Progress on angled barrier simulations

Judah Ryoo

Columbia University

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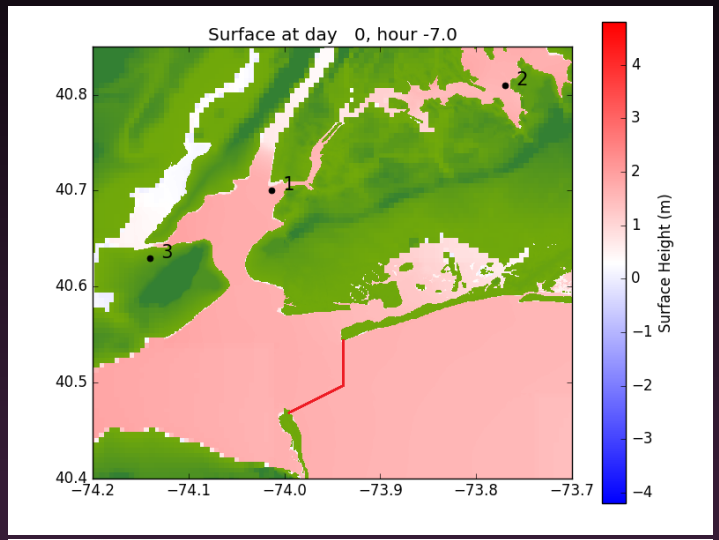
Objective and Obstacles

Objective: Want to simulate wave interactions with barrier in 2D shallow water equations

Obstacle: Small cell problem



Application

Weatherwatch: latest sea level rise forecast alarms scientists

Warming of oceans due to greenhouse gas absorpt accelerate rise to beyond 1 metre by 2100

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Air Force wants to know if key Pacific airfield could disappear under rising sea

Bloomberg Green

Energy & Science

Sea Levels Are Rising Faster Than Most Pessimistic Forecasts

New research indicates economies have to emit even less carbon than budgeted to keep oceans from rising.

By Jonathan Tirone
February 2, 2021, 12:00 AM EST

[redacted]

Why Are Rising Sea Levels a Bad Thing for Humanity's Future?

BY **ANDREW KROSOFSKY**

FEB. 2 2021, PUBLISHED 2:09 P.M. ET

ay be lampooned as one of the worst movies of all time, but the cautionary tale
conflict is becoming more relevant with each passing year. According to NASA,
over the past 100 years have led to a quantifiable rise in sea levels of about 6 to

Shallow Water Equations

$$h_t + (hu)_x + (hv)_y = 0 \tag{1}$$

$$(hu)_t + (1/2gh^2 + hu^2)_x + (huv)_y = -ghb_x \tag{2}$$

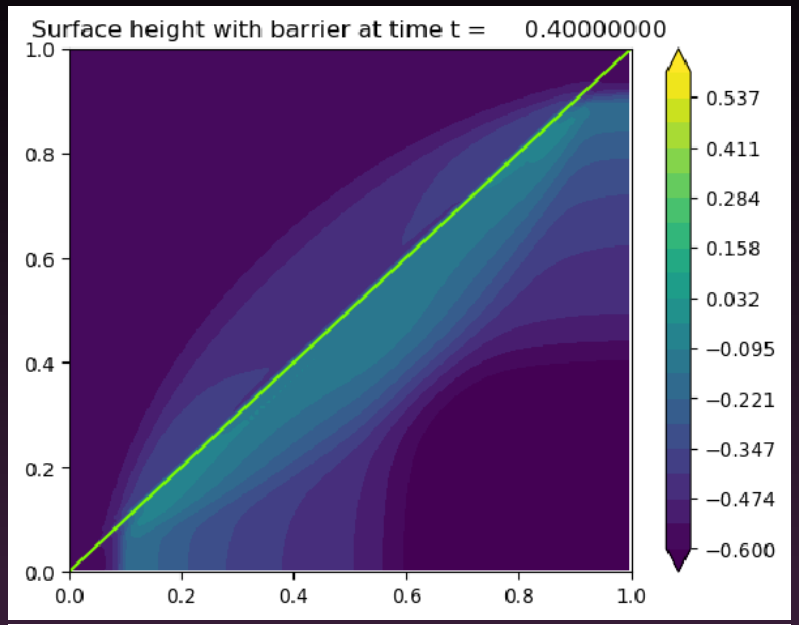
$$(hv)_t + (huv)_x + (1/2gh^2 + hv^2)_y = -ghb_y \tag{3}$$

