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Minority turnout and representation under cumulative voting. An experiment.[☆]

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ABSTRACT

Under majoritarian election systems, securing the participation and representation of minorities remains an open problem, made salient in the US by its history of voter suppression. One remedy recommended by the courts is the adoption of Cumulative Voting (CV) in multi-member districts: each voter has as many votes as open positions but can cumulate votes on as few candidates as desired. Historical experiences are promising but also reflect episodes of minority activism. We present the results of a controlled lab experiment that isolates the role of the voting rule from other confounds. Although each voter is treated equally, theory predicts that CV should increase the minority's turnout relative to the majority and the minority's share of seats won. The experimental results strongly support both theoretical predictions.

1. Introduction

The fragility of American democracy, rooted historically in slavery, manifests itself in persistent efforts to disenfranchise racial and linguistic minorities, Black Americans first and foremost. Almost 60 years after the Voting Rights Act (VRA), the disputes we continue to witness are reminders of the heightened importance of voters' participation. In 2012, the Pew Research Center concluded: "The Growing Electoral Clout of Blacks Is Driven by Turnout".[1] The date is not coincidental: 2012 was the election year for President Obama's second term, when for the first time, Black turnout was higher than White turnout.[2]

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1 Taylor (2013).

2 According to the US Census Bureau (2017), Black non-Hispanic turnout increased from 60% to 65% and then to 67% from 2004 to 2008 to 2012, while, over the same period, the turnout of non-Hispanic Whites went from 67% to 66% and then 64%.

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Guaranteeing high electoral participation by minorities requires rules about fair and equal access to voting. But that is not enough: as the surge in Black political engagement during the Obama years shows, it also requires giving minorities the realistic chance of a desired outcome. America's majoritarian electoral system makes this difficult. Without resorting to proportional representation, the courts have mandated modifications to electoral rules in jurisdictions where majoritarian systems effectively disenfranchise the minority. The traditional remedy has been the design of single member districts in which the minority constitutes a majority of the electorate. In 1993, however, the Supreme Court judged unconstitutional districting plans driven by considerations of race (*Shaw v. Reno*), and such districts have since had a troubled legal history. Our focus is on a different alternative proposed by the courts: Cumulative Voting (CV), a solution built not on controlling district borders but on the voting rule itself.[3]

CV applies to elections in multi-member districts. The core idea is to allow voters to vary the number of votes cast for each candidate. Under CV, each voter has as many votes as there are open seats, and the candidates with more votes win, as under simple plurality. However, each voter is allowed to distribute the votes freely among any number of candidates. CV treats every voter equally; yet, a cohesive minority can ensure itself some victories by cumulating its vote. CV does not rely on fixed patterns of geographical segregation and thus does not require adjustments as social and political conditions change. As a result, it is not subject to the type of litigation that has weakened the VRA. In addition, although CV can deliver semi-proportional outcomes, it does so through a relatively minor modification of a majoritarian system. The US, the UK, and Scotland—all countries with long-held skepticism of proportional representation—have a history of accepting CV.[4]

This paper contributes to the debate on minority representation by analyzing, both theoretically and experimentally, CV's potential to increase both the relative voting participation of the minority and its share of seats in elected bodies. Both effects have been observed in actual applications, but evaluating historical evidence is complicated by the non-random adoption of CV. Thus, the existing evidence must be accompanied by experimental testing. Our main conclusion is that, in the controlled environment of the lab, CV's predicted outcomes are realized. Across all experimental parametrizations, the relative participation in voting of the minority group increases and so does its share of electoral victories. Part of the impact on minority victories stems immediately from the allowed cumulation of votes: to prevent spreading votes too thinly, the majority must limit the number of candidates it fields, leaving openings for minority candidates. But another important contributor to the minority's success is that CV increases the differential turnout of the minority, relative to the majority. Both theoretically and in the lab, the improved prospects brought by CV work to increase the fraction of minority voters among those who turn out. As in the Obama election or the Pew Research report conclusion, the realistic promise of representation encourages political participation.

CV was used for more than 100 years, from 1870 to 1980, to elect representatives to the Illinois State House and is the rule now in the election of local commissions in tens of local jurisdictions. Case studies have also been promising. In Alamogordo, New Mexico, Latinos amounted to 20% of the electorate but had long been unrepresented in the City Commission. In 1987, after the adoption of CV, the City Commission welcomed its first Latino representative in twenty years. In Amarillo, Texas, minorities made up 24% of the electorate, but lacked representation on the school board. In 2000, after the adoption of CV, Amarillo welcomed its first Black and first Latina school board representatives. In Chilton County, Alabama, the Black community (11% of the electorate) struggled for years with unpaved roads. In 1988, after the adoption of CV, the first Black county commission member was elected since Reconstruction. The roads were finally paved.[5] Empirically, then, CV correlates with an increase in the number of elected minority representatives (Brockington et al., 1998; Bowler et al., 2003), and in the public goods provided to minority communities (Pildes and Donoghue, 1995). In addition, its use appears to increase minority participation in the political system: in a study analyzing the impact on local US jurisdictions, CV is associated with an increase of approximately 5 percentage points in overall turnout (Bowler et al., 2001).

As inspiring as these results are, CV's implementation has typically followed voting rights litigation, indicating heightened sensitivity to minority representation and stronger minority involvement. Understanding the specific role of the voting rule is helped by complementing the historical experiences with the study of a move to CV under the controlled conditions of the lab. We run different experimental treatments, comparing standard bloc voting (one-vote-per-open-seat) and CV, and varying both the number of seats and the relative size of the minority. Because our main focus and our more innovative contribution concern turnout, we focus on a voter's incentive to overcome obstacles to voting. We run a canonical costly voting experiment where payoffs depend on one's own group achieving electoral success but voting is individually costly (Palfrey and Rosenthal, 1985; Levine and Palfrey, 2007). Do participants on the minority side overcome those costs more often than participants on the majority side when votes can be cumulated?

[3] For a brief panoramic summary of voter suppression in the US and the historic role of the VRA, see Grofman et al. (1992). For long-term effects of the VRA, see Ang (2019). For theory and evidence on the majority's strategic design of electoral rules and modern-day voter suppression, see Trebbi et al. (2008) and Ricca and Trebbi (2022). For discussion of theories of representation and empowerment, see Banducci et al. (2004).

[4] Similarly to CV, Limited Vote (LV) also results in semi-proportional outcomes. Under LV, voters have fewer votes than the number of candidates and cast one vote per chosen candidate. LV is considered simpler than CV but less reliable in generating minority representation. See for example Arrington and Ingalls (1998). For a broad discussion of alternative rules and proportional representation, see Lijphart and Grofman (1984).

[5] See Bowler et al. (2003) for a short history of CV. Other useful sources are Bowler et al. (1999), Engstrom (2010), Pildes and Donoghue (1995). For a strong defense of CV, see Guinier (1994). Updated information on the current use of CV is reported in fairvote.org. Outside local politics, CV is used to elect corporate boards in approximately 10% of S&P 500 companies, again with the goal of protecting minority representation.

Empirically, unless complemented by sophisticated formulations of bounded rationality, costly voting models fail to predict the level of turnout in large elections. However, in studies both of historical and of experimental data, their comparative predictions have fared better: turnout is predicted to increase when elections are closer, when the stakes are higher, when voting costs are lower, when the electorate is smaller (for example, Bursztyn et al., 2017; Levine and Palfrey, 2007). It is this type of comparative effect that interests us here: when the voting rule changes to CV, are minority voters more represented among overall voters? Precise theoretical predictions depend on complex calculations of pivotality, but the logic underlying the results is in fact much simpler. When votes can be cumulated, and only when votes can be cumulated, the minority can win seats even if realized turnout rates are similar between minority and majority voters. In equilibrium, the higher chance of having one's voice heard encourages minority participation: CV increases the differential turnout of the minority, relative to the majority. The result, predicted by the theory, is observed in the lab and is robust: we find it across all different parametrizations we test. Surprisingly, however, we see it accompanied by higher turnout for both groups than theory predicts, a regularity suggesting that subjects are responding to the increased competitiveness introduced by CV with an emotional intensity only partly captured by the pivotality calculations.

To our knowledge, there is no existing theoretical or experimental study of turnout under CV. Previous laboratory experiments on CV (Gerber et al., 1998 and Cooper and Zillante, 2012) focus on the coordination problem the voting rule poses, and neglect the impact of the voting rule on voters' participation decision. We take the opposite approach. We focus on voters' turnout decisions and assume that the coordination problem is addressed by the parties' leadership, and addressed primarily through the leaders' choice of the number of party candidates.

We make this assumption because it mirrors our reading of CV's historical experiences. For example, Bowler et al. (1999) is a very lively study of CV in Victorian England, an interesting environment for its experimental spirit, the richness of cases, and the availability of historical documents. Focusing exactly on the strategic problems posed by CV, the authors find: "a willing demand for party organization from voters, as much as a willing supply of it from the parties themselves." Strategic mistakes were made, typically in the form of over-nominations by the majority party, but their responsibility was attributed to party leaders and quickly corrected. Consider the following exchange from 1884 Parliamentary hearings on an election run with CV (cited by Bowler et al., p. 911):

Mr. Courtney: If a party ran too many candidates it might not gain its due proportion of power.
Mr. Sanford: Quite so. That is its own fault.[6]

Similar sentiments recur in other episodes, whether the majority and minority identities are party-based, as in Victorian England, or correspond to racial or linguistic divisions, like in Pildes and Donoghue (1995)'s detailed chronicle of the first adoption of CV in Chilton County, Alabama, following VRA litigation. Because they are so costly, nomination mistakes are corrected rapidly, and granting party leaders their optimal choice of candidates seems a good working assumption.[7] Note also that a common finding in the literature is that nomination mistakes are more common on the majority side, for whom the need to concentrate votes is less obvious. If our analysis underestimates the parties' difficulties in coordinating votes, it is likely to also underestimate the extent to which the minority benefits from CV.

