2D Barrier Simulations:
h-Box and State Redistribution Methods

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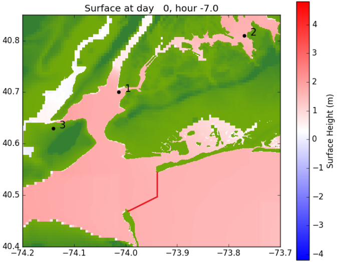
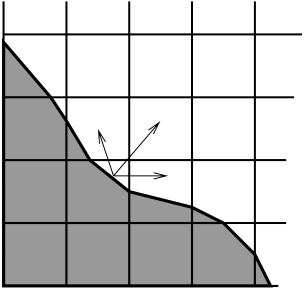
Objective and Obstacles

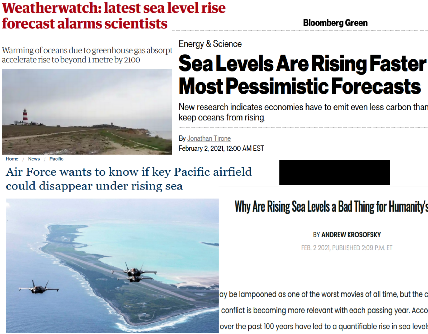


Objective: Want to simulate wave interactions with barrier in 2D shallow water equations

Obstacle: Small cell problem (CFL)



Application



Weatherwatch: latest sea level rise forecast alarms scientists

Warming of oceans due to greenhouse gas absorption could accelerate rise to beyond 1 metre by 2100

Bloomberg Green

Energy & Science

Sea Levels Are Rising Faster Than Most Pessimistic Forecasts

New research indicates economies have to emit even less carbon than keep oceans from rising.

By Jonathan Tirone
February 2, 2021, 12:00 AM EST

Home / News / Pacific

Air Force wants to know if key Pacific airfield could disappear under rising sea

[redacted]

Why Are Rising Sea Levels a Bad Thing for Humanity's

BY ANDREW KROSOFSKY
FEB. 2 2021, PUBLISHED 2:09 P.M. ET

ay be lampooned as one of the worst movies of all time, but the c
conflict is becoming more relevant with each passing year. Acco
over the past 100 years have led to a quantifiable rise in sea level

Shallow Water Equations

$$h_t + (hu)_x + (hv)_y = 0 \quad (1)$$

$$(hu)_t + (1/2gh^2 + hu^2)_x + (huv)_y = -ghb_x \quad (2)$$

$$(hv)_t + (huv)_x + (1/2gh^2 + hv^2)_y = -ghb_y \quad (3)$$

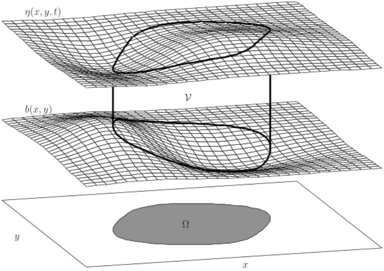
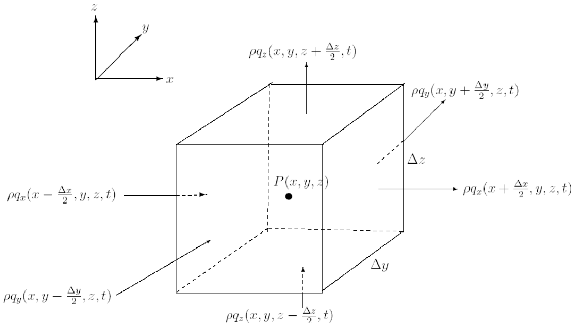


Figure 4.1: Deriving the shallow water equations from conservation principles. The three dimensional control volume $\mathcal{V}$ and the two dimensional region Ω that remains after the vertical dimension is integrated out, are shown. The shallow water equations are a hyperbolic conservation law in the two dimensional x-y plane.

