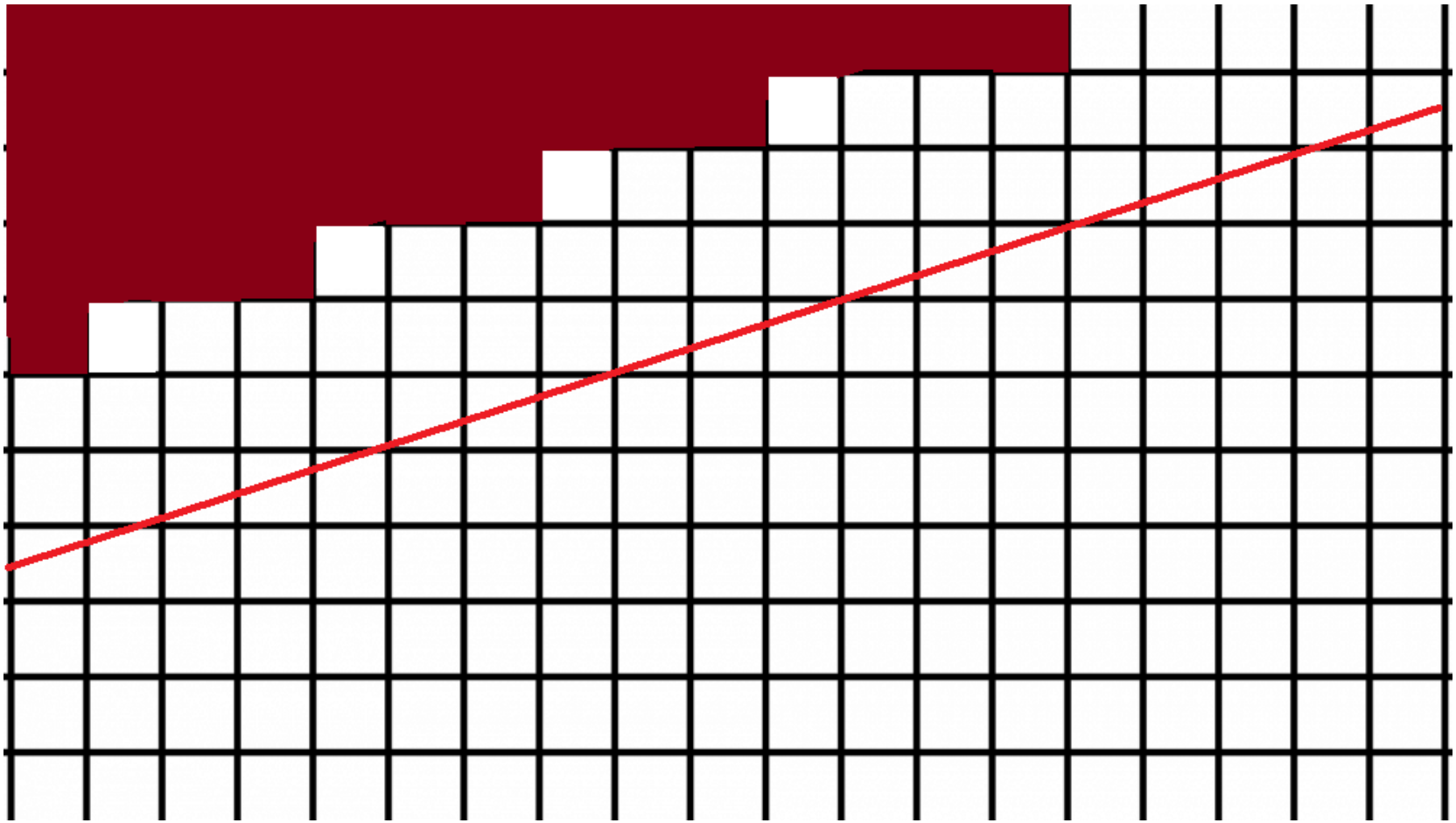
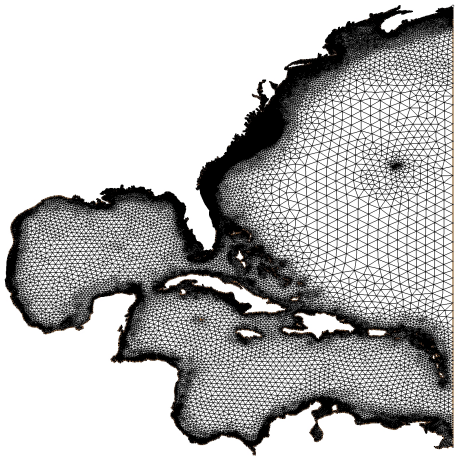
Problem Setup