Because CV is an example of "semi-proportional" voting rules—rules whose results approach proportional representation without imposing proportionality—parallel to our work are the experiments in Herrera et al. (2014) and Kartal (2015), which compare turnout under single-winner majoritarian and proportional elections. However, although CV leads to quasi-proportional outcomes, the turnout decision is quite different: under proportional representation, the value of a marginal single vote is proportional to the change in the party's vote share, and pivotality, in its usual sense, is moot.[8] With CV, instead, pivotality continues to drive turnout decisions. The difference, relative to majoritarian voting, is that the possibility of cumulating votes implies richer pivotality calculations. This said, the conclusions are similar: both Herrera et al. and Kartal find that proportional representation increases the turnout rate of the minority relative to the majority's, as well as the minority's expected share of power. The same results hold under CV.

Currently, CV is limited to elections of local committees, and it is natural to ask whether there are realistic chances of applications at a higher level. One possible reason for skepticism is that CV applies to multi-member district elections and, outside municipal elections, multi-member districts are relatively infrequent, not least because of their long history of legal challenges, exactly on the grounds of discrimination against racial minorities. The problem, however, is the combination of multi-member districts and bloc voting: in the absence of cumulation, a group that has a minority position in all districts can potentially win no representation at all. CV, on the contrary, favors minority victories, in line with the courts' recommendation of its adoption following voting rights litigation. At the moment, ten US states use multi-member district elections to elect at least one of their state chambers.[9] Such elections could, in principle, adopt CV; indeed, there have been

[6] *Report and Minutes of Evidence of the Select Committee on School Board Voting*, P.P. (1884/85, p. 78). School board elections attracted much attention because they decided religious education in schools, and in particular the inclusion or exclusion of Catholicism.

[7] When CV becomes established, if anything the often voiced concern is the possibility of collusion between party leaders, reducing voters' choices, as was remarked for example during the long experience with CV in the Illinois State House elections (Sawyer and MacRae, 1962).

[8] Indeed, Herrera et al. comment on the similarity in turnout decisions between proportional voting models and non-instrumental models of voting.

[9] Arizona, Idaho, Maryland, New Hampshire, New Jersey, North Dakota, South Dakota, Vermont, Washington, and West Virginia. Ten more states explicitly allow multi-member districts by law.

repeated calls to that effect.[10] With broader stakes, the focus would presumably be less on the representation of racial and linguistic minorities and more on parties' dynamics. The need to understand better the effects of the voting rule remains.

The paper proceeds as follows. The next section describes the basic model in the absence of voting costs and compares equilibrium minority victories under one-vote-per-seat and CV. Section 3 discusses theoretical predictions under voting costs. Section 4 describes the experiment, and Section 5 discusses the experimental results. Section 6 concludes. Additional theoretical material is left to the Appendix. An online Appendix reports supplementary empirical results, as well as a copy of the experimental instructions.

2. Base model

An electorate of N potential voters selects $K > 1$ representatives for a commission. All positions are identical, and all are simultaneously filled in the election. The N voters are divided into two parties: M, the majority party with M members, and m, the minority party with $m < M$ members, where $M + m = N$. Parties are led by party leaders whose role is to propose the party's list of candidates.

Within each party, all potential candidates are identical, and party leaders and voters share the same objective: to maximize the number of positions won by their party. The utility derived from one's party winning k positions is $u(k)$, increasing in k. We denote by V the value of controlling all positions and assume $u(k) = (k/K)V$. Linearity captures the focus on the number of positions and simplifies both the lab implementation and the theoretical analysis. But we adopt it on substantive grounds as well: any "place at the table" has value. The assumption mirrors an exercise of committee power that is proportional to the number of seats a party has won.

Each voter has K votes, and the K candidates with most votes are elected. If there are ties, after the highest voted candidates are elected, the remaining open positions are filled by selecting winners randomly among the tied candidates. We call x_p the profiles of votes cast by members of party p, where x_{ip}^k is the number of votes cast by voter $i \in p$ for candidate k, and x_{ip} the vector of all votes cast by i.

We study two electoral systems, *multi-seat plurality* (MP) and *cumulative voting* (CV). MP corresponds to standard bloc voting in multi-member districts: under MP, each voter casts at most one vote for each candidate: $x_{ip}^k \in \{0, 1\}$ for all i, k, and p, and each party nominates K candidates.[11] Under CV, each voter can distribute the K votes in any manner the voter desires, as long as the overall budget of K votes is satisfied: $\sum_k x_{ip}^k \leq K$. The possibility of cumulating votes creates a coordination problem that party leaders help address by selecting the number of candidates, G for the majority party, and g for the minority party. In line with historical experience,[12] we allow for fractional votes, but voters, candidates, and positions are constrained to be integers.

The game has two stages. In the first stage, party leaders announce the party list; in the second stage, voters distribute their votes over the party candidates. We focus on equilibria in weakly undominated strategies where voters cast all their votes and cast votes on their party's candidates only. Under MP, each voter casts one vote for each party candidate. Under CV, the equilibrium is a pair of vote profiles $\{x_M(G, g), x_m(G, g)\}$ and a pair of party lists $\{G(x_M, x_m), g(x_M, x_m)\}$ such that each party member's votes maximize the number of seats won by the party, given the parties' lists and the other voters' voting choices, and each party list maximizes the number of seats won by the party, given the opposite party's list and all voters' voting strategies.

Although our main focus is on turnout, and thus the theory will require some positive costs of voting, it is helpful to begin by understanding the functioning of the two voting rules without the complication of voting costs.

2.1. Minority representation without voting costs

With no reasons to abstain and no leeway in distributing votes, under MP, party M wins all seats: each M candidate receives M votes, and each m candidate receives $m < M$ votes.

CV grants the minority the possibility of winning some seats. Suppose for example that all voters in party m concentrate all their votes on a single candidate, who thus receives mK votes. The minority wins one seat if its candidate beats the Kth weakest majority candidate, that is, the majority party's candidate who ranks Kth in terms of votes received. If the majority nominates fewer than K candidates, the minority candidate is elected. If the majority targets all positions and nominates K candidates, the weakest majority candidate will have most votes when the MK total majority votes are distributed equally among the K majority candidates, and each receives $MK/K = M$ votes. Hence the minority can *guarantee* itself a seat if $mK > M$, or $m > M/K$. This ratio, known in the literature as the *threshold of exclusion*, is a fraction of M: for example, a minority that is half the size of the majority can guarantee itself a seat if the number of open seats is three or more.

Academics and lawyers have extended this logic to a handy formula that delivers a party's guaranteed number of seats for each $\{m, M, K\}$.[13] On its face, the formula does not address what we are really interested in: not how many seats can

[10] See for example IGPA, Univ. of Illinois (2001), for a return to CV in Illinois state elections, or the advocacy of fairvote.org.

[11] MP is used by all but two of the US states electing their legislatures from multi-member districts.

[12] For example, half votes were allowed in the Illinois State House; half, third, and quarter votes are allowed in the Peoria, IL elections.

[13] The formula is so widely known and used that CV-calculators can be found online. See, for example, https://www.lawjock.com/tools/cumulative-voting-calculator/, or Wikipedia: https://en.wikipedia.org/wiki/Cumulative_voting. Early influential references in political science are Cole (1950), Glasser (1959), Sawyer and MacRae (1962), Brams (1975), and Glazer et al. (1984).

the minority make sure to win, but how many seats will it win when both parties play their optimal strategies. Yet the answer the formula yields can be grounded in a strategic analysis. In line with our focus on the coordinating role of the party leaders, we call *party-optimal* those equilibria that for each party maximize the number of seats won. We denote by z the number of seats won by party m. As we prove in the Appendix:

Proposition 1. *In the absence of voting costs, in all party-optimal equilibria of the CV voting game: (i) for all $m < M/K$, the minority never wins any seat; (ii) for all $m \geq M/K$:*

$$z = \left\lfloor \frac{Km+m}{M+m} \right\rfloor \qquad \text{if } \frac{Km+m}{M+m} \notin \mathbb{Z}$$

$$z = \begin{cases} \frac{Km+m}{M+m} - 1 & \text{with prob } m/(m+M) \\ \frac{Km+m}{M+m} & \text{with prob } M/(m+M) \end{cases} \qquad \text{if } \frac{Km+m}{M+m} \in \mathbb{Z}$$

Given M, m, and K, the proposition yields the equilibrium number of minority seats. Suppose, for example, $m = M/2$. Then $z = 1$ if $K = 4$; $z = 2$ if $K = 6$; and $z = 1$ with probability 1/3 or $z = 2$ with probability 2/3 if $K = 5$.

What makes the result powerful is the unique equilibrium prediction on the number of minority seats. As we discuss in more detail in the Appendix, for given M, m, and K, the game admits a large number of party-optimal equilibria. And yet the multiplicity is irrelevant to the outcome: *all* party-optimal equilibria must yield the same number of minority victories.

The result follows from two main reasons. First, because voters and leaders share a common goal, party-optimal equilibria correspond to the equilibria of a two-player game where the two party leaders directly control the distributions of the votes over the party candidates. Second, the linearity of the objective function, $u(k)$, renders the game constant-sum. As a result, all party-optimal equilibria must result in maximin payoffs, the payoffs the two parties can guarantee themselves. Extending the reasoning described earlier then yields the proposition.

Note an immediate corollary that will shape intuition for what follows. There always exists an equilibrium where $g = \lfloor \frac{Km+m}{M+m} \rfloor$, $G = K - \left\lceil \frac{Km+m}{M+m} - 1 \right\rceil$, and $x^k_{im} = K/g$, $x^k_{iM} = K/G$: all voters spread their votes equally over their party's candidates, and the two parties nominate just enough candidates to fill all open positions if $\frac{Km+m}{M+m} \notin \mathbb{Z}$, or exceed the number of positions by 1 if $\frac{Km+m}{M+m} \in \mathbb{Z}$.

3. Voting costs

Suppose now that each voter i faces a cost of voting c_i, drawn randomly and independently across voters from a common distribution $F(c)$ everywhere continuous and atomless over support $[\underline{c}, \overline{c}]$, with $\underline{c} \geq 0$. Realized costs are private information, but the distribution $F(c)$ is common knowledge and does not depend on party affiliation. The cost c_i represents the cost of going to the polls and is independent of the number of votes cast. A voter i whose party wins k positions has utility $U_i(k)$, given by:

$$U_i(k) = \begin{cases} u(k) - c_i & \text{if voter } i \text{ voted} \\ u(k) & \text{if voter } i \text{ abstained} \end{cases}$$

3.1. Multi-winner plurality (MP)

Under MP, voters who have turned out cast a single vote for each of the party's K candidates. Although multiple positions are in play, the analysis mirrors closely the standard approach to costly voting in single winner elections.[14] Following Palfrey and Rosenthal (1985) and the subsequent literature, we focus on semi-symmetric Bayesian equilibria in threshold strategies: there exist cost cutpoints c_M and c_m such that any voter i in party M (m) turns out to vote if $c_i < c_M$ ($c_i < c_m$) and abstains if $c_i > c_M$ ($c_i > c_m$).