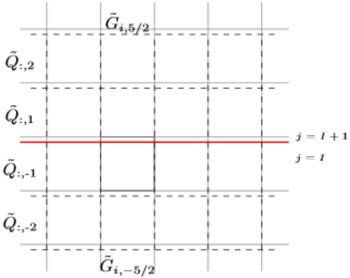
Finite volume method

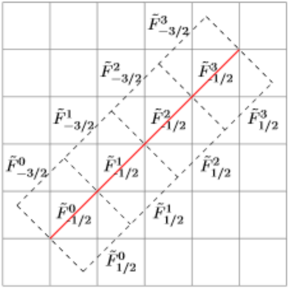
Cells are boxes that can: take flux in and pump flux out
Update = original + net flux



Using *h*-box method

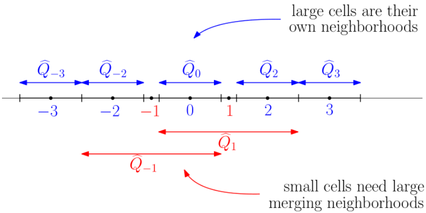
Virtual grid while considering conservation





(a) Normal fluxes.

Using *SRD*[1] method



$$\widehat{Q}_{-1} = \underbrace{\frac{1}{h/2 + \alpha h + h/3}}_{\text{weighted volume}} \underbrace{\left(\frac{h}{2}\widehat{U}_{-2} + \alpha h \widehat{U}_{-1} + \frac{h}{3}\widehat{U}_{0} \right)}_{\text{weighted mass}}.$$

$$\widehat{Q}_i = \widehat{U}_i \quad \text{for } i = -3, -2, 0, 2, 3.$$

l solution average at time t^{n+1} on a cell in the base g
f all the weighted neighborhood averages that overla
ree neighborhoods we have

$$U_0^{n+1} = \frac{1}{3}(\widehat{Q}_{-1} + \widehat{Q}_0 + \widehat{Q}_1).$$

apped by two neighborhoods, we have

$$U_{-2}^{n+1} = \frac{1}{2}(\widehat{Q}_{-1} + \widehat{Q}_{-2}) \text{ and } U_2^{n+1} = \frac{1}{2}(\widehat{Q}_1 + \widehat{Q}_2).$$

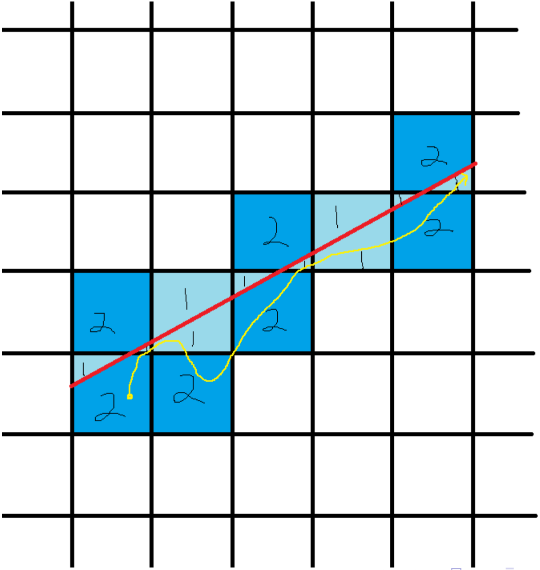
verlapped by only one neighborhood, we have

$$U_i^{n+1} = \widehat{Q}_i \text{ for } i = -3, -1, 1, 3.$$

[1] Berger and Giuliani, 2020, *JCP*

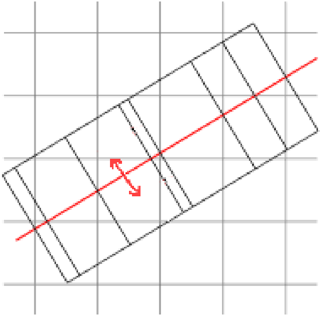
Slanted barrier problem with arbitrary angle $\alpha \in [0, \pi/2]$