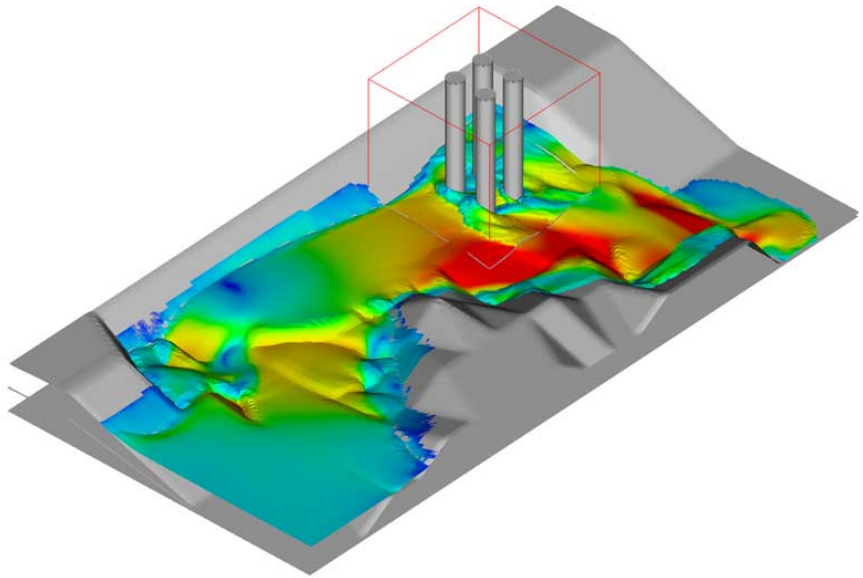


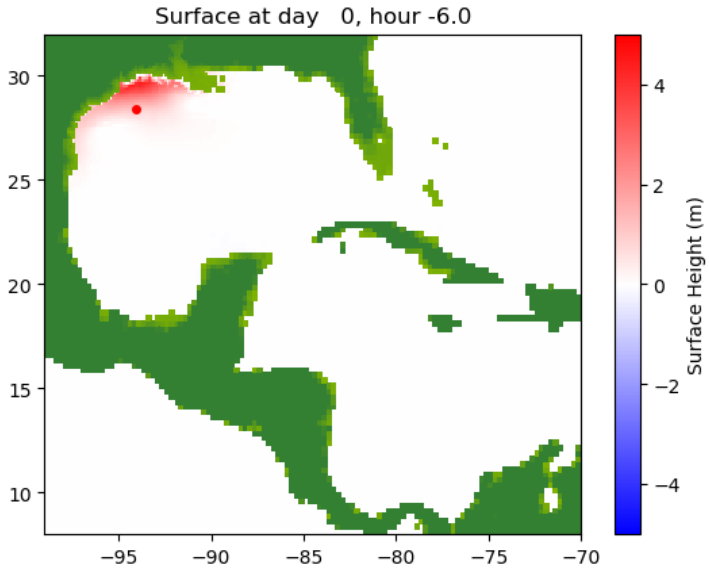
Literature: Storm related simulations



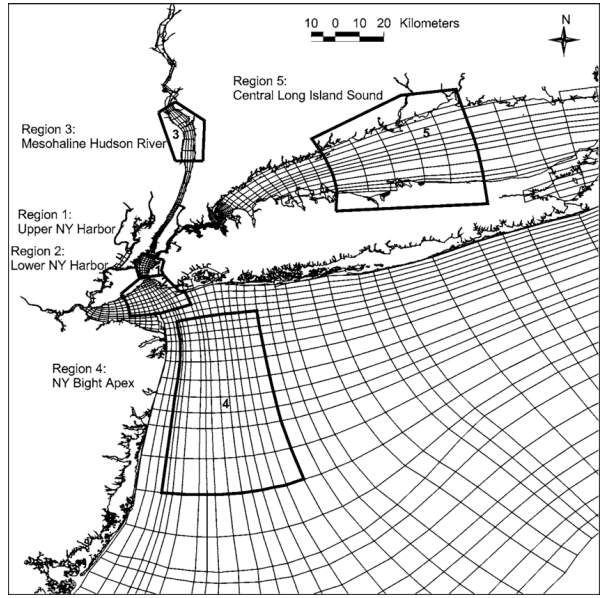
ADCIRC

flow3D



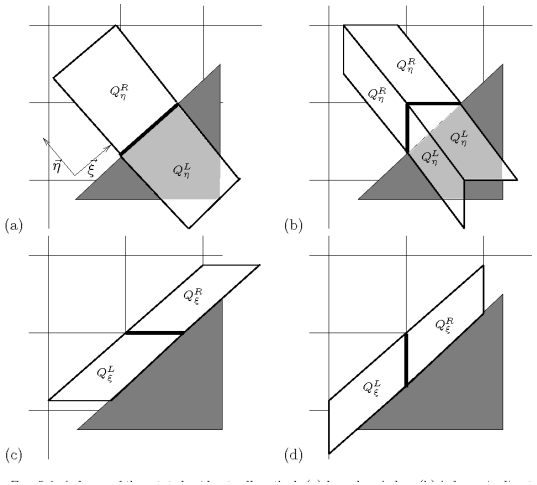


GeoClaw

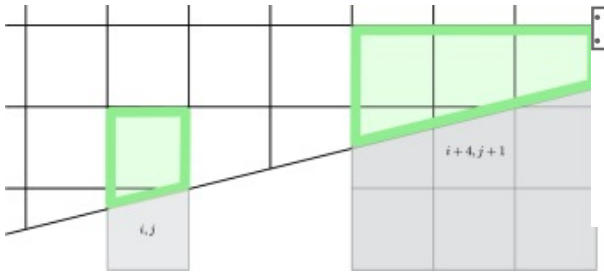


NYHOPS

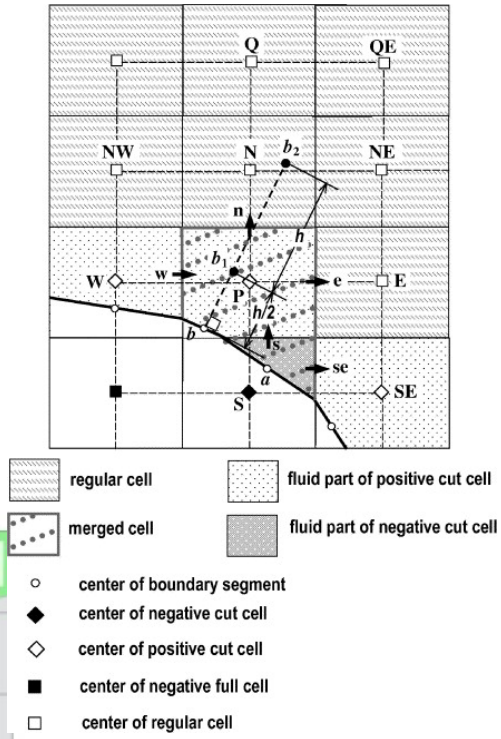
Literature: Cut cell methods



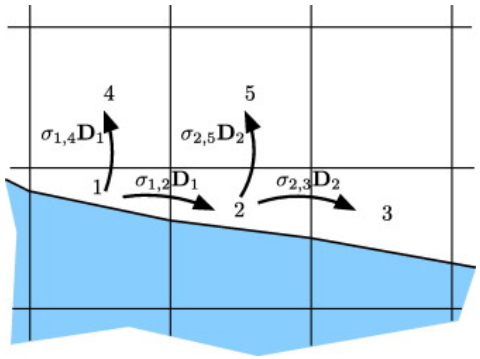
h-box



SRD



cell merging



flux redistribution

Literature

Storm related simulations

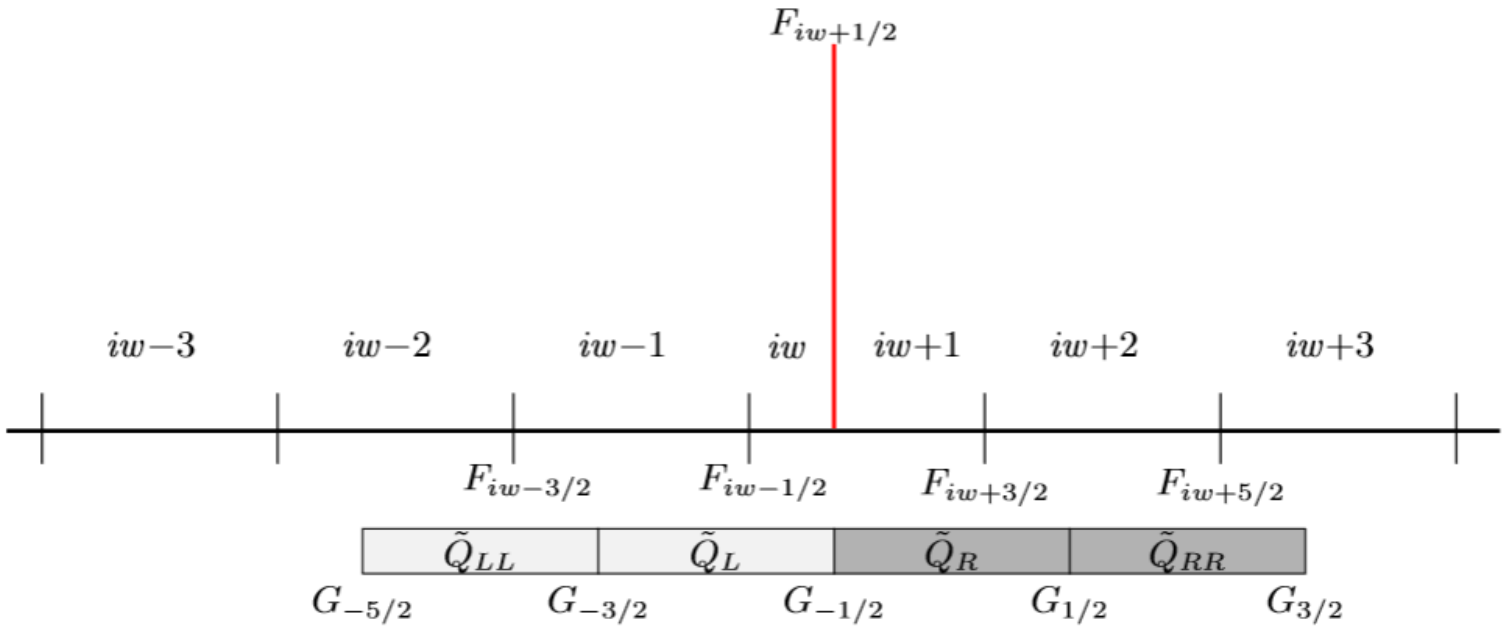
- Big CFD softwares e.g. ADCIRC, flow3D, GeoClaw, NYHOPS
- Unstructured meshing, fitted curvilinear grids
- Calculates many things, e.g. salinity, temperature

Cut cell methods

- Cell merging (Chung 2006)
- h-box (Berger, Helzel, Leveque 2003)
- SRD (Berger, Giuliani 2020)

Using h-box method

Key Idea: Virtual grid while considering conservation



Comparison: Large Time-Stepping

Key Idea: Track the waves at each edge

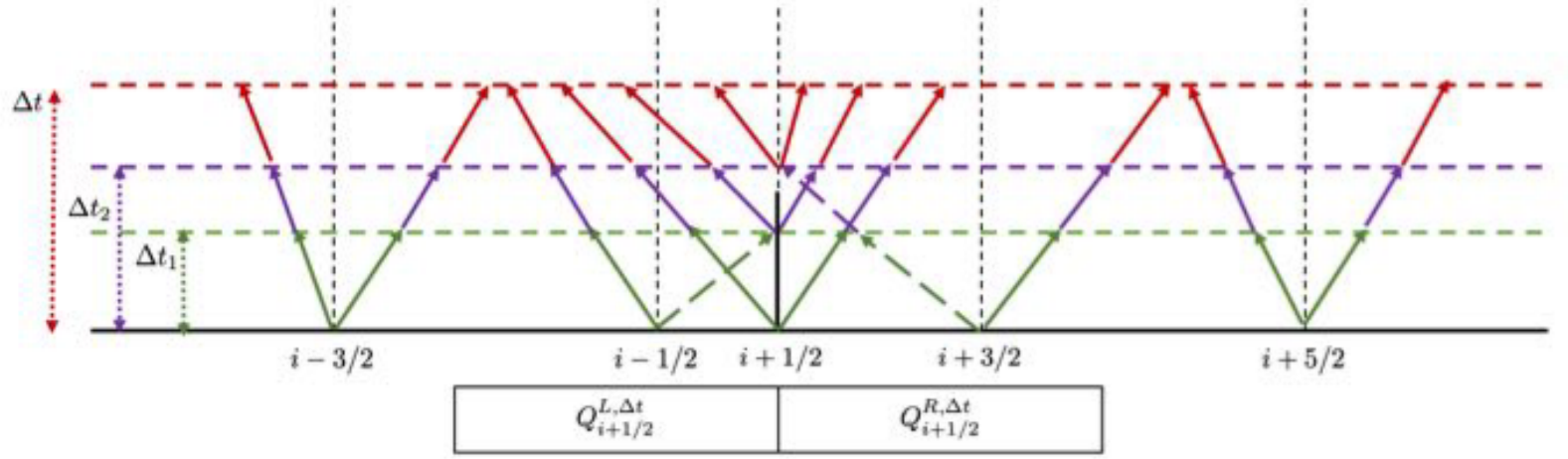
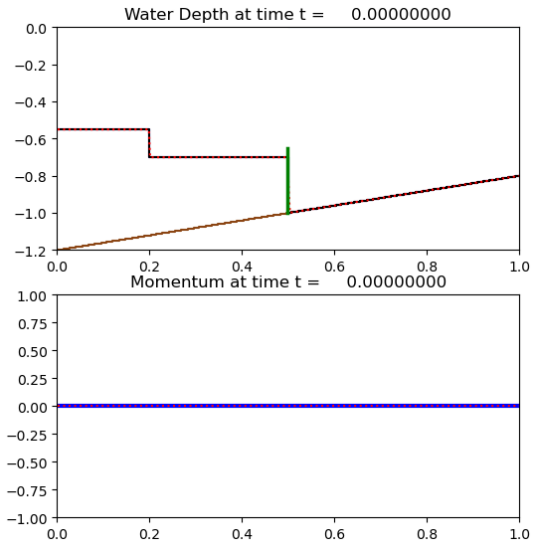