Call S_p the number of voters who turn out for party p. Each M candidate receives S_M votes, and each m candidate receives S_m votes. Thus only three outcomes are possible: either $S_M > S_m$, and all K positions are won by M candidates; or $S_M < S_m$, and all K positions are won by m candidates; or $S_M = S_m$, and all K positions are tied, with K majority and K minority candidates all having the same number of votes. Under a tie, the K winners are chosen randomly among all tied candidates. We denote by Eu^{MP}_T the expected utility gain from winning seats under MP in case of a tie. Then:

$$Eu^{MP}_T = \sum_{k=0}^{K} \frac{\binom{K}{k}\binom{K}{K-k}}{\binom{2K}{K}} u(k) = V/2$$

[14] Arzumanyan and Polborn (2017) study costly voting with multiple candidates but a single winner. Our model is closer to the traditional two-candidate, one-winner set-up, with each party list being the parallel to the party candidate.

where the second equality follows from $u(k) = (k/K)V$.

As in single-winner elections, a voter from party p facing opposite party p' must weigh her cost of voting against the expected utility gain from influencing the outcome. Denoting by S_{-ip} the number of voters who turn out in party p ignoring i, voter i can influence the outcome either by breaking ties (when $S_{-ip} = S_{p'}$; an event whose probability we denote by π_p^T) or by making ties (when $S_{-ip} = S_{p'} - 1$, with probability π_p^{T-1}). Thus if the cutpoints $\{c_\mathrm{M}, c_\mathrm{m}\}$ are interior, they solve the system of equations:

$$c_\mathrm{m} = [u(K) - Eu_T^{MP}]\pi_\mathrm{m}^T(c_\mathrm{M}, c_\mathrm{m}) + [Eu_T^{MP} - u(0)]\pi_\mathrm{m}^{T-1}(c_\mathrm{M}, c_\mathrm{m}) \tag{1}$$

$$c_\mathrm{M} = [u(K) - Eu_T^{MP}]\pi_\mathrm{M}^T(c_\mathrm{M}, c_\mathrm{m}) + [Eu_T^{MP} - u(0)]\pi_\mathrm{M}^{T-1}(c_\mathrm{M}, c_\mathrm{m}) \tag{2}$$

or:

$$c_\mathrm{m} = (V/2)\pi_\mathrm{m}(c_\mathrm{M}, c_\mathrm{m}) \tag{3}$$

$$c_\mathrm{M} = (V/2)\pi_\mathrm{M}(c_\mathrm{M}, c_\mathrm{m}) \tag{4}$$

where $\pi_p \equiv \pi_p^T + \pi_p^{T-1}$ is the pivotal probability for a voter of party p.

The linearity of the utility function implies that the equilibrium equations (3) and (4) do not depend on K. The problem is then formally identical to the classic costly voting problem with a single winner and two alternatives. It is well-known, and we leave the expressions for the pivot probabilities to the Appendix. Given equilibrium $\{c_\mathrm{m}, c_\mathrm{M}\}$, we can derive the probabilities of winning different numbers of positions. The derivation is straightforward, and again is left to the Appendix.

3.2. Cumulative voting (CV)

With voting costs, the interests of voters and party leaders need not coincide any longer. The game now has $M + m + 2$ players and three stages: a nomination stage, when each of the two leaders chooses the number of candidates; a turnout stage, when, after observing privately the realization of the voting cost, each of the $M + m$ voters decides whether or not to vote; and finally a voting stage, when voters at the polls choose how to cast their votes.[15]

We focus on pure strategy semi-symmetric perfect Bayesian equilibria such that within each party, all voters follow the same strategy. We denote by x_{-ip} the profile of votes cast by voters other than i who have turned out and belong to p. The equilibrium is a pair of party lists $\{g, G\}$, a pair of cost cutpoints $\{c_\mathrm{M}, c_\mathrm{m}\}$, and a pair of voting profiles $\{x_\mathrm{M}, x_\mathrm{m}\}$ such that: (i) at the voting stage, voter i in party p who has gone to the polls sets $x_{ip}(G, g, c_M, c_m, x_{-ip}, x_{p'})$ so as to maximize the expected number of positions won by p, and in equilibrium $x_{ip}^k = x_p^k$ for all i and $k \in p$; (ii) at the turnout stage, all $i \in p$ with $c_i < c_p(G, g, c_{p'}, x_M, x_m)$ strictly prefer to vote, and all $i \in p$ with $c_i > c_p(G, g, c_{p'}, x_M, x_m)$ strictly prefer to abstain; and (iii) at the nomination stage, the two party leaders set $g(G, x_M, x_m, c_M, c_m)$ and $G(g, x_M, x_m, c_M, c_m)$ so as to maximize their party's expected number of positions. The term "equilibrium" in what follows, refers to such equilibria.

For any positive turnout, if $g < K$, party M is guaranteed $\min[G, K - g]$ seats, and similarly, if $G < K$, party m is guaranteed $\min[g, K - G]$ seats. The positions contested are $\max[0, g + G - K]$.[16]

The voters' turnout decision complicates greatly the characterization of the equilibrium. Intuitively, there are three main reasons. First, turnout is stochastic and in evaluating how to distribute votes, each voter needs to account for the probability of different turnout rates, both among voters of her own party and among opponents. Second, the number of candidates nominated by the party leaders will affect not only the distribution of votes among voters at the poll, but the decision to turnout itself—the cost cutpoints. And because such influence is mediated by the voting profiles, the link in general is complex. Third, the multiplicity of equilibria noted in the absence of voting costs continues to exist when voting is costly. And because the game cannot be assimilated to a zero-sum two-player game any longer, the multiplicity of equilibria will in general translate into multiplicity in outcomes.

This said, two limited results must hold and will help the experimental design. We summarize them in one proposition. Recall that an equilibrium is strict when deviation implies a non-zero loss.

Proposition 2. *(i) There exists no strict equilibrium with $g + G < K + 1$. (ii) If there exists an equilibrium with $g + G = K + 1$, then the equilibrium has equal spreading of votes: $x_{i,\mathrm{m}}^k = K/g$ and $x_{i,\mathrm{M}}^k = K/G$.*

Proof. (i) If $g + G < K + 1$, there are no contested seats. With no contested seats, no voter with positive voting costs goes to the polls. Because non-contested positions are ensured, for given opposite party list, increasing the number of party candidates cannot cost any seat. (ii) If $g + G = K + 1$, there is a single contested seat, and the competition between the two parties is over protecting the least voted of their respective candidates. We are focusing on semi-symmetric equilibria,

15 Because party leaders influence turnout through the number of candidates, note the connection to models of leaders' enforced social norms in voting (Levine and Mattozzi, 2020).

16 We assume that non-contested positions are assigned to candidates nominated by the parties even in the absence of voters' turnout.

where therefore $x_{ip}^k = x_p^k$ for all $i \in p$: in equilibrium all voters in p cast the same number of votes on party candidate k. Can there be an equilibrium where there exist two candidates from the same party, k and k', such that $x_p^k > x_p^{k'}$? All party candidates with the exception of the single least voted candidate are guaranteed election. Thus, if there exist k and k' such that $\sum_{-i\in p} x_{-i,p}^k > \sum_{-i\in p} x_{-i,p}^{k'}$, voter i in party p benefits from deviating. Rather than casting $x_{ip}^k > x_{ip}^{k'}$ and reinforcing the difference in votes across the candidates, i should counter it, and cast her votes so as to maximize the votes total of the least voted of the party's candidates: $Max_{\{x_{ip}\}}(Min_k(\sum_{i\in p} x_{ip}^k))$. Because by construction such a deviation increases the votes of the least voted candidate, it cannot cost any seat and is profitable if the voter is pivotal. Hence in any semi-symmetric equilibrium, if $g + G = K + 1$, it must be that $x_p^k = x_p^{k'}$ for all k, k', or $x_{i,\mathrm{m}}^k = K/g$ *and* $x_{i,\mathrm{M}}^k = K/G$. □

In what follows, we identify and use as theoretical references equilibria with equal spreading of votes. We do so when $g + G = K + 1$, but also when $g + G > K + 1$. Distributing votes equally is an easy default for the voters, but we focus on such equilibria for two additional reasons. First, observers have documented that equal spreading of votes over all party candidates was the norm in the Illinois State House elections (Sawyer and MacRae, 1962; Goldburg, 1994). With more than a century of experience, it seems plausible that such behavior condensed CV's lessons when parties play their coordinating roles. Second, the explicit constraint that votes must be spread equally is part of a modified CV rule ("Equal and even CV") applied in city council elections in Peoria, IL and at times proposed, because of its simplicity, as a possible model for wider adoption.[17]