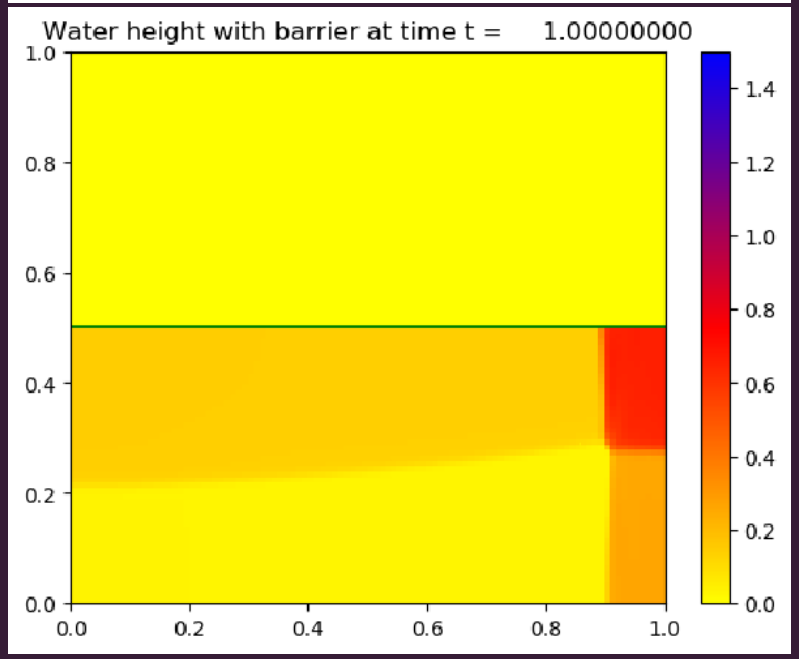
Thus far...

H-box methods for simple cases:

- Parallel barriers
- Diagonal barriers

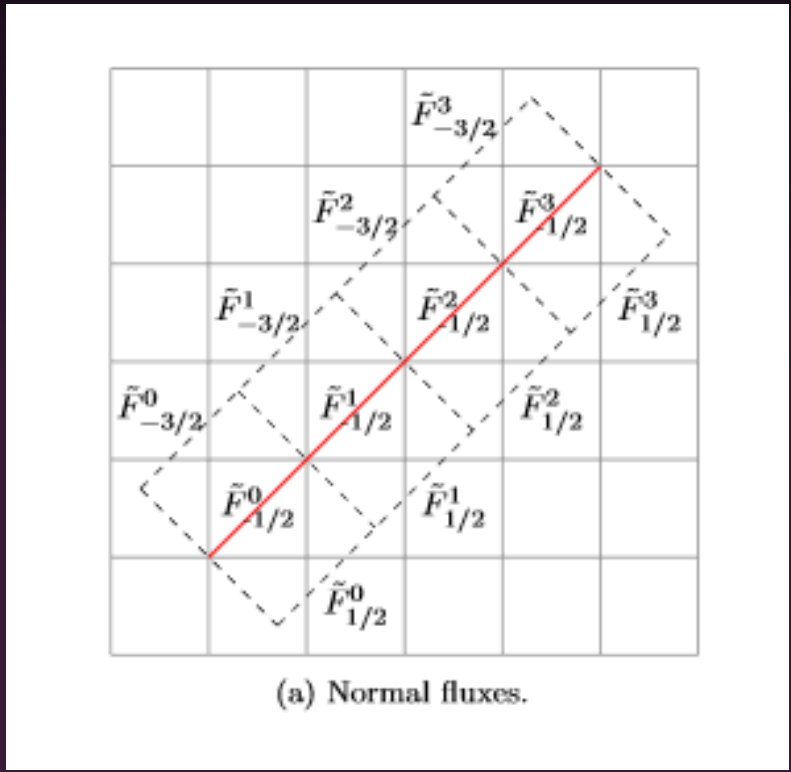
SRD methods for parallel barriers





Brief recap on *h*-box method

- Extend off barrier by regular grid size Δx
- Take average
- Update this average
- Update small cells $\{s\}$ & regular cells underneath it