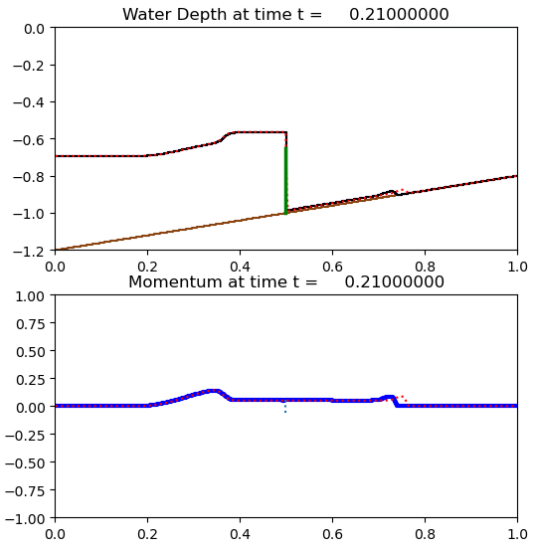
FIG. 12. *Step 6: at* Δt*, update* $Q^{\Delta t}_{i-1}$, $Q^{\Delta t}_{i}$, $Q^{\Delta t}_{i+1}$ *and* $Q^{\Delta t}_{i+2}$ *accordingly.*

1D h-box Results with sloping beach

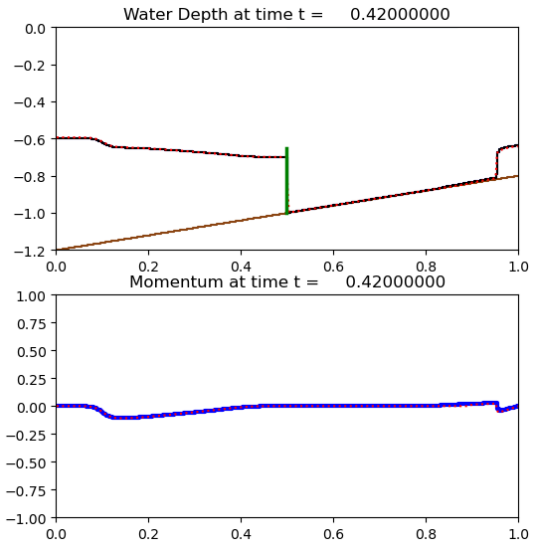
$\boldsymbol{t = 0.0}$:



$t = 0.21$:



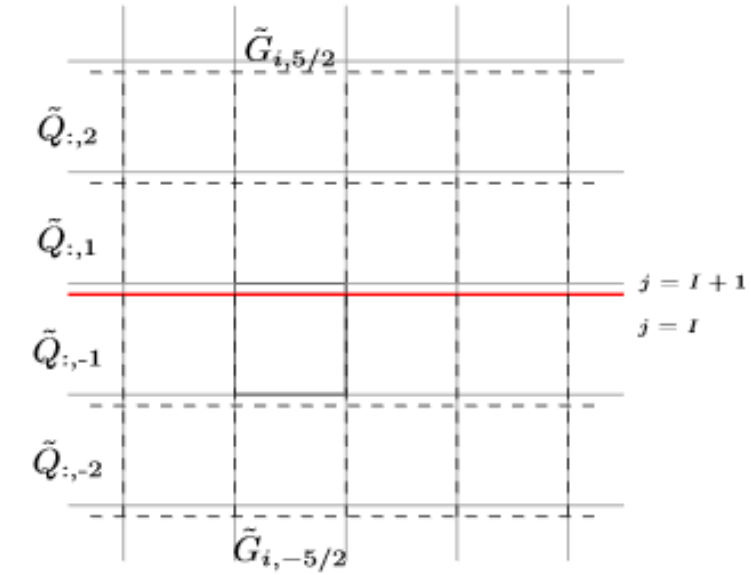
$t = 0.42$:

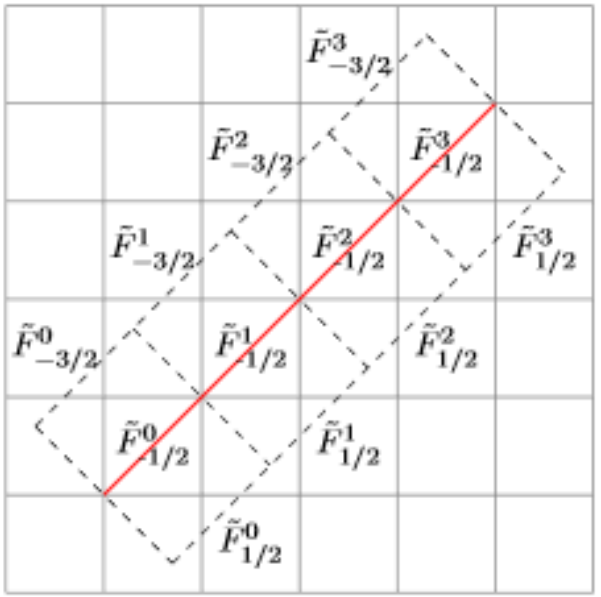


$\Delta x = \Delta y = 1/400$; $\beta = 0.35$, $\alpha = 0.1$

Using h-box method

Key Idea: Virtual grid while considering conservation
Single layer for angled barriers

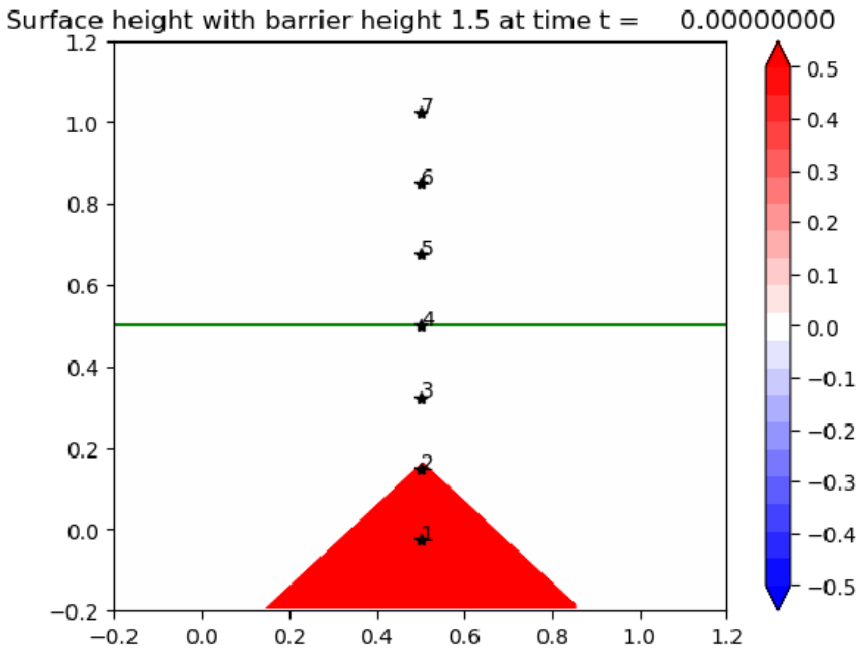




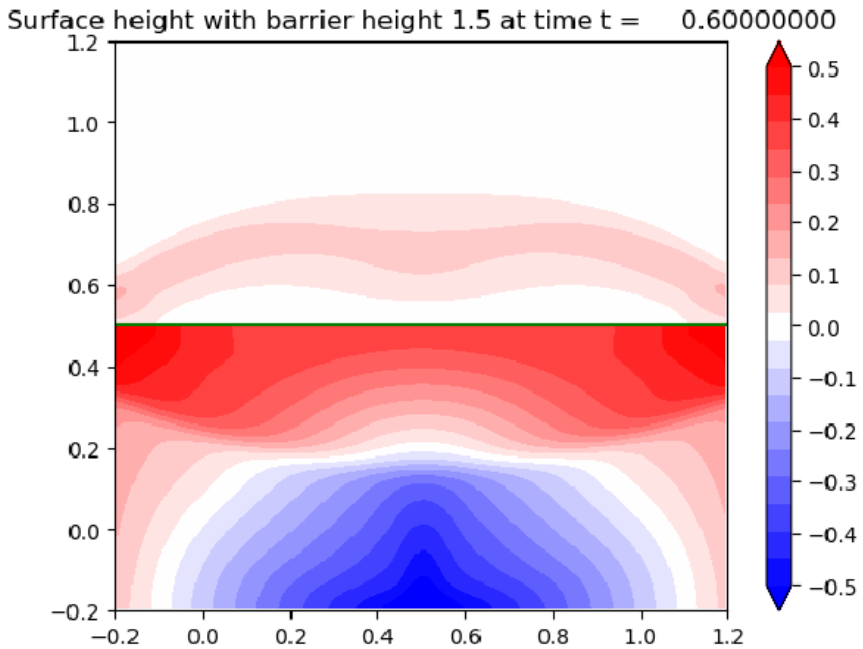
(a) Normal fluxes.