To characterize equilibria for the experimental parametrizations, we begin by discussing the derivation of the equilibrium cost cutpoints, and thus turnout, and showing how such derivation differs from the usual approach. For given g and G, equilibrium cost cutpoints continue to trade off costs of voting and expected utility gains from influencing the election. As before, a voter may break an existing tie or cause a tie, but if the party's candidates are fewer than the number of seats, by casting more than a single vote on each, the voter may also move the outcome from a loss to a win of all contested positions. Consider the problem for $i \in \mathrm{M}$. By voting, i breaks a tie if $(K/G)S_{\mathrm{M-i}} = (K/g)S_{\mathrm{m}}$, or $S_{\mathrm{M-i}} = S_{\mathrm{m}}(G/g)$; i causes a tie if $(K/G)(S_{\mathrm{M-i}} + 1) = (K/g)S_{\mathrm{m}}$, or $S_{\mathrm{M-i}} = S_{\mathrm{m}}(G/g) - 1$. In addition, voter i can shift M from losing to winning all contested positions if both $(K/G)S_{\mathrm{M-i}} < (K/g)S_{\mathrm{m}}$ and $(K/G)(S_{\mathrm{M-i}} + 1) > (K/g)S_{\mathrm{m}}$, or $S_{\mathrm{M-i}} \in (S_{\mathrm{m}}(G/g) - 1, S_{\mathrm{m}}(G/g))$. Denoting by π_p^W the probability that the votes of a member of party p move party p from losing to winning all contested positions, if c_{M} is interior and $G + g > K$, c_{M} must solve:

$$c_{\mathrm{M}} = [u(G) - Eu_{T,\mathrm{M}}^{CV}(G,g)]\pi_{\mathrm{M}}^T + [Eu_{T,\mathrm{M}}^{CV}(G,g) - u(K-g)]\pi_{\mathrm{M}}^{T-1} + [u(G) - u(K-g)]\pi_{\mathrm{M}}^W$$

where[18]:

$$Eu_{T,\mathrm{M}}^{CV}(G,g) = \sum_{x=0}^{G} u(x)\binom{G}{x}\binom{g}{K-x}\Big/\binom{G+g}{K} = \frac{G}{g+G}V.$$

Or:

$$c_{\mathrm{M}} = \frac{V(g+G-K)}{K}\left[\frac{G}{g+G}\pi_{\mathrm{M}}^T + \frac{g}{g+G}\pi_{\mathrm{M}}^{T-1} + \pi_{\mathrm{M}}^W\right] \tag{5}$$

The problem is analogous for minority voters. The equilibrium condition for an interior cutpoint c_{m} is:

$$c_{\mathrm{m}} = \frac{V(g+G-K)}{K}\left[\frac{g}{g+G}\pi_{\mathrm{m}}^T + \frac{G}{g+G}\pi_{\mathrm{m}}^{T-1} + \pi_{\mathrm{m}}^W\right] \tag{6}$$

The pivot probabilities and the probabilities of winning different numbers of position in case of ties can be derived as under MP, taking into account that the number of candidates, in each party, may differ from the number of seats. We leave them to the Appendix.

Given (5) and (6), and positing $x_{i\mathrm{m}}^k = K/g$, $x_{i\mathrm{M}}^k = K/G$, we can find party leaders' optimal choice of G and g. Given G, g, c_{M}, c_{m}, and the conjecture that all other voters spread votes equally, we can verify that equal spreading is a best response for a voter at the polls. Hence the solution is an equilibrium.

3.3. Equilibria for the experimental parametrizations

Fig. 1 shows the equilibrium cost cutpoints in the two parties, $\{c_{\mathrm{m}}, c_{\mathrm{M}}\}$, and the expected fraction of seats won by the minority under the two voting systems for a set of parameters that include those used in the experiment. The distribution $F(c)$ is Uniform over $[0,1]$, and thus the cost cutpoints are equal to the two parties' turnout rates. The first column

[17] See, for example, the discussion by fairvote.org at https://fairvote.org/cumulative-voting-a-step-towards-proportional-representation/ (accessed June 20, 2023).

[18] Throughout the paper, we use the convention $\binom{r}{y} = 0$ if $y > r$.

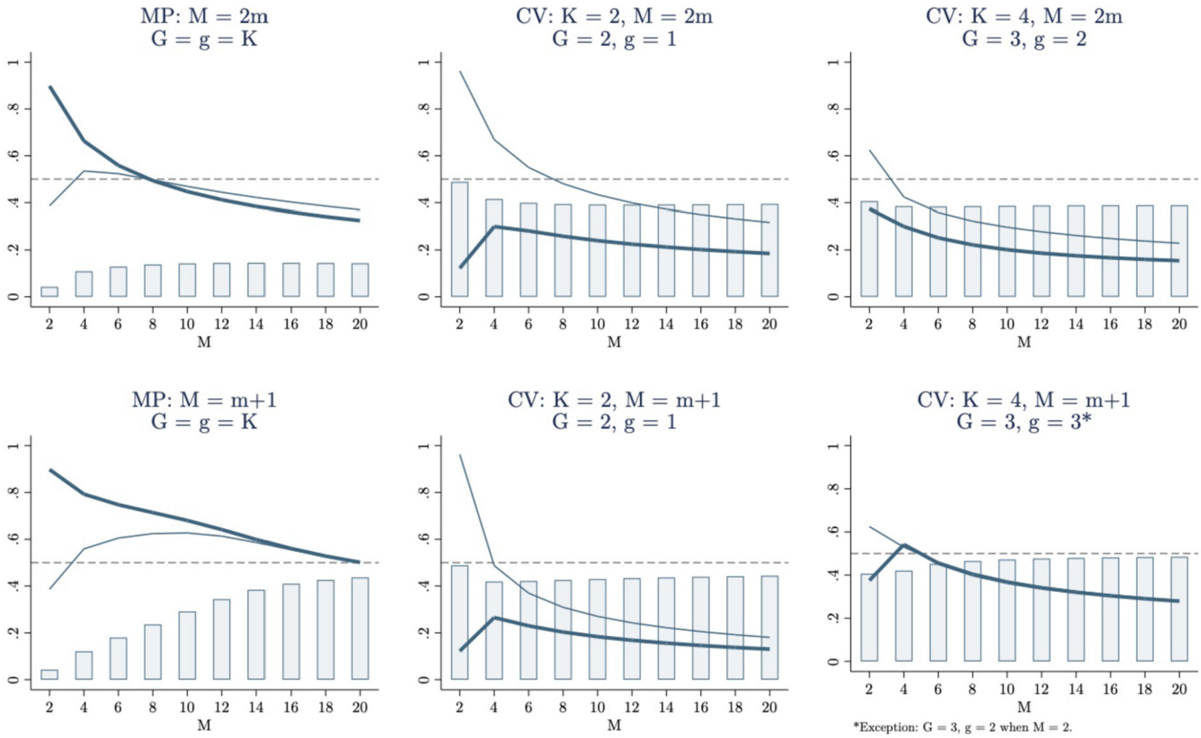

Fig. 1. *Expected turnout rates and share of minority seats, MP and CV.* The thick lines correspond to c_M, the thin lines to c_m; the bars correspond to the expected share of minority seats. F is uniform over $[0, 1]$; $V = 4$.

corresponds to MP, the second and third to CV (for $K = 2$ and $K = 4$, respectively; recall that K does not affect outcomes under MP). The number of candidates, G and g, equals K for MP and is set at equilibrium value for CV. In each panel, the horizontal axis corresponds to different values of M, while upper and lower panels refer to different relative sizes of the two parties.[19]

The figure highlights two main regularities. First, the differential between the minority's and the majority's turnout rates, $c_m - c_M$, is consistently higher under CV than under MP: CV leads to a higher expected presence of minority voters among those going to the polls. The results hold whether the minority is half the size of the majority or barely smaller; whether the number of open seats is just enough for CV to differentiate itself from MP ($K = 2$) or is higher ($K = 4$); whether the electorate is small or large, unless the difference in size of the two parties becomes negligible.[20] Second, the expected fraction of seats won by the minority is consistently higher under CV. The effect is most striking when the minority is relatively small ($M = 2m$), and its expected share of seats never rises above 14% under MP, less than half its share of the electorate, as opposed to being consistently close to 40% under CV.

In all cases, the minority party sets $g < K$ under CV, and thus exploits the possibility to cumulate votes. When $G = K$, the minority's cumulation of votes results in a higher probability of affecting the outcome, incentivizing turnout; when $G < K$, the difference in turnout probabilities is reduced, but the share of minority victories is boosted by the seats left uncontested by the majority.

The simulations focus on small size electorates because their purpose is to generate the hypotheses we test in the experiment. But how would the voting rules compare when the electorates are large? For MP, given its correspondence to single winner plurality systems, the theoretical predictions are known: turnout rates fall with the increase in population, but less so for the minority, whose probability of success increases with population size (for given population share) while remaining below 50 percent (Levine and Palfrey, 2007; Herrera et al., 2014). There is no corresponding theoretical analysis of turnout in large populations under CV. However, the regularities we see in our simulations match the theoretical results found by Herrera et al. for proportional representation: when the relative difference in size between the two groups persists in large electorates, the difference in turnout between the minority and the majority is consistently larger under proportional representation than under plurality. In our simulations, we observe the same results under CV when $M = 2m$, for larger values of M.[21] As mentioned earlier, the logic behind the turnout decision is different under proportional representation

[19] We found a unique equilibrium in all cases. We discuss in the Appendix the surprising lack of a consistent underdog effect ($c_m > c_M$) in the MP model. For both MP and CV, raising K to 6 does not change the qualitative results.

[20] If $M = m + 1$ and M is large, turnout equalizes for the two parties under both MP and CV.

[21] We have also run additional simulations with $M = 30$ and $M = 40$, confirming the qualitative results.

Table 1
Experimental Predictions. F uniform over $[0, 100]$; $V = 400$.

M, m	K	Rule	G, g	τ_m	τ_M	$(\tau_m - \tau_M)$	Expected Share Minority Seats
4, 2	2	MP	2, 2	0.54	0.66	-0.12	0.11
4, 2	2	CV	2, 1	0.67	0.30	0.37	0.42
4, 2	4	MP	4, 4	0.54	0.66	-0.12	0.11
4, 2	4	CV	3, 2	0.42	0.30	0.12	0.38
4, 3	2	MP	2, 2	0.56	0.79	-0.23	0.12
4, 3	2	CV	2, 1	0.49	0.27	0.22	0.42
4, 3	4	MP	4, 4	0.56	0.79	-0.23	0.12
4, 3	4	CV	3, 3	0.53	0.54	-0.01	0.42

and CV, but in both cases the comparison to MP reflects the smaller impact of a large electorate on the minority decision, because the marginal impact of an additional vote is larger in a smaller group in the case of proportional representation, or because of the positive impact of cumulated votes on pivotality in the case of CV.[22]

Finally, we can compare the results to minority victories in the absence of voting costs, and thus of turnout effects. Under MP and costless voting, as we know, the minority never wins any seat, as opposed to the small but positive share predicted with costly voting. Under CV and costless voting, the expected share of minority victories is $1/2$ if $m = M - 1 > K/2$, and either $1/3$ (if $K = 2$) or $1/4$ (if $K = 4$) if $m = M/2$, as opposed to being close to 40% in all such cases if voting is costly. Changing the voting system from MP to CV always helps the minority, but the dramatic effect expected if voting is costless is mitigated when voting is costly and turnout is not universal. When voting is costly, the minority achieves substantive representation under CV but is always expected to maintain its minority status in the allocation of seats.