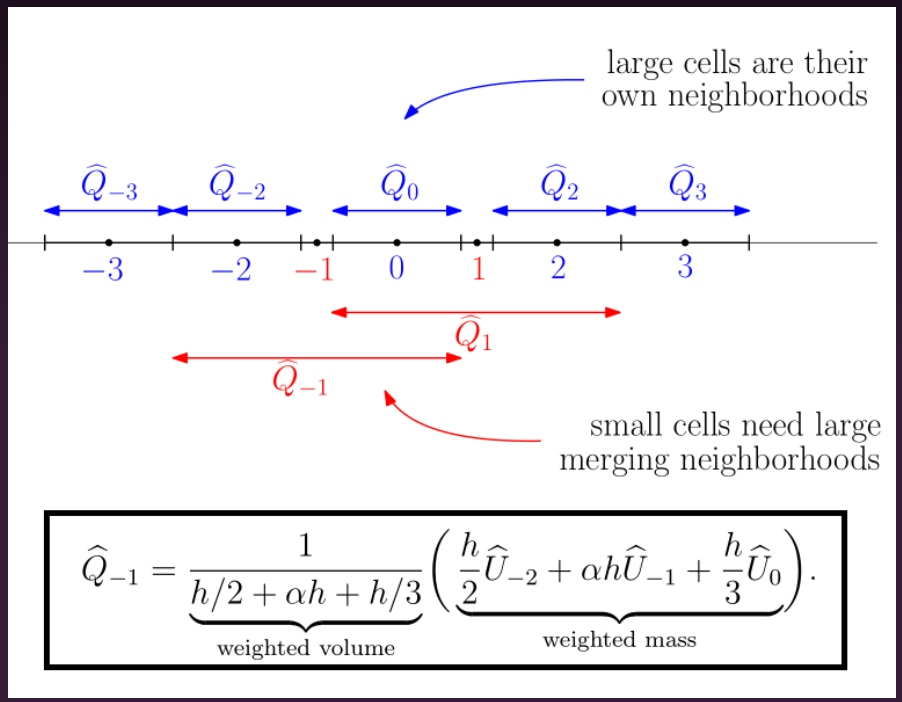


(a) Normal fluxes.

Brief recap on SRD[1] method

- Update all cells without worrying about small cell problem (CFL violation temporarily allowed)
- $\forall s$: find neighborhood N such that accumulated area $> 0.5 \Delta x \Delta y$
- Set new averages for N by weight-averaging the updated cell values using the area and "overlap count"
- Update s with N averages that cover it (discounted by overlap count)

Cont'd



$$\widehat{Q}_{-1} = \underbrace{\frac{1}{h/2 + \alpha h + h/3}}_{\text{weighted volume}} \left(\underbrace{\frac{h}{2}\widehat{U}_{-2} + \alpha h \widehat{U}_{-1} + \frac{h}{3}\widehat{U}_0}_{\text{weighted mass}} \right).$$

$$\widehat{Q}_i = \widehat{U}_i \quad \text{for } i = -3, -2, 0, 2, 3.$$

l solution average at time t^{n+1} on a cell in the base g
: all the weighted neighborhood averages that overla
ree neighborhoods we have

$$U_0^{n+1} = \frac{1}{3}(\widehat{Q}_{-1} + \widehat{Q}_0 + \widehat{Q}_1).$$

apped by two neighborhoods, we have

$$U_{-2}^{n+1} = \frac{1}{2}(\widehat{Q}_{-1} + \widehat{Q}_{-2}) \text{ and } U_2^{n+1} = \frac{1}{2}(\widehat{Q}_1 + \widehat{Q}_2).$$