h-box Results with parallel barrier

$\boldsymbol{t = 0.0}$:



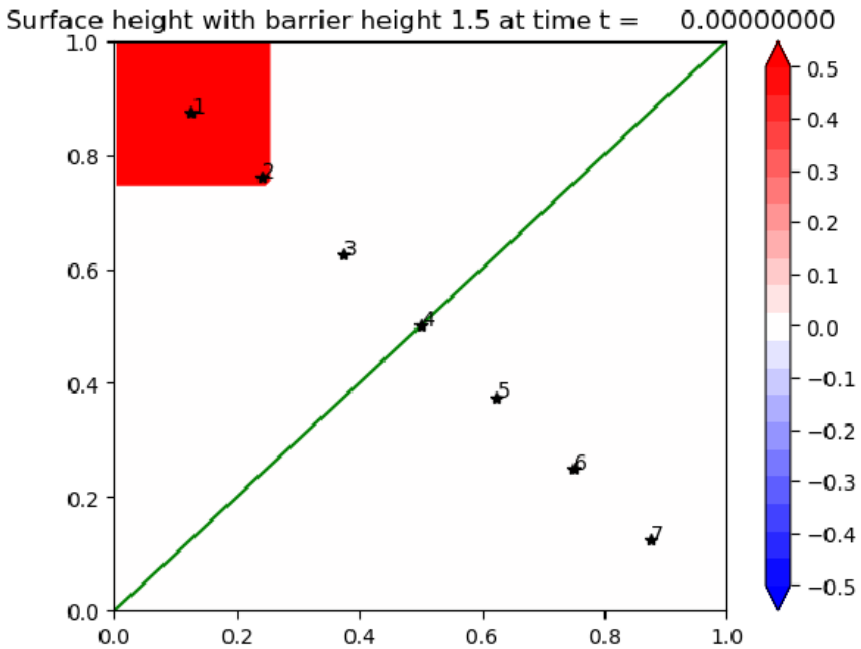
$t = 0.6$:



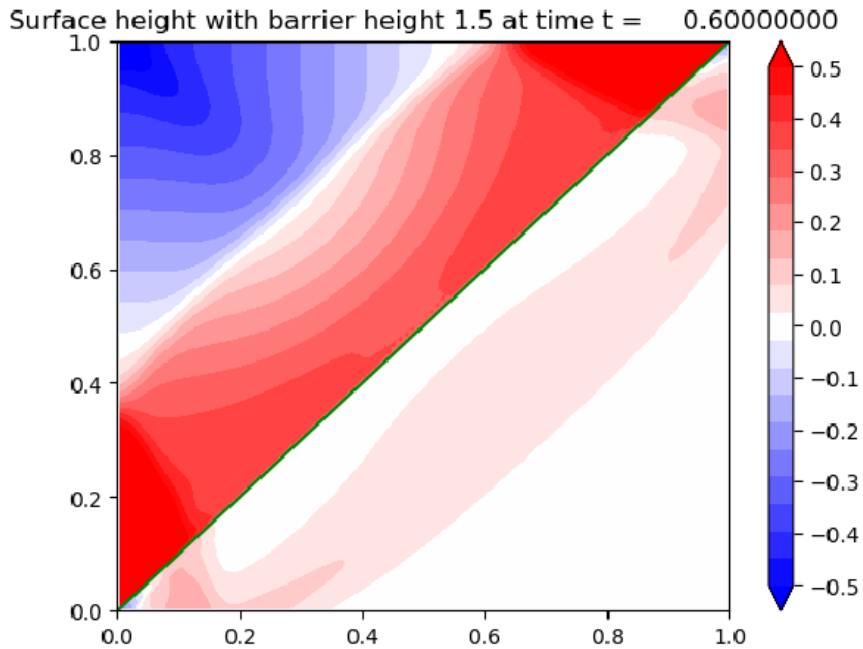
$\Delta x = \Delta y = 10^{-2}$; $\beta = 1.5$, $\alpha = 0.1$
Reflection and overtopping at $t = 0.6$

h-box Results with diagonal barrier

$\boldsymbol{t = 0.0}$:



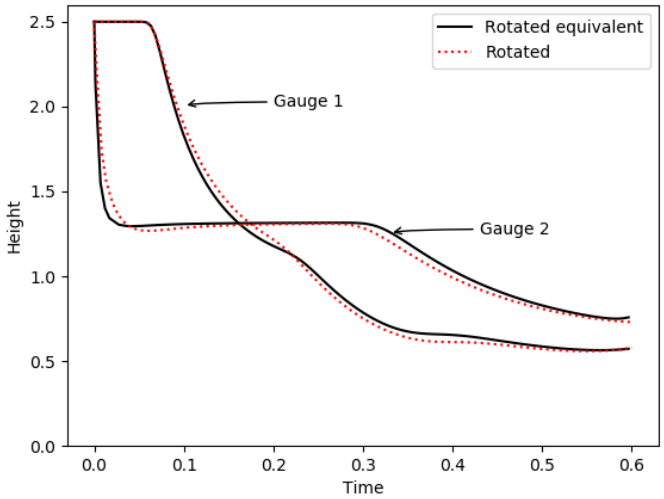
$t = 0.6$:

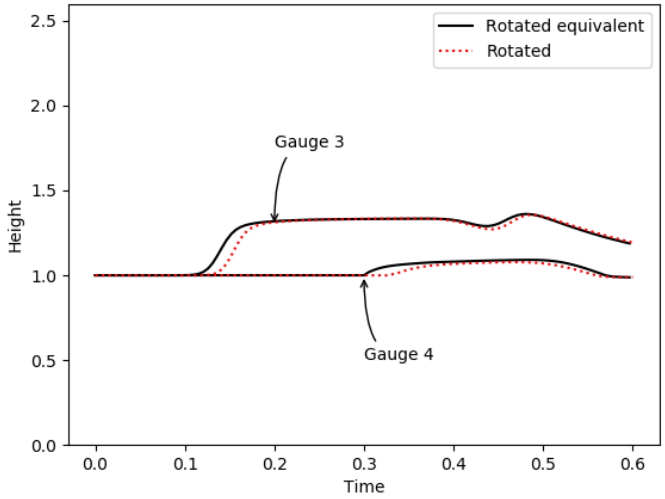


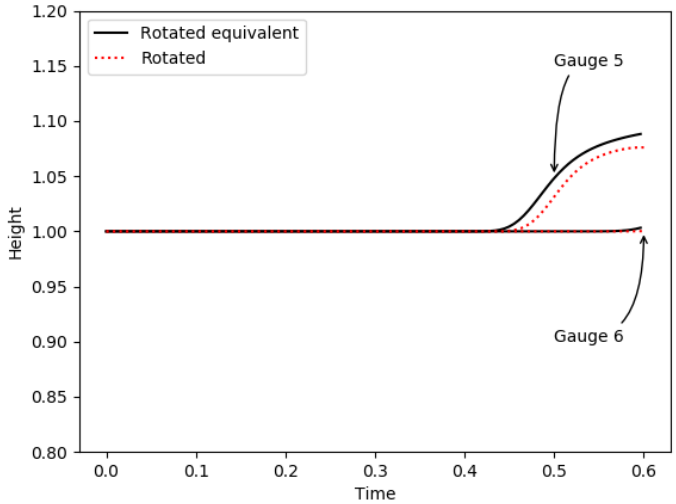
$\Delta x = \Delta y = 10^{-2}$; $\beta = 1.5$, $\alpha = 0.5$
Reflection and overtopping at $t = 0.6$

h-box Results with diagonal barrier

Gauge comparison:

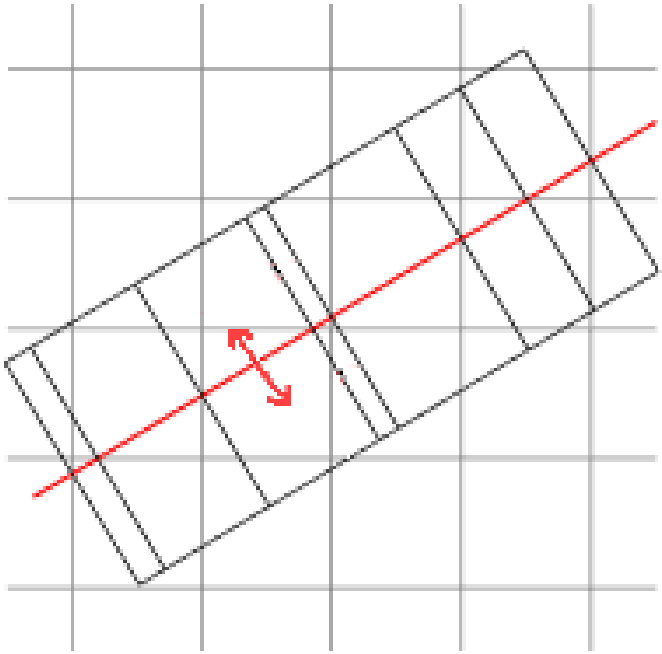




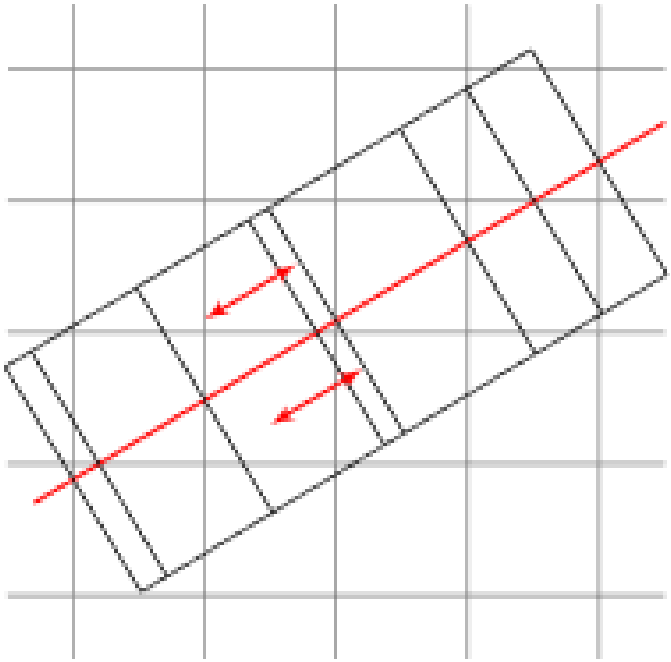


Using hybrid h-box and SRD method

Wave redistribution in normal direction



SRD in transverse direction

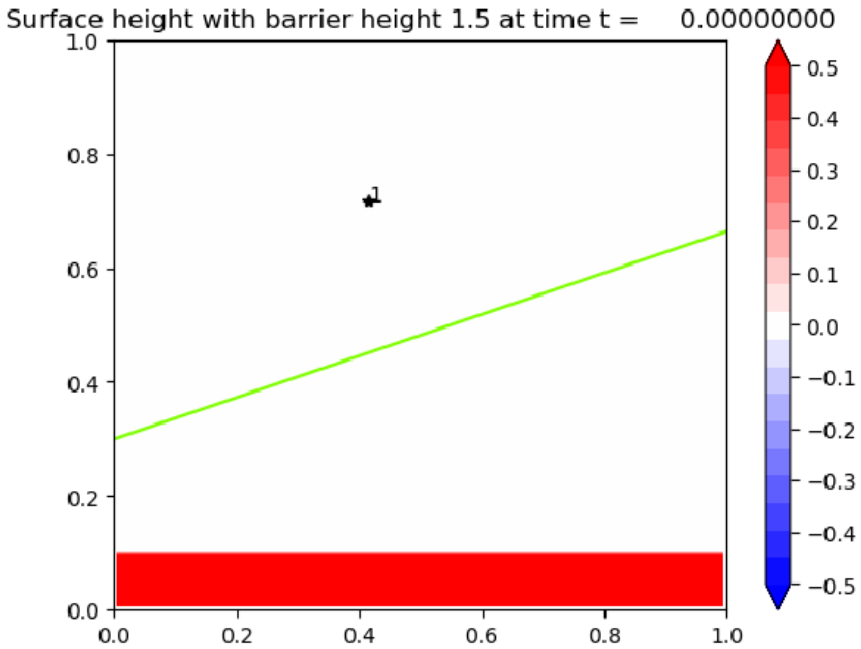


Finally conservation calculations

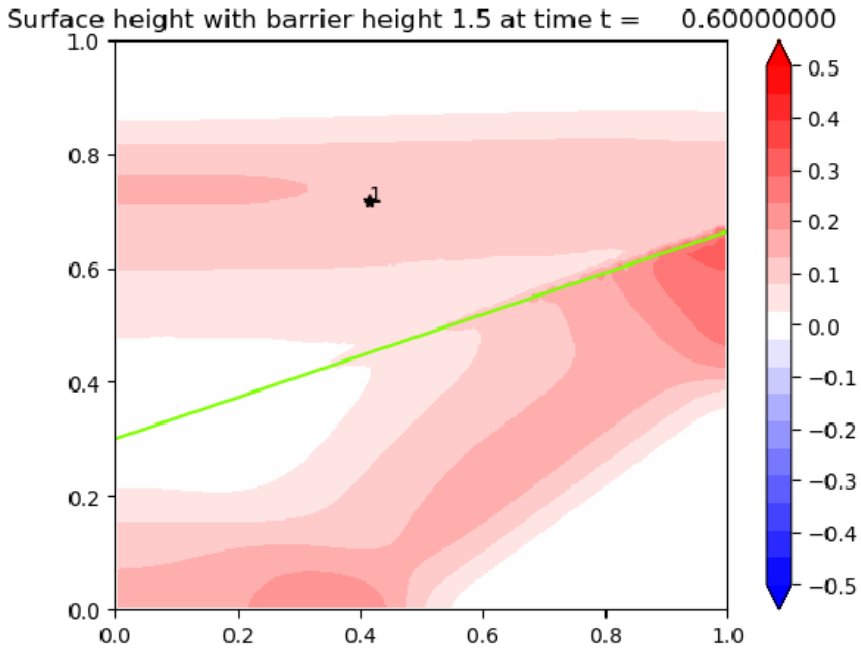
$$\mathcal{F}_o^i = -\sum_{ps} \alpha_i \Delta Q_{ps}^{n+1}.$$

Hybrid h-box Results with 20° barrier (Overtopping)

$\boldsymbol{t = 0.0}$:



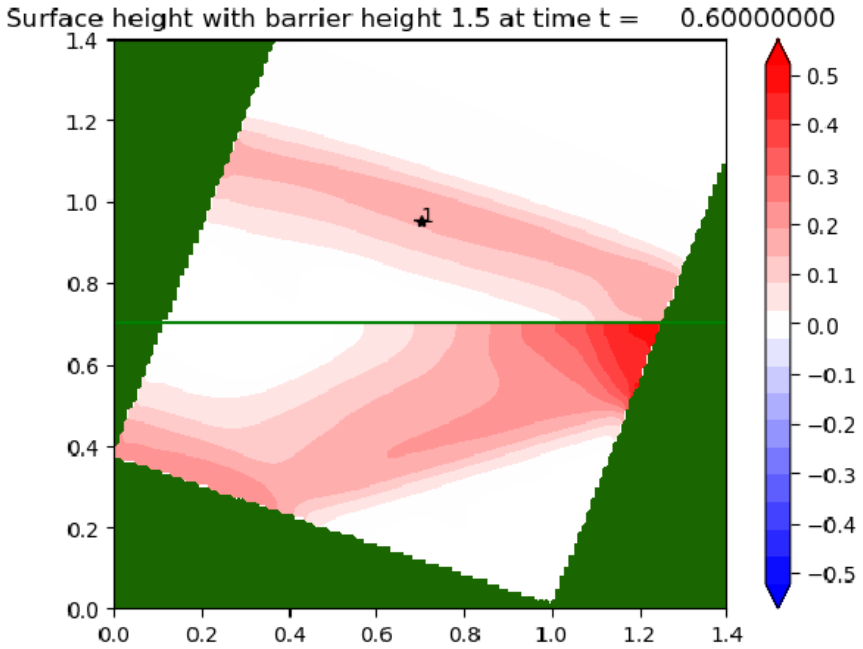
$t = 0.6$:



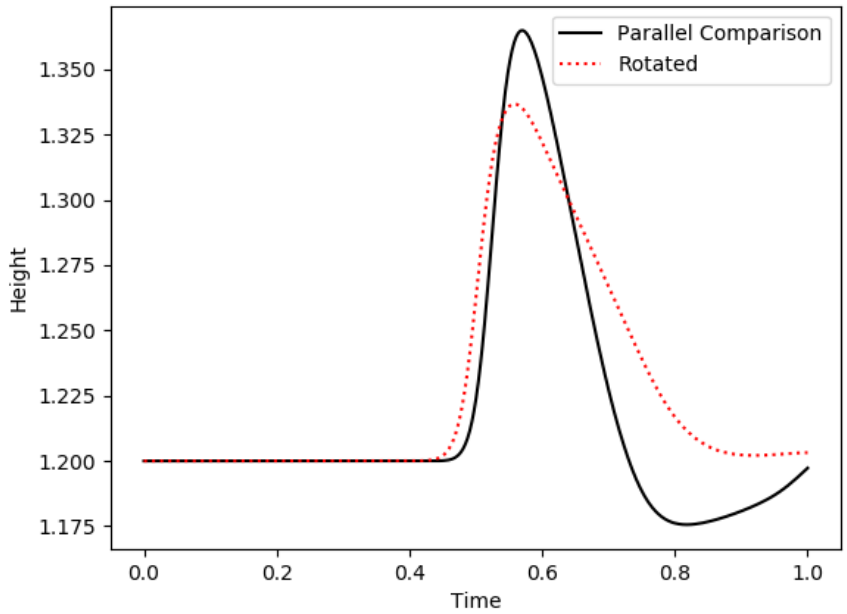
$\Delta x = \Delta y = 10^{-2}$; $\beta = 1.5$, $\alpha_{\min} \sim 10^{-5}$

Hybrid h-box Results with parallel barrier (OT)

Parallel barrier with rotated initial condition:

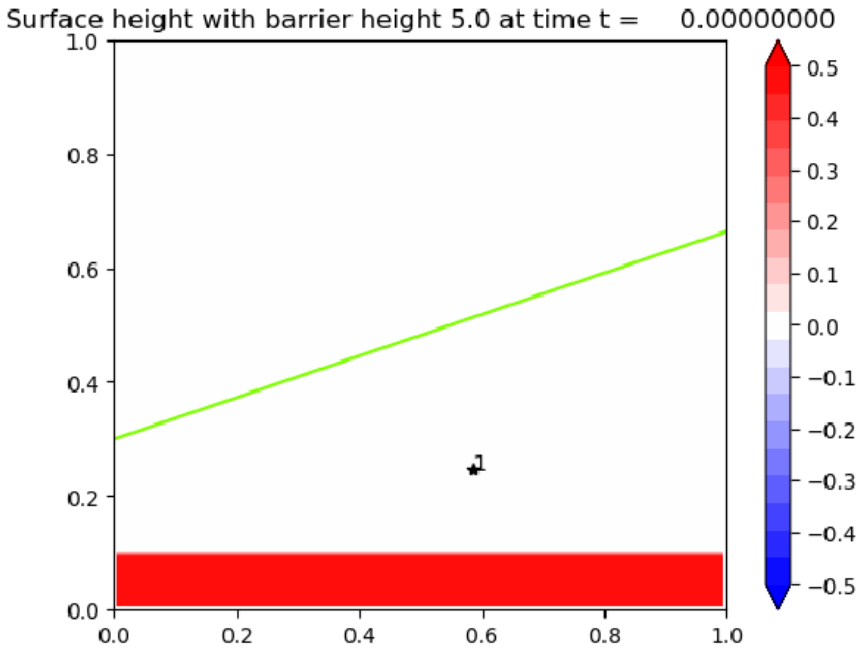


Gauge comparison:

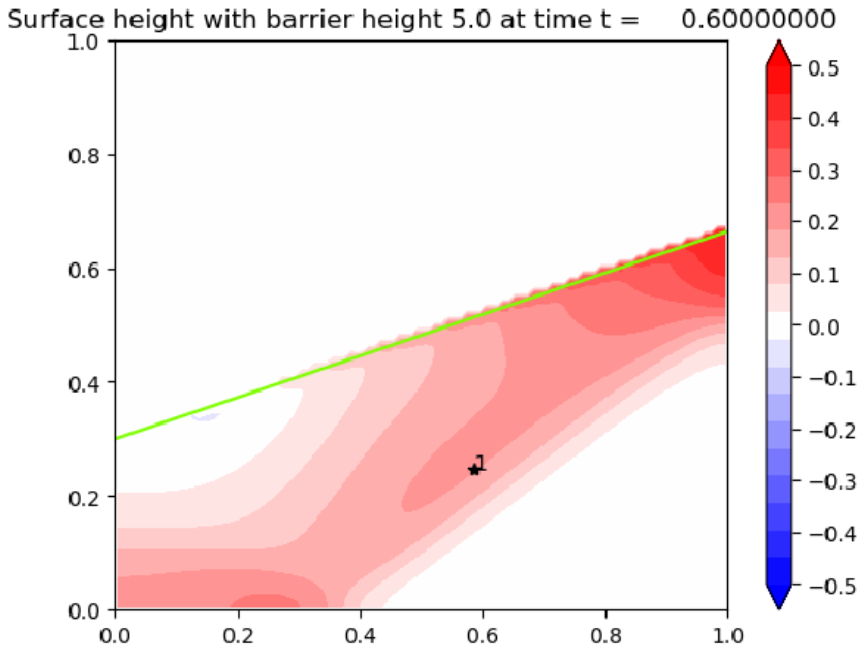


Hybrid h-box Results with 20° barrier (Reflection)

$\boldsymbol{t = 0.0}$:



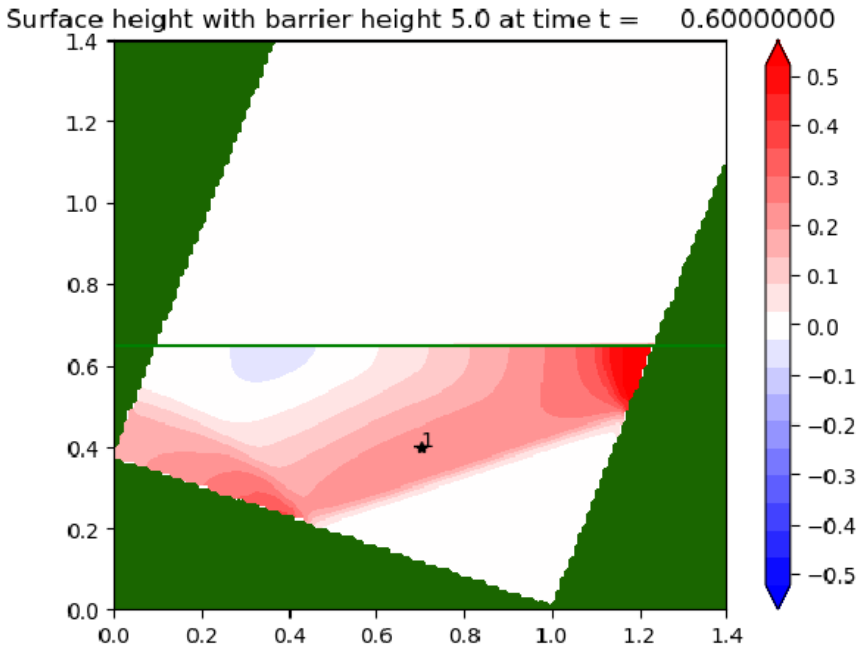
$t = 0.6$:



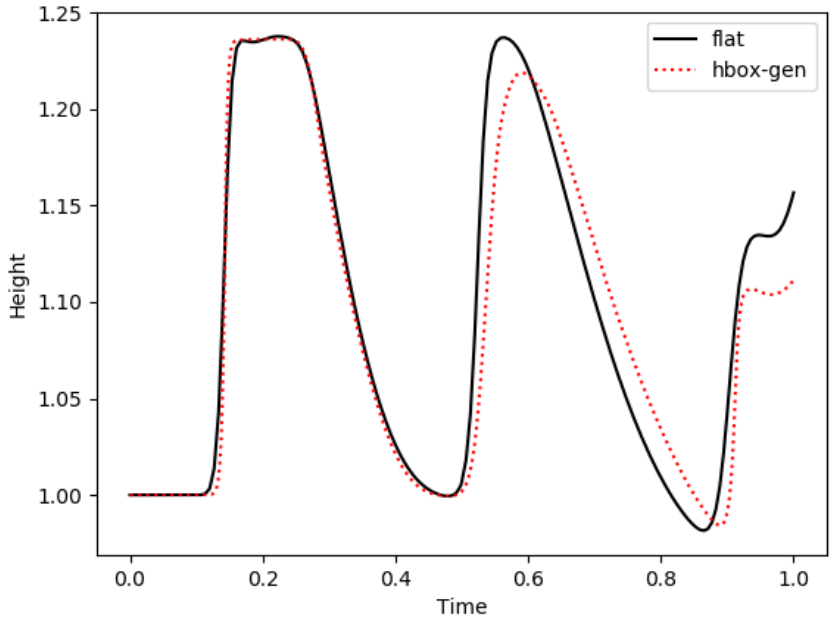
$\Delta x = \Delta y = 10^{-2}$; $\beta = 5$, $\alpha_{\min} \sim 10^{-5}$