4. The experiment

The experiment reproduces exactly the theoretical model. Our main focus is the impact of the voting rule on turnout, and especially on differential minority-majority turnout, and on the fraction of minority victories. To evaluate the robustness of the results and to test the power of the theoretical framework we implemented four different parametrizations: while we kept $M = 4$ throughout the experiment, we varied m between 2 and 3; for each m, we set $K = 2$ and $K = 4$. In all treatments, voting costs were drawn independently across participants from a uniform distribution with support $[0, 100]$, and V, the value of controlling all positions, was set at 400.

The number of candidates fielded by each party under CV was set at the theoretical equilibrium value for each parametrization, and we constrained voters who turned out to spread their votes equally over their party candidates. With our parameters, not only is equal spreading of votes an equilibrium response, but in the absence of distinguishing features among candidates or seats, a voter's unequal distribution of votes among the party's candidates could only reflect noise.[23] Participants acted as eligible voters: in each round, each drew an independent voting cost and decided whether or not to vote. The design thus mimics the numerical simulations, with $M = 4$. Denoting by τ_p the turnout rate of voters from party p, we reproduce in Table 1 the theoretical predictions for the experimental parametrizations.

The table replicates results from Fig. 1, reporting precise numerical values. It dictates the hypotheses to test. Our main focus is on comparative predictions on the effect of a change in voting rule, from MP to CV, on the minority versus the majority. We will discuss three hypotheses in particular. First, the differential of minority-majority turnout rates $(\tau_m - \tau_M)$ is strictly higher under CV than under MP. Second, a related but tighter hypothesis states that such differential is strictly negative under MP in all parametrizations; it is strictly positive under CV in three of the four parametrizations, and barely negative in the fourth. Third, in all parametrizations the expected share of seats won by the minority is higher under CV than under MP. CV benefits the minority through the difference in turnout and, when $K = 4$, because the majority should not, and in our experiment does not, contest all seats.

We conducted the experiment between August and October 2020, with participants recruited using the Columbia Experimental Laboratory for the Social Sciences (CELSS)' ORSEE website.[24] Most subjects were undergraduate students at Columbia University or Barnard College. All sessions were online due to the COVID-19 pandemic: participants received instructions and communicated with experimenters using the Zoom videoconferencing software, and accessed the experiment interface on their personal computer's web browser. The experiment was programmed in z-Tree (Fischbacher, 2007) and run virtually using z-Tree unleashed (Duch et al., 2020). Each experimental session lasted about 90 minutes with average earnings of $23. With the exception of a more visual style for the instructions, the experiment developed very similarly to in-person experiments in the lab.[25]

[22] Casella and Gelman (2008) study pivot probabilities in large electorates when voters can choose to cast more than a single vote. The problem analyzed–simultaneous referenda over multiple binary decisions–is different, but the effect of cumulation on pivot probabilities seems likely to generalize.

[23] Note that there is no communication among voters.

[24] Greiner (2015).

[25] The online Appendix contains a reproduction of the instructions.

Table 2
Experimental Design.

Sessions	m	Subjs	Rounds	K	Rule	Order	Sessions	m	Subjs
1,9	2	12	15	2	CV	1	5,13	3	14
			15	2	MP				
			15	4	MP				
			15	4	CV				
2,10	2	12	15	2	MP	2	6,14	3	14
			15	2	CV				
			15	4	CV				
			15	4	MP				
3,11	2	12	15	4	MP	3	7,15	3	14
			15	4	CV				
			15	2	CV				
			15	2	MP				
4,12	2	12	15	4	CV	4	8,16	3	14
			15	4	MP				
			15	2	MP				
			15	2	CV				

Table 3
Experimental Results: Summary Statistics. Standard errors are in parentheses.

M, m	K	Rule	G, g	τ_m	τ_M	$(\tau_m - \tau_M)$	Average Share Minority Seats
4, 2	2	MP	2, 2	0.40 (0.022)	0.64 (0.015)	-0.23 (0.029)	0.11 (0.018)
4, 2	2	CV	2, 1	0.64 (0.022)	0.64 (0.015)	0.00 (0.029)	0.26 (0.016)
4, 2	4	MP	4, 4	0.59 (0.022)	0.62 (0.016)	-0.02 (0.027)	0.16 (0.019)
4, 2	4	CV	3, 2	0.72 (0.020)	0.66 (0.015)	0.06 (0.027)	0.35 (0.008)
4, 3	2	MP	2, 2	0.57 (0.018)	0.69 (0.015)	-0.12 (0.024)	0.21 (0.023)
4, 3	2	CV	2, 1	0.65 (0.018)	0.57 (0.016)	0.07 (0.024)	0.40 (0.013)
4, 3	4	MP	4, 4	0.56 (0.018)	0.75 (0.014)	-0.20 (0.022)	0.13 (0.019)
4, 3	4	CV	3, 3	0.62 (0.018)	0.74 (0.014)	-0.12 (0.024)	0.35 (0.012)

During each session, party sizes were kept fixed, and participants played 15 consecutive rounds of each of four treatments, CV and MP for each of $K = 2$ and $K = 4$. Having multiple treatments within a session has two main purposes: it provides some control over idiosyncratic individual behavior and, equally important in this type of experiment, keeps the subjects engaged in what is otherwise a monotonous series of decisions. We controlled for the exact sequence of the treatments by varying their order: for given m, either 2 or 3, we ran two experimental sessions for each of four orders of treatments. Thus eight sessions were conducted with $m = 2$ (12 subjects per session), and eight with $m = 3$ (14 subjects per session), for a total of 208 experimental subjects. Table 2 reproduces the experimental design.

Party affiliations were kept constant within each treatment to facilitate learning but were assigned randomly across treatments. In each round, two groups were formed randomly, each composed of m minority and M majority members. At the end of the round, an outcome screen reported the party affiliations of the K winning candidates and the number of members of each party who had voted. Each participant's final earnings corresponded to the sum of their earnings from one randomly drawn round from each treatment (in addition to the \$5 show-up fee).

5. Experimental results

We begin by summarizing experimental results in Table 3, in the same format used for Table 1, for ease of comparison to the theoretical predictions.[26] We then discuss in more detail the experimental results on aggregate turnout rates–the most

[26] The table is descriptive and standard errors reported here are not corrected for possible correlations.

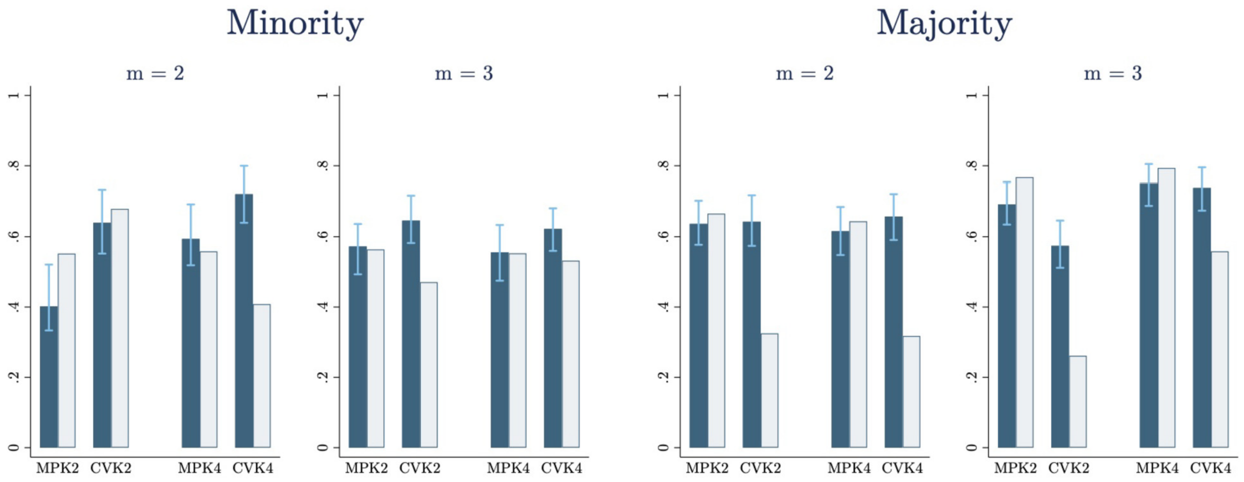

Fig. 2. *Turnout Frequencies.* The darker columns correspond to the data, the lighter ones to the theory. The 95% confidence intervals are calculated from 10,000 Monte Carlo simulations that allow for correlation in turnout decisions at the individual level.

novel contribution of this study, and on the share of seats won by the minority–the core outcome variable. We conclude by analyzing the source of these outcomes, i.e. individual turnout decisions.

Comparing the results to Table 1, some observations are immediate. First, with the exception of the minority when $m = 2$ and $K = 2$, aggregate turnout in the lab under MP is close to the theory in all parametrizations and for both parties; under CV, on the other hand, turnout is consistently and substantially higher than predicted in both parties, with larger disparity for the majority. Second, in line with the theory, changing the voting rule from MP to CV leads to an increase in differential minority-majority turnout: although the quantitative effect is smaller than predicted, $(\tau_m - \tau_M)$ is always higher under CV than under MP. In fact, as the theory predicts, the differential is negative in all MP parametrizations and positive, if weakly so, in three of the four CV parametrizations. Third, again as predicted, the share of seats captured by the minority is consistently higher under CV. We examine these results in more detail in what follows, and find them robust. They are a good summary of the experiment's main lessons.

5.1. Turnout

Fig. 2 reports aggregate turnout rates for minority and majority voters in the different treatments. The darker columns refer to the experimental data; the lighter columns to the theoretical predictions, calculated from the realized voting cost draws. To account for the presence of multiple decisions by the same participant, the 95% confidence intervals are calculated from 10,000 Monte Carlo simulations that allow for correlation in turnout decisions at the individual level.[27]

For all experimental values of K and m, minority turnout is higher under CV than under MP. The effect is particularly strong for $m = 2$, but remains positive, if more muted, for $m = 3$. In the case of the majority, turnout is effectively unchanged under CV or MP, with the exception of $m = 3$ and $K = 2$, where we see a decline under CV. In this latter parametrization, the minority fields one candidate, and the majority two; the majority is certain of one victory but, with a relatively large minority concentrating all votes on a single candidate, the chances of a second majority victory are low, discouraging turnout.