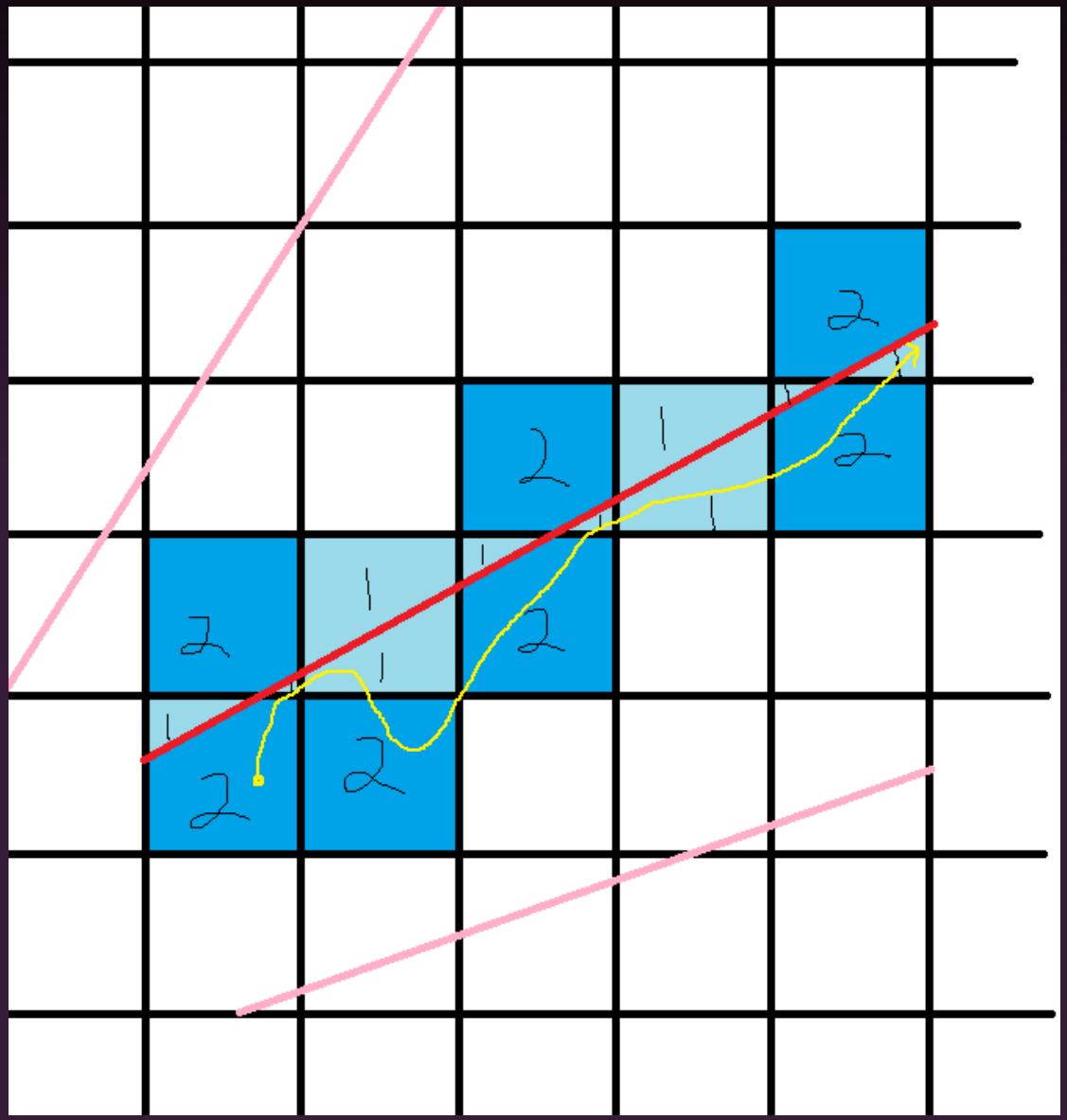
verlapped by only one neighborhood, we have