Neighbors are the directly above or below cells and overlap count = 2

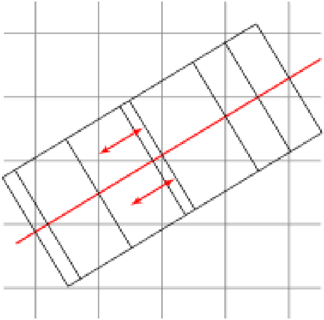


Using hybrid *h*-box and SRD method

Wave redistribution in normal direction



SRD in transverse direction

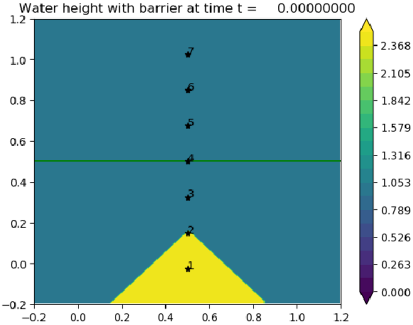


Finally conservation calculations

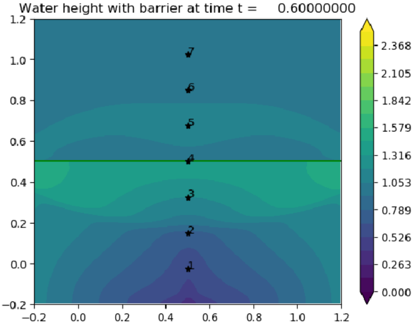
$$\mathcal{F}_o^i = -\sum_{ps} \alpha_i \Delta Q_{ps}^{n+1}.$$

h-box Results with parallel barrier

$t = 0.0$:



$t = 0.6$:

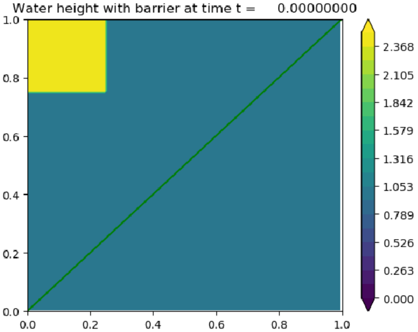


$\Delta x = \Delta y = 10^{-2}$; $\beta = 1.5$

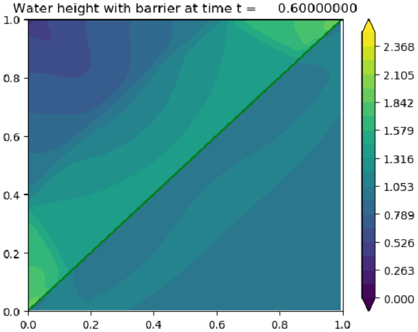
Reflection and overtopping at $t = 0.6$

h-box Results with diagonal barrier

$t = 0.0$:



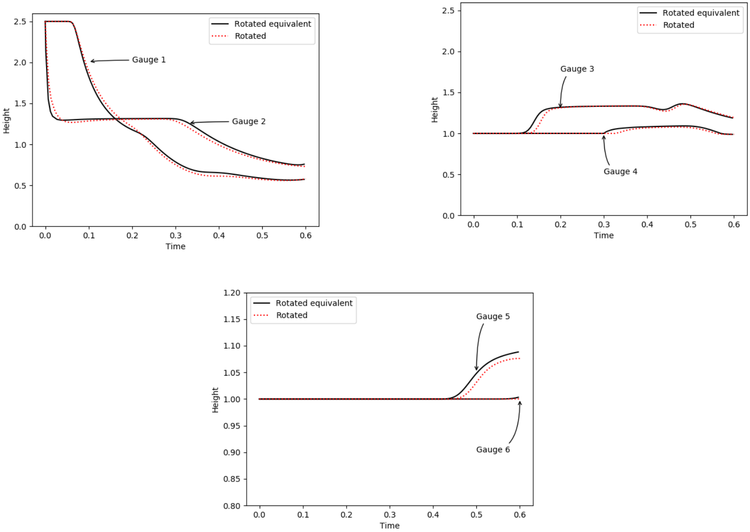
$t = 0.6$:



$\Delta x = \Delta y = 10^{-2}$; $\beta = 1.5$
$\alpha = 0.5$
Reflection and overtopping at $t = 0.6$

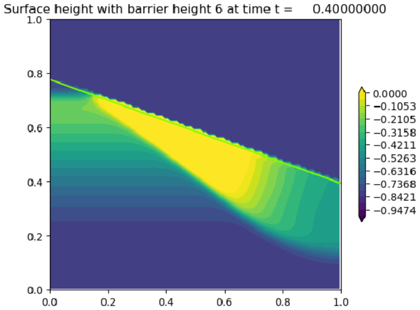
h-box Results with diagonal barrier

Gauge comparison:

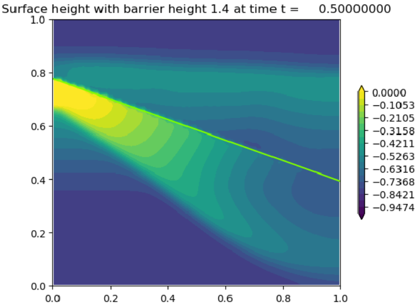


SRD Results with $\approx 22^\circ$ barrier

Complete block:



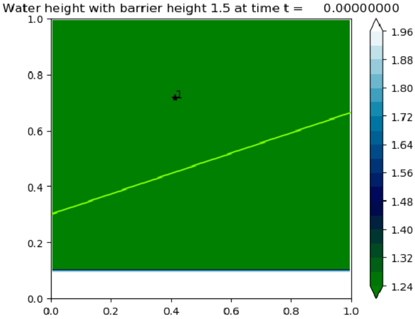
Overtopping:



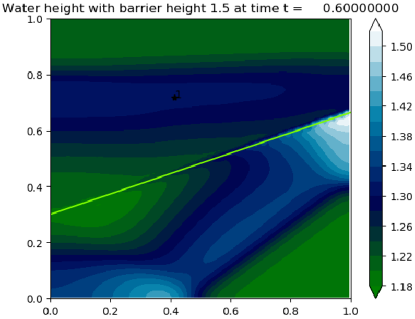
$\Delta x = \Delta y = 10^{-2}$
$\beta_1 = 6,\ \beta_2 = 1.4$
$\alpha_{\min} \sim 10^{-5}$

Hybrid *h*-box Results with 20° barrier

$t = 0.0$:



$t = 0.6$:

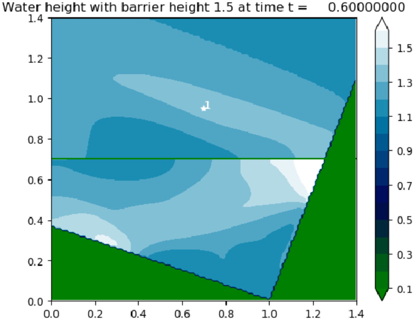


$\Delta x = \Delta y = 10^{-2}$; $\beta = 1.5$

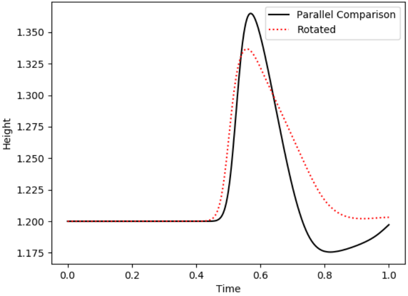
$\alpha_{\min} \sim 10^{-5}$

Hybrid *h*-box Results with parallel barrier

Parallel barrier with rotated initial condition:



Gauge comparison:



Need to do

- Finish using SRD to solve "V" shaped barrier problem
- Develop cell-merging method (e.g. acute "V" shaped barrier problem)