Hybrid h-box Results with parallel barrier (RF)

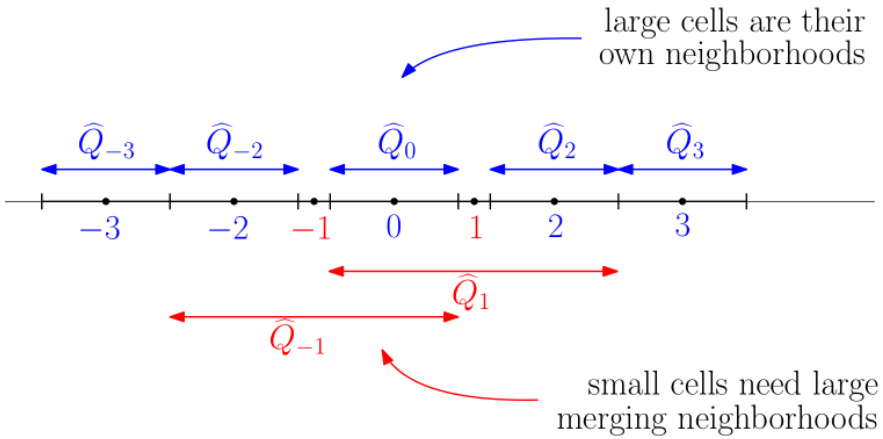
Parallel barrier with rotated initial condition:



Gauge comparison:



Using *SRD*[1] method

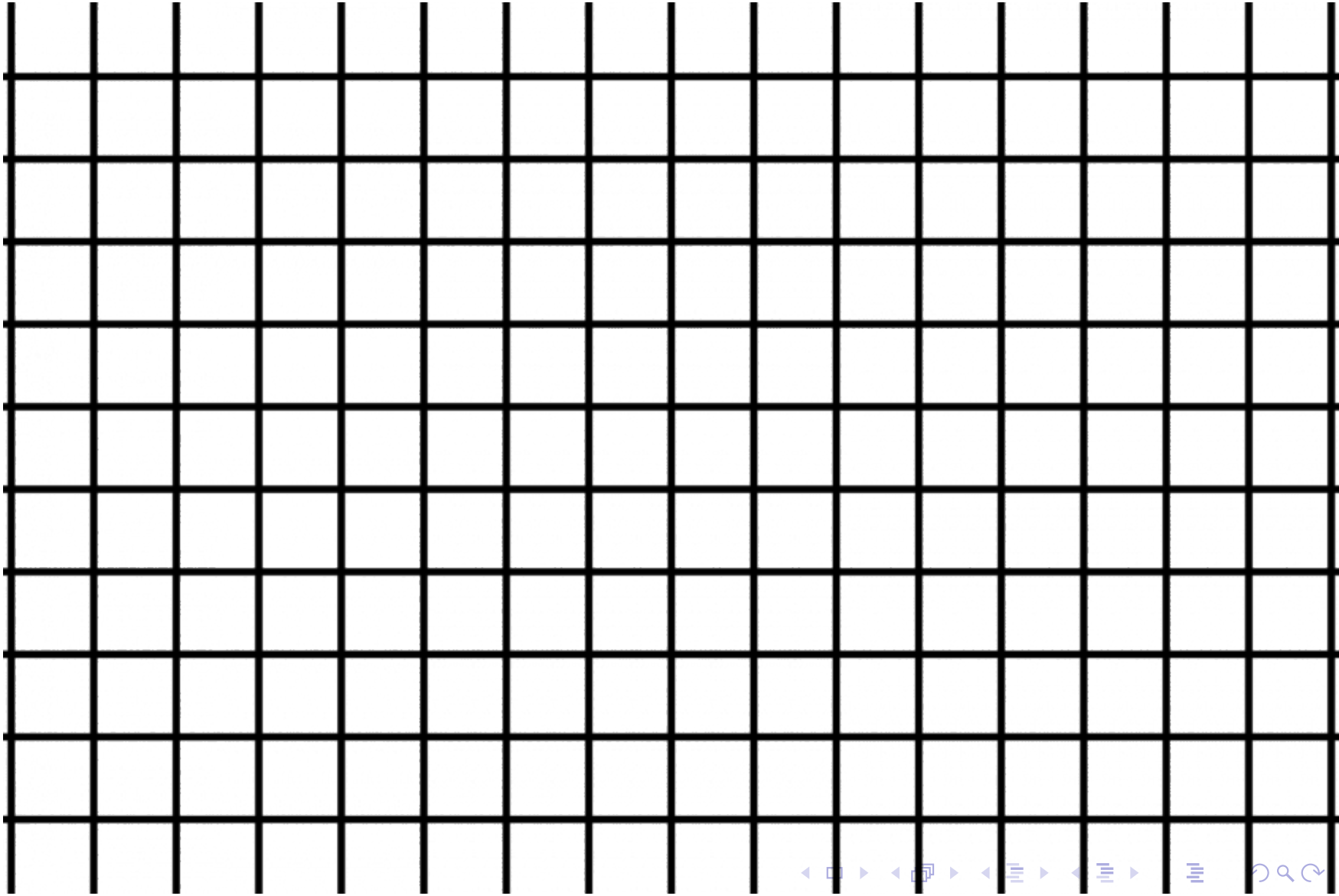


$$\hat{Q}_{-1/1} = \frac{1}{h/2+\alpha h+h/3}\left(\frac{h}{2}\hat{U}_{-2/2} + \frac{h}{3}\hat{U}_0 + \alpha h\hat{U}_{-1/1}\right)$$
$$\hat{Q}_i = \hat{U}_i \text{ for } i = -3, -2, 0, 2, 3$$
$$U_0^{n+1} = \frac{1}{3}(\hat{Q}_{-1} + \hat{Q}_0 + \hat{Q}_1)$$
$$U_{-2/2}^{n+1} = \frac{1}{2}(\hat{Q}_{-1/1} + \hat{Q}_{-2/2})$$
$$U_i^{n+1} = \hat{Q}_i \text{ for } i = -3, -1, 1, 3$$

[1] M. Berger and A. Giuliani, *A state redistribution algorithm for finite volume schemes on cut cell meshes*, Journal of Computational Physics, (2020)

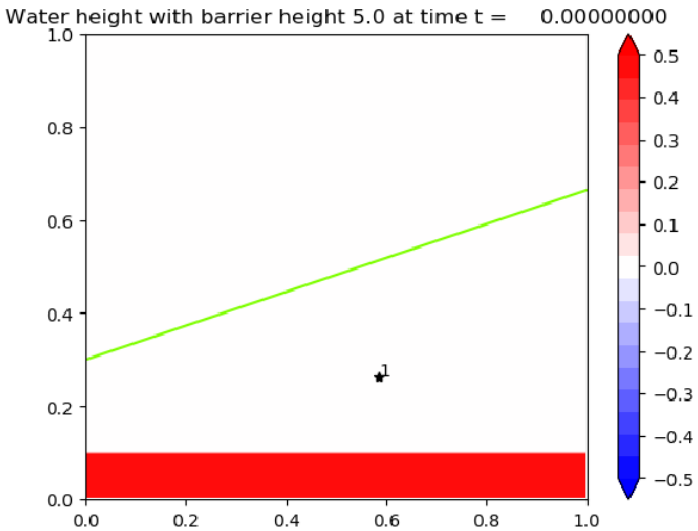
Slanted barrier problem with arbitrary angle $\alpha \in [0, \pi/2]$

Neighbors are the directly above or below cells and overlap count $= 2$

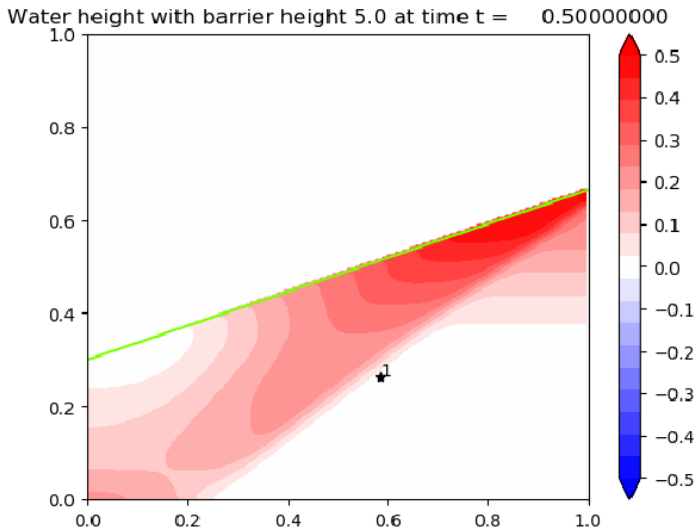


SRD Results with $\alpha = 20°$ (Reflection)