In fact, the observed decline in majority turnout when $m = 3$ and $K = 2$ is much smaller than the theory predicts. The disparity is less pronounced in the other parametrizations, but, with the exception of the minority when $m = 2$ and $K = 2$, turnout under CV is consistently higher than theory predicts for both parties. In particular, the grey columns in the figure, which report the theoretical predictions, show an expected decline in turnout for members of both parties in all parametrizations (again with the only exception of the minority when $m = 2$ and $K = 2$). Such generalized decline in turnout is absent from the data. The robustness of turnout under CV is the most unexpected finding of the experiment, and we return to it in Section 5.3, when we look in more detail at individual turnout decisions.

Given our focus on the potential role of CV in supporting the minority, the core theoretical prediction on turnout is the increase CV is expected to induce in *differential* minority to majority turnout rates $(\tau_m - \tau_M)$: in all parametrizations, such differential turnout is expected to be higher under CV than under MP. Fig. 3 reports the experimental results (the darker

[27] We populate the simulations with a subject's full set of 15 choices for the relevant treatment. Each simulation corresponds to drawing, with replacement, the correct number of minority and majority subjects corresponding to the treatment, with all their decisions, and generating one number for differential turnout (the difference in the frequency of Vote decisions in each party). Over 10,000 simulations, we construct a distribution of differential turnout. The 95% confidence intervals correspond to the boundaries of the 95% probability mass centered on the empirical ratio. Results remain substantively unchanged if we use standard automated programs for clustering standard errors.

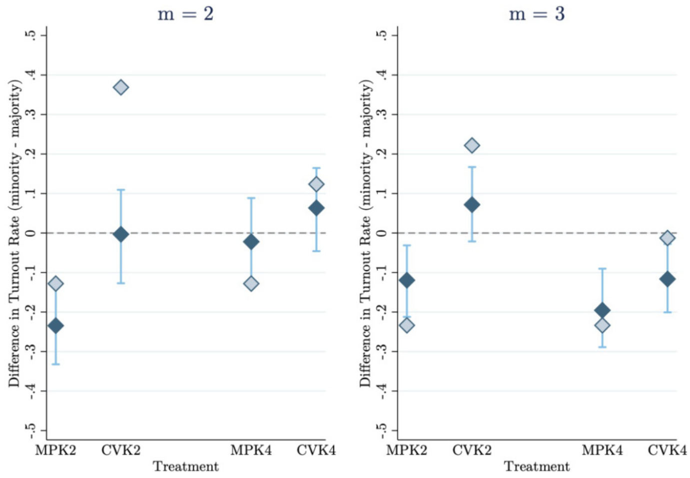

Fig. 3. *Differential Turnout*: $\tau_m - \tau_M$. Darker diamonds correspond to $\tau_m - \tau_M$ in the experiment, lighter diamonds to $\tau_m - \tau_M$ in the theoretical prediction. The 95% confidence intervals are calculated from 10,000 Monte Carlo simulations that allow for correlation in turnout decisions at the individual level.

diamonds) and the theoretical predictions (the lighter diamonds). In all cases, the differential minority to majority turnout is higher under CV than under MP.

The absolute magnitudes of the experimental effects are muted, relative to the theory, particularly in the $K = 2$ treatments, reflecting the higher than expected majority turnout. Comparative magnitudes, however, are roughly in line with predictions—more pronounced in $K = 2$ treatments, and more muted when $K = 4$. The theory also predicts that the minority's turnout rate should be lower than the majority's in all MP treatments, higher in three of the four CV treatments, and barely lower in the fourth. This too is observed in the experimental results.

We construct a non-parametric test of the significance of these results by comparing them to the corresponding simulations under the null hypothesis of no difference in behavior between minority and majority voters. For each treatment t—i.e., for each parametrization and voting rule—we call $n_1(t)$ the size of the minority sample, and $n_2(t)$ the size of the majority sample. We then combine minority and majority subjects in a single sample and construct two random groups, 1 and 2, labeled $g_1(t)$ and $g_2(t)$, by drawing subjects with replacement from the joint sample and assigning $n_1(t)$ random draws to group 1 and $n_2(t)$ to group 2.[28] We treat the samples in the two groups as if they corresponded to the minority and to the majority, and calculate differential turnout ($\tau_1 - \tau_2$). We repeat the procedure 10,000 times and obtain a distribution of differential turnout, under the hypothesis of no systematic difference in turnout between minority and majority subjects. The relevant p-value is the probability mass of the distribution at $(\tau_1 - \tau_2) \geq (\tau_m - \tau_M)$, where the latter is the difference in turnout rates observed in the data. We find: $p = 0.003$ ($m = 2, K = 2$; and $m = 3, K = 2$); $p = 0.137$ ($m = 2, K = 4$); $p = 0.135$ ($m = 3, K = 4$). The test confirms the visual evidence of the figure: the impact of CV on differential minority-majority turnout is clear in the $K = 2$ parametrizations; it is still present but smaller and thus less sharply identified when $K = 4$.

Further analysis supports these conclusions. In Table 4, for given parametrization, we report the estimation of a linear probability model where the subjects' turnout decisions are regressed on a dummy variable for minority status, a dummy variable for CV, and the interaction of the two dummies, controlling for voting costs, for the relative order of CV and MP, and for the round number.

The table tells us that, in all parametrizations, belonging to the minority decreases turnout under MP, relative to the majority voters, while shifting to CV has no impact on majority voters' turnout, with the exception of the case $m = 3$, $K = 2$, when the majority turnout declines. At the center of our predictions, the positive parameter of the interaction term tells us that shifting to CV has a larger positive effect (or a smaller negative effect) on the turnout of minority voters relative to majority voters, The effect is precisely estimated when $K = 2$, although not in the more complex treatments with $K = 4$. As expected, higher voting costs decrease turnout, but whether subjects see CV before or after MP has no noticeable effect on the decision to vote, although as a session proceeds, turnout slightly decreases.

These qualitative results are robust. As we report in the online Appendix, they remain unchanged if we estimate the model using only data where the relevant treatment was the one the subjects saw first—i.e., where we exploit the changing order of treatments across sessions to create an in-between subjects design. Results also remain unchanged if we estimate a probit model, rather than a linear probability model, with or without random effects.[29]

[28] To allow for correlation of individual decisions, each subject is drawn with all 15 rounds of turnout decisions and voting costs.

[29] Interacting the order of treatments with CV (i.e., adding a variable CV × CVfirst) has no effect. Changing the level at which standard errors are clustered affects statistical significance but does not change the main message. Focusing on the interaction term CV × Minority, and thus on the effect of CV on differential turnout, clustering at the group level leads to a decline in all estimated standard errors, and thus an increase in significance; clustering at the session level increases estimated standard errors for the $m = 2$ parametrizations and reduces them when $m = 3$. See the online Appendix for all alternative results and more discussion.

Table 4
Individual turnout decisions. The default sample is majority subjects under MP. Standard errors are clustered at the individual level.

	(1)	(2)	(3)	(4)
	m=2, K=2	m=2, K=4	m=3, K=2	m=3, K=4
Minority	-0.229***	-0.042	-0.144***	-0.202***
	(0.052)	(0.052)	(0.045)	(0.052)
CV	0.008	0.029	-0.130***	-0.027
	(0.035)	(0.035)	(0.038)	(0.030)
CV × Minority	0.227***	0.118	0.218***	0.091
	(0.078)	(0.072)	(0.069)	(0.061)
Voting Cost	-0.007***	-0.008***	-0.009***	-0.008***
	(0.001)	(0.001)	(0.000)	(0.001)
CVfirst	-0.057	-0.029	0.029	-0.006
	(0.047)	(0.047)	(0.040)	(0.040)
Round	-0.008***	-0.003	-0.008***	-0.005***
	(0.002)	(0.002)	(0.002)	(0.002)
Constant	1.095***	1.048***	1.201***	1.189***
	(0.039)	(0.047)	(0.033)	(0.037)
R^2	0.239	0.228	0.296	0.268
Number of observations	2880	2880	3360	3360
Number of clusters	96	96	112	112

Standard errors in parentheses.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

The regression results mirror closely what we see in the figures. Lower minority turnout under MP is less precisely estimated when $m = 2$ and $K = 4$; the decline in majority turnout under CV is only detectable for the case $m = 3$, $K = 2$, and the positive effect of CV on differential minority-majority turnout is strongly significant for the $K = 2$ treatments, but not when $K = 4$. These are the regularities we also see in the figures.

We use the simulations reported earlier, estimating differential turnout under the null hypothesis of no difference in behavior across minority and majority subjects, to construct a placebo test. The simulations yield turnout rates for group 1 and group 2, the two groups generated randomly from our data, for each treatment, by drawing subjects without distinguishing by party. For each simulation, we replicate the regression in Table 4; over 10,000 simulations, we generate a distribution of the parameter of the interaction term, CV × group 1, under the null of no difference in the population. Fig. 4 reports such distribution for each parametrization, as well as the 95% confidence interval around the distribution mean, and, with a thicker black line, the parameter estimated in the original regression.

Under the null of no systematic differences in turnout decisions between minority and majority voters, the probability of estimating a differential effect of CV on the minority equal or larger to the estimate in our original regression is effectively zero in both $K = 2$ treatments ($p < 0.001$); such probability is slightly larger in the $K = 4$ treatments but still below conventional significance levels ($p = 0.031$ when $m = 2$; $p = 0.048$ when $m = 3$). The placebo test suggests strongly that minority and majority members do in fact respond differently to the shift in voting rule from MP to CV. As a result, differential minority-majority turnout is higher under CV than under MP, an effect we cannot attribute to randomness.

5.2. Minority victories

Did CV help the minority secure more seats? Fig. 5 shows that the answer is positive.