$$U_i^{n+1} = \widehat{Q}_i \text{ for } i = -3, -1, 1, 3.$$

[1]credit: Marsha Berger, NYU

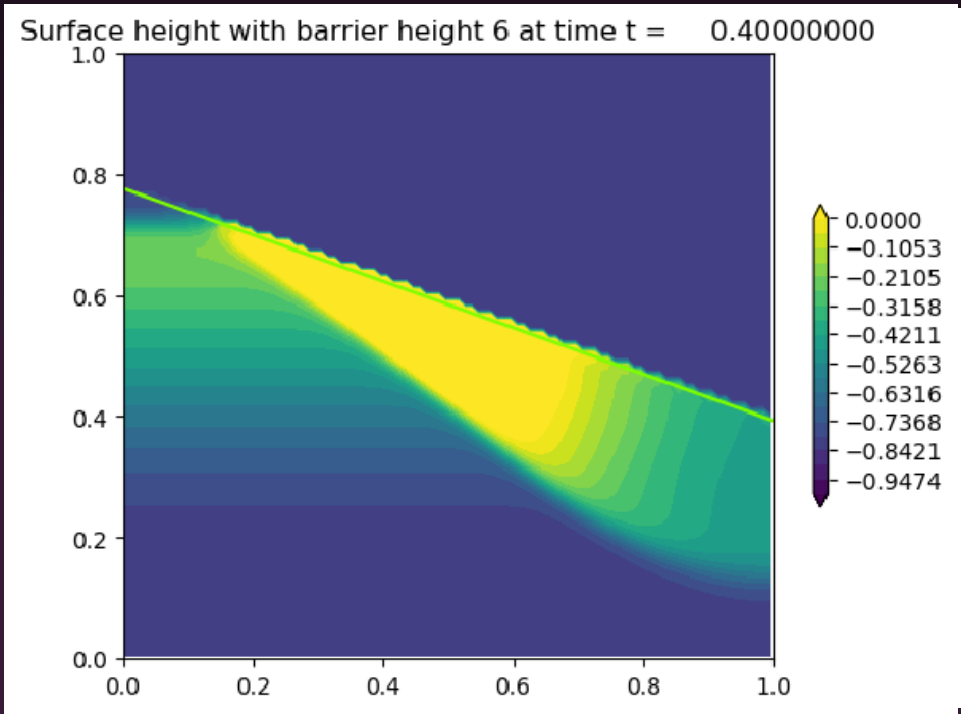
Slanted barrier problem with arbitrary angle $\alpha \in [0, \pi/2]$

Neighbors are the directly above or below cells and overlap count = 2

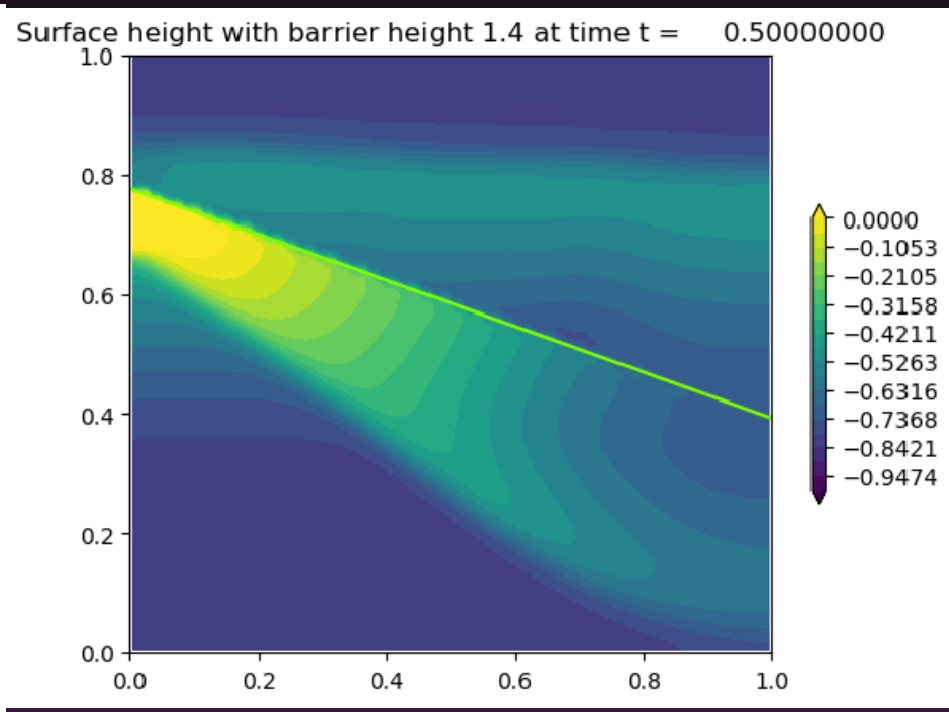


SRD Results with $\alpha \approx 22^\circ$

Complete block:



Overtopping:



Here, $\Delta x = \Delta y = 10^{-2}$
Observe no small cell problem even with size $10^{-5}\Delta x \Delta y$

Need to do

- Expand on *h*-box method for general angled barrier case
- Use SRD to solve ”V” shaped barrier problem