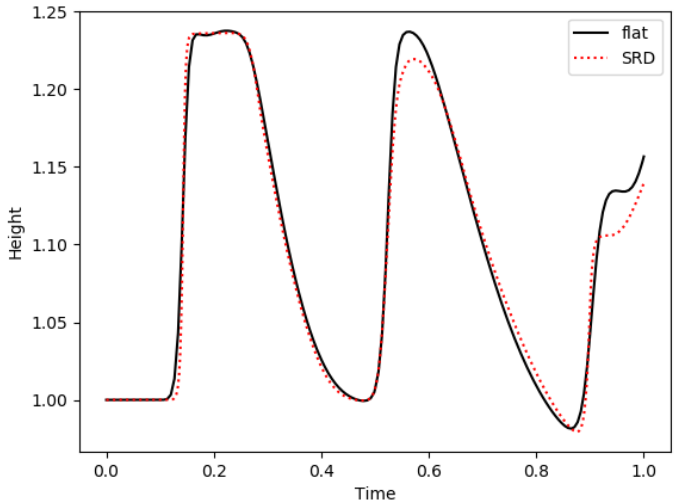
$\boldsymbol{t = 0.0}$:



$t = 0.5$:

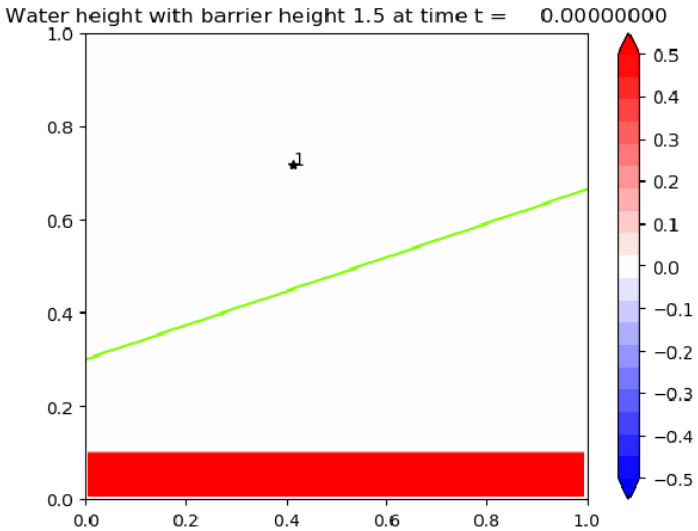


$\Delta x = \Delta y = 0.5 \times 10^{-2}$; $\beta = 5.0$, $\alpha_{\min} \sim 10^{-5}$

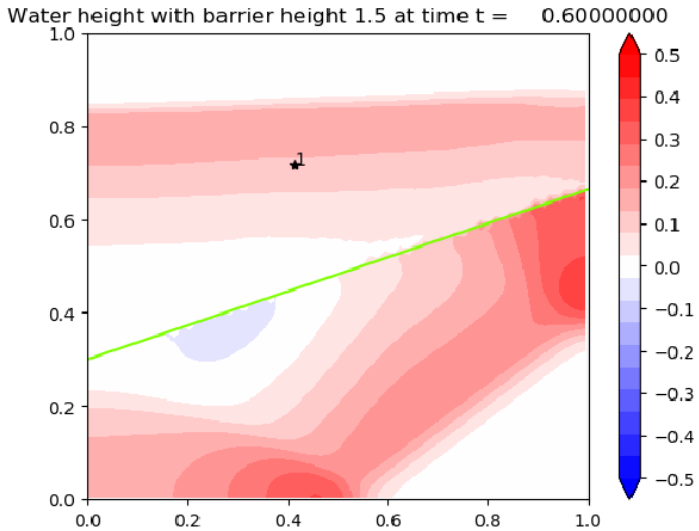


SRD Results with $\alpha = 20°$ (Overtopping)

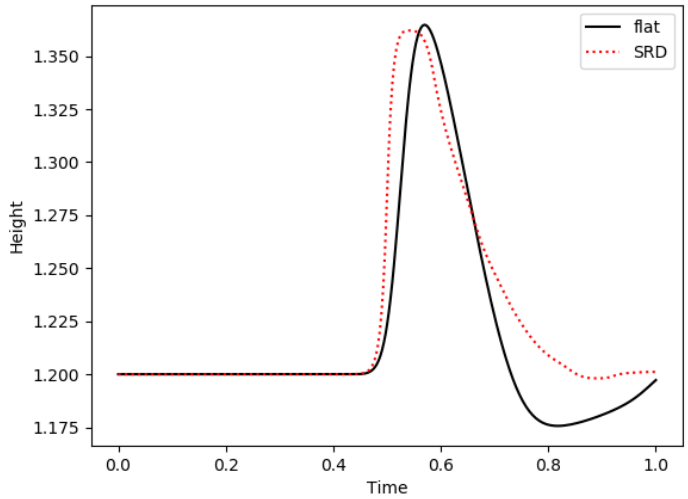
$t = 0.0$:



$t = 0.6$:

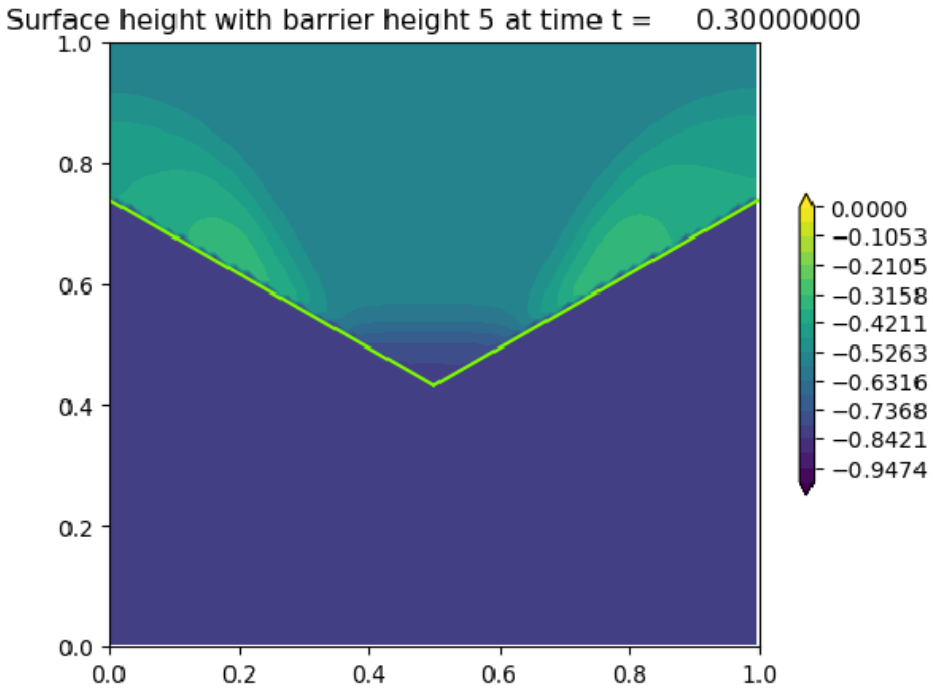


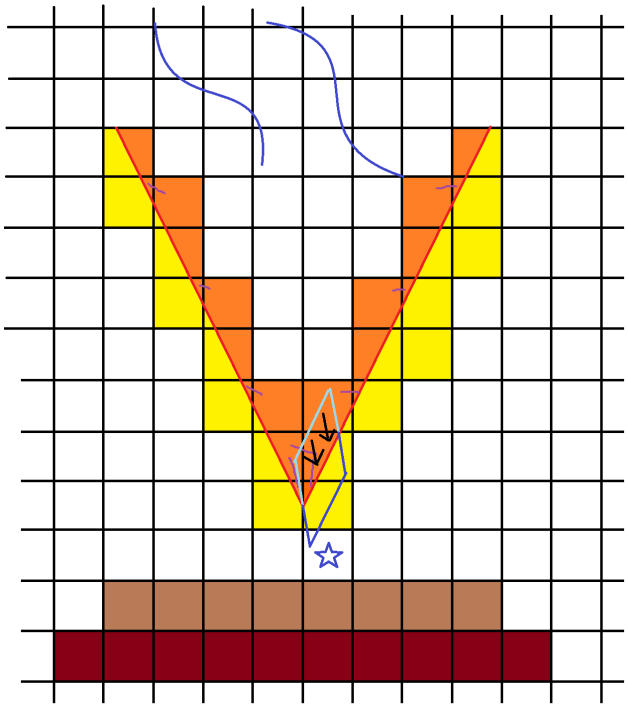
second order corrector on
$\Delta x = \Delta y = 10^{-2}$; $\beta = 1.5$, $\alpha_{\min} \sim 10^{-5}$



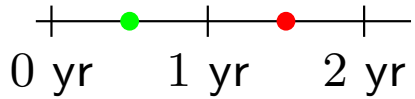
Need to do

- Finish using SRD to solve "V" shaped barrier problem
- Develop cell-merging method (e.g. acute "V" shaped barrier problem)





Potential Timeline



+0.5: In half a year, wrap up SRD.
+1.5: In one and a half year, start and finish cell merging method

Thesis Outline:

1. h-box
2. SRD
3. cell merging

Long term goal of project: Implement with adaptive mesh refinement in GeoClaw