For every parametrization, CV increases the share of seats won by the minority, and does so very significantly. There are some disparities relative to the theory: the minority fares better than expected under MP in the $m = 3$, $K = 2$ treatment, and less well than expected under CV in the $m = 2$, $K = 2$ treatment. On the whole, however, the results are in line with predictions: not only do minority victories increase under CV in all cases, but the magnitude of the change is large: in all parametrizations, the share of seats won by the minority doubles or more than doubles when shifting from MP to CV.[30]

The message of the figure is confirmed by the statistical analysis. Table 5 reports the results of regressing, for each parametrization, the share of seats won by the minority on a dummy variable for CV, controlling for the relative order of CV and MP, for an interaction term between the order and CV (CV × CVfirst), and for the round number. In all parametrizations, CV consistently and substantially increases the share of minority victories. The order of treatments per se is not significant. In the case of the $m = 2$ and $K = 2$ parametrization, the effect of CV on increasing minority victories is stronger in sessions where CV was run after MP. However, a Wald test confirms that even when $m = 2$ and $K = 2$, the joint coefficient of (CV + CV × CVfirst) remains significantly different from zero at conventional significance levels ($p = 0.0412$). More transparently still, regressions run on data restricted to the first treatment in each session confirm that the positive and significant effect

[30] Although, as predicted, such share remains under 50% in all cases.

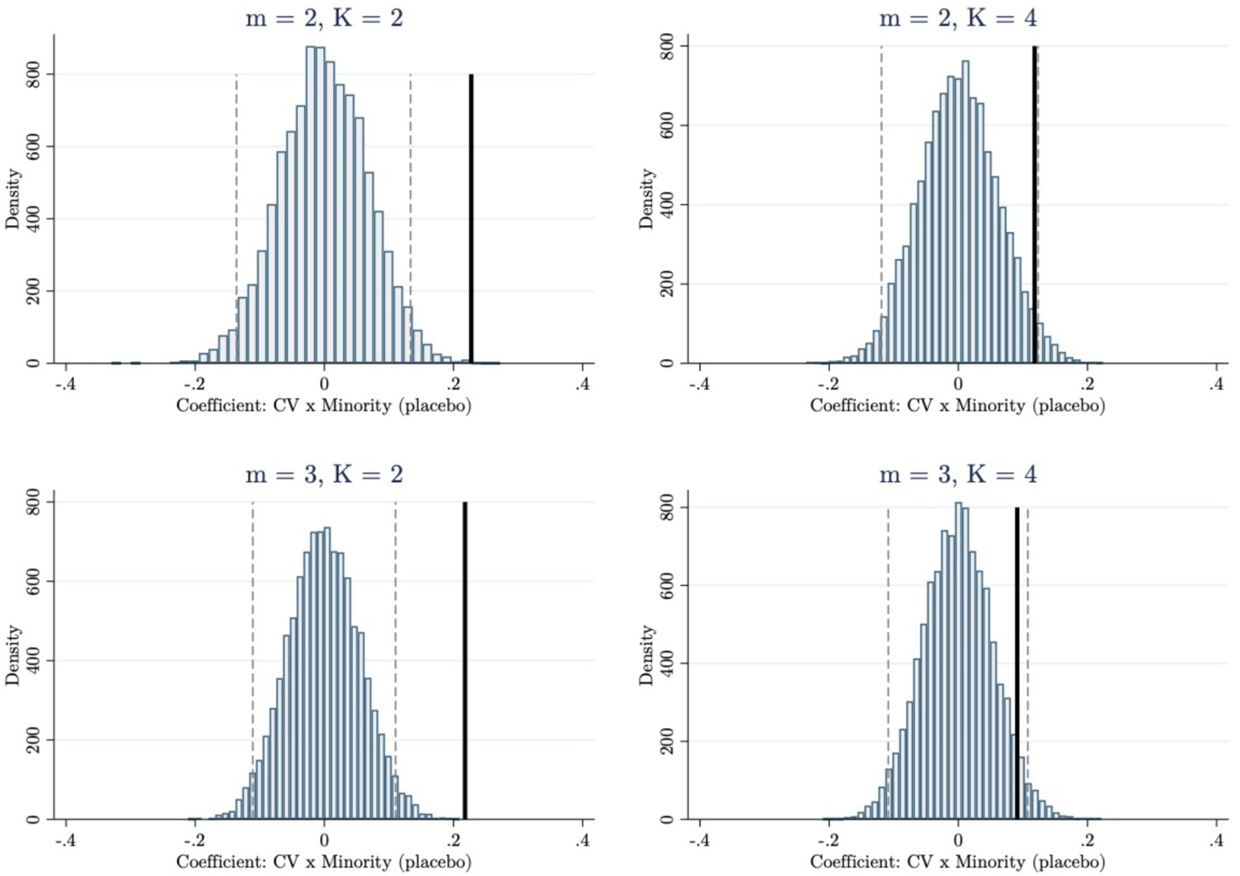

Fig. 4. *The effect of CV on differential minority-majority turnout rates. A placebo test.* Results of the original regressions (the thicker black lines) versus 10,000 replications with random group formation. The dotted lines correspond to the 95% confidence interval around the distribution mean.

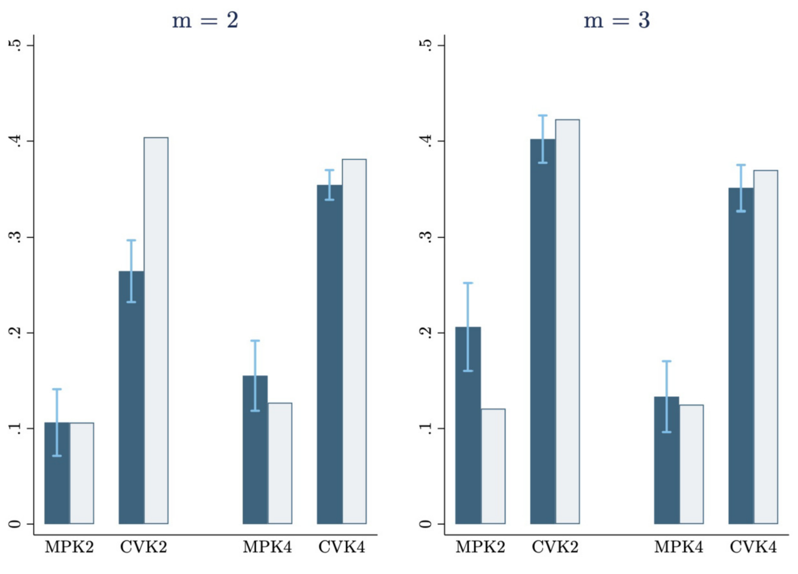

Fig. 5. *Share of seats won by the minority.* Darker columns correspond to the data, lighter columns to the theory. The 95% confidence intervals are calculated from standard errors clustered at the level of the voting group.

Table 5
Share of seats won by the minority. Standard errors are clustered at the level of the voting group.

	(1)	(2)	(3)	(4)
	m=2, K=2	m=2, K=4	m=3, K=2	m=3, K=4
CV	0.241***	0.202***	0.230***	0.224***
	(0.032)	(0.031)	(0.037)	(0.028)
CVfirst	0.045	-0.003	0.069	0.059
	(0.036)	(0.037)	(0.047)	(0.038)
CV × CVfirst	-0.164***	-0.005	-0.068	-0.011
	(0.050)	(0.040)	(0.054)	(0.045)
Round	-0.001	-0.000	-0.002	0.001
	(0.003)	(0.002)	(0.003)	(0.002)
Constant	0.094**	0.161***	0.188***	0.098***
	(0.036)	(0.032)	(0.039)	(0.030)
R^2	0.094	0.165	0.082	0.131
Number of observations	1649	1641	1870	1861
Number of clusters	387	414	448	448

Standard errors in parentheses.
$^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

of CV on minority victories is very robust: as we show in the online Appendix, the finding is confirmed in such restricted data set for all treatments, including when $m = 2$ and $K = 2$.[31] Under CV, the possibility to cumulate votes combines with the increase in differential minority-majority turnout to deliver larger influence to the minority.

5.3. Individual turnout decisions: monotonicity violations and cutpoints

Aggregate group outcomes—turnout rates and shares of seats won—are the main variables of interest. But group outcomes are rooted in individual turnout decisions. In this subsection, we briefly discuss the experimental evidence on individual behavior.

In costly voting experiments, and more broadly in experiments where the equilibrium is in monotone cutpoint strategies, violations of monotonicity are informative not only about the accuracy of the theoretical predictions but also about the participants' understanding of the rules of the game. In our experiment, this is particularly important because a common objection to CV is that its strategic complexity is a difficult obstacle for voters. Although experimental participants limit themselves to the decision to turn out or not, the fact that turning out under CV implies casting multiple votes for each candidate, and in one case fractional votes, could be confusing.

For each participant, we calculated the number of monotonicity violations, defined as the minimum number of decisions that would need to be modified for that participant's turnout to be fully monotonic in the voting cost realization in a given treatment. If i chooses to turn out for a cost realization $c_i = c'$ then i should turn out for all $c_i < c'$, and if i chooses to abstain for a cost realization $c_i = c''$ then i should abstain for all $c_i > c''$. In our data, monotonicity violations are not common: in all treatments more than half of participants have at most a single violation. In fact, over the full data set, 75 percent of participants have at most one violation. Most importantly, there is no systematic difference between the frequency of violations under MP and under CV. As we show more formally in the online Appendix, at least in the simplified structure of our experiment, the hypothesis that CV is more confusing for voters is not supported by any evidence of more random behavior.[32]

We can use the minimization of monotonicity violations as a guide to estimating individual cost cutpoints. Fig. 6 reports, for each subject, the cutpoint that minimizes the frequency of violations.[33] Because the theory predicts different behavior depending on party affiliation, the figure reports estimated cutpoints separately for each party. In all cases, the darker diamonds correspond to the average of the individual cutpoints, and the lighter diamonds to the theoretical prediction.

The figure shows clearly the high heterogeneity in behavior: estimated cutpoints vary across individuals, in both parties and for both voting rules. The theoretical semi-symmetric equilibrium predicts a single cutpoint for each party, a prediction clearly violated by the data. The heterogeneity we see, however, is in line with previous findings from similar experiments.[34]

[31] See the online Appendix, where we also document that results are unchanged when clustering standard errors at the session level.

[32] We report in the online Appendix the fraction of individuals with different numbers of monotonicity violations, represented as separate CDF's for MP and CV. The two CDF's are barely distinguishable, and Kolmogoroff-Smirnov (K-S) two-sample tests, corrected for discreteness, cannot reject the hypothesis of a common population.

[33] See, for example, Casella et al. (2006) or Levine and Palfrey (2007) for a similar approach. When, for a given subject, multiple cutpoints are consistent with minimizing monotonicity violations, the figure reports the mean cutpoint. In a few cases (15 subjects out of 208), the multiplicity concerns ranges of possible cutpoints; in these cases reporting the mean would muddle behavior, and we have chosen the range that is closest to equilibrium. In all cases, we have verified that alternative choices do not change the qualitative results.

[34] For example, Levine and Palfrey (2007). We checked whether restricting data to the last 10 rounds of each treatment would result in a narrower range of estimates. We found no such effect: the range of estimated cutpoints is virtually identical.

Fig. 6. *Estimated Cost Cutpoints.* The darker diamonds correspond to the average of the individual cutpoints, and the lighter diamonds to the theoretical prediction. The size of each circle is proportional to the number of subjects it represents.

Comparing the dispersion of cutpoints across the two voting rules, we find very similar dispersion across voting rules and treatments, both for the minority and for the majority. The visual impression is confirmed by the standard deviations of the corresponding distributions of cutpoints.

When we aggregate individual behavior into average cutpoints, regularities emerge. Under MP, the average of the estimated individual cutpoints is remarkably close to the equilibrium cutpoint in all parametrizations, with the single exception of the minority when $m = 2$ and $K = 2$. With the same single exception, under CV, the average cutpoint is above the equilibrium cutpoint in both parties and in all parametrizations, indicating more frequent participation in voting than theory predicts. When $m = 2$ and $K = 2$, the average minority cutpoint in the data is below the equilibrium cutpoint for MP (suggesting less participation), and coincides with the equilibrium for CV.

A different way of visualizing estimated individual cutpoints conveys clearer lessons on the impact of the voting rule. Fig. 7 reports the CDF's of the individual cutpoints, comparing the CDF's under MP (the lighter line) and under CV (the darker line), for both parties and all parametrizations.

The minority's higher propensity to vote under CV operates throughout the distribution of individual cutpoints. The move to CV causes a shift rightward of the whole distribution: the minority cutpoints distribution under CV FOSD's the distribution under MP. Formal tests strongly reject the hypothesis of equal distributions for the minority when $m = 2$ and

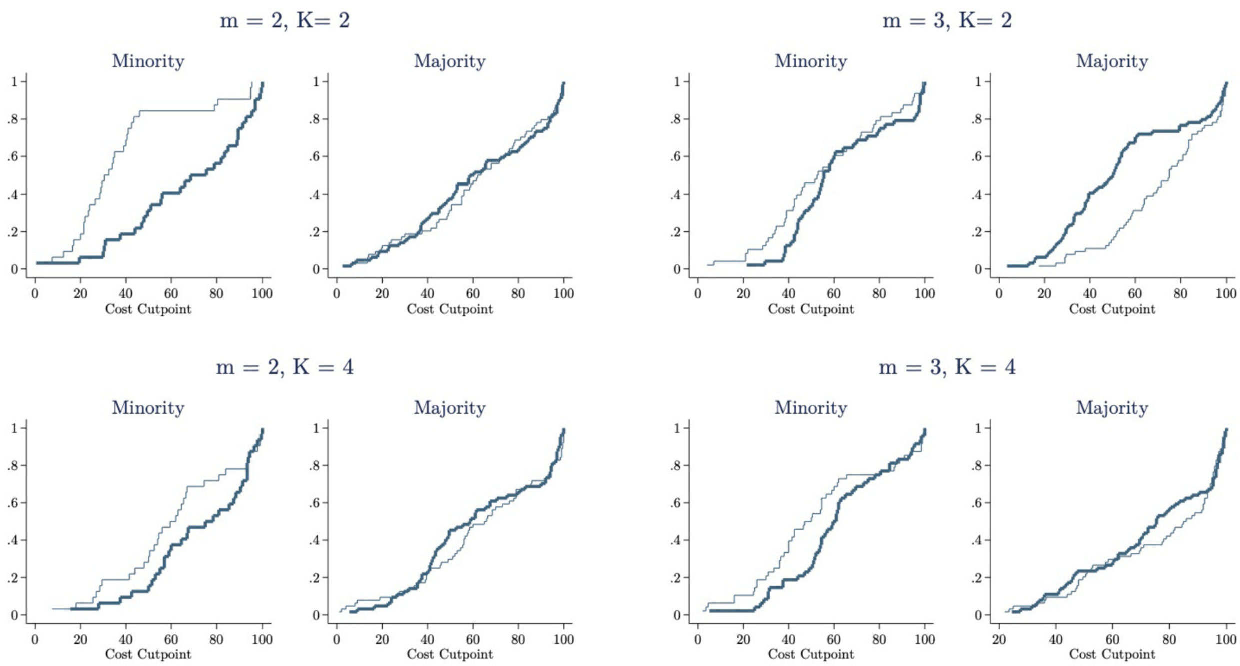

Fig. 7. *CDF's of Cost Cutpoints.* The darker lines correspond to CV; the lighter lines to MP.

$K = 2$ (a two-sample K-S test yields $p < 0.001$), with weaker evidence when $m = 3$ and $K = 4$ ($p = 0.05$) and when $m = 2$ and $K = 4$ ($p = 0.26$).

As for the majority, with $m = 2$, majority members barely modify their propensity to vote. With $m = 3$, the conclusion is similar when $K = 4$; when $K = 2$, however, we see a consistent decline in voting under CV throughout the majority's cutpoint distribution. When $m = 3$, $K = 2$, as we discussed earlier, under CV the majority's chances of winning both seats are very small and that realization depresses turnout: the distribution of majority cutpoints under MP FOSD's the distribution under CV, and the corresponding one-sided K-S test yields $p < 0.001$.[35]

The analysis of individual behavior in the experiment thus delivers four main lessons. First, we do not see more random behavior under CV than under MP, as captured by the frequency of monotonicity violations. Such violations are few under both voting rules. Second, estimated individual cutpoints are heterogeneous, with comparable dispersion across parametrizations, parties, and voting rules. Third, but for a single exception (the minority, when $m = 2$ and $K = 2$), average cutpoints are close to the theoretical predictions for MP, but consistently higher than predicted under CV, especially, but not exclusively, for the majority. Fourth, plotting CDF's of cutpoints highlights participants' differential behavior under the two voting rules. The distribution of minority party subjects' cutpoints shifts to the right (i.e., towards higher turnout) under CV, relative to MP; it moves less, and when it does it is in the opposite direction, for majority party subjects.

6. Conclusions

Cumulative Voting (CV) is a voting system for multi-member districts that allows each voter to cumulate votes freely on a single or a subset of candidates. It has been in use since the 19th century in different countries, for example in the US, England, and Scotland, for the election of both political bodies and corporate boards. Because it delivers semi-proportional outcomes without imposing a proportional representation system, CV favors the representation of minorities while remaining familiar and acceptable to majoritarian democratic systems. In the US, it is one of the remedies imposed by the courts to resolve violations of the Voting Rights Act. Adoption of CV correlates empirically with higher participation and success of minorities. However, because such adoption typically follows litigation, and thus heightened engagement of minorities, it is important to complement the historical experiences with experiments. This is the purpose of the present study.

We use a traditional experimental design with costly voting to test predictions on turnout and on the electoral success of the minority under two voting systems: Multi-seat Plurality (MP), or bloc voting, where each voter can only cast one vote per candidate, and CV. Inspired by historical episodes, we assume that the coordination problem posed by CV is solved by the party leaders. The leaders choose the number of party candidates optimally, in an equilibrium in which votes are spread evenly over all party candidates. Experimental subjects are voters who face a private, individual cost of voting, and must decide whether or not to turn out.

Although we see more individual heterogeneity in behavior than expected, aggregate data under MP are well predicted by the theoretical model. We find, however, that the model significantly underpredicts turnout under CV for both parties, with a larger quantitative discrepancy for the majority. Given the good performance of the model under MP, the finding is puzzling. Why the underprediction under CV only? Without claiming to have an answer, one possible conjecture is that experimental subjects correctly perceive CV as a more competitive system, giving the minority a higher chance of winning seats. The enhanced competition then leads to higher participation, in the same spirit as overbidding in auction experiments.[36] We leave the conjecture open for future work.

The higher turnout under CV does not invalidate the model's predictions about the relative impact of the voting system on minority and majority voters. For the cases we bring to the lab, the theory says that CV should be associated with an increase in the turnout of the minority, relative to the majority, and with an increase in the share of seats won by the minority. Both predictions are supported by our data in all the experimental parametrizations we study. In our experiment, CV works as expected to magnify the voice of the minority.

As we write, debates over voting rights rage in Congress, in state legislatures, in the courts, in the media. Initiatives aimed at limiting access to the polls and partisan redistricting following the 2020 census increase fears of disenfranchisement. Encouraging minority turnout is a high priority. Voting rules like CV have the potential to help. Although much remains to be studied, in the lab we find that such potential is fulfilled.

Declaration of competing interest

The Columbia Program for Economic Research and the Columbia Experimental Laboratory for the Social Sciences provided financial support for this research. The experiment was approved under Columbia IRB Protocol AAAS8614.

Alessandra Casella, Jeffrey D. Guo and Michelle Jiang all declare no conflict of interest relating to this research.

[35] The test cannot reject the hypothesis of equal distributions of majority cutpoints in the other cases.

[36] A variation of the "spite" argument invoked to explain overbidding in second price auctions (Andreoni et al., 2007), could then justify the majority's higher excess turnout.

Data availability

All our data and programs are available on the Open Science Framework at: https://osf.io/ytusv/.

Appendix A

A.1. CV when voting is costless

Proposition 1. *In the absence of voting costs, in all party-optimal equilibria of the CV voting game: (i) for all $m < M/K$, the minority never wins any seat; (ii) for all $m \geq M/K$:*

$$z = \left\lfloor \frac{Km+m}{M+m} \right\rfloor \qquad \text{if } \frac{Km+m}{M+m} \notin \mathbb{Z}$$

$$z = \begin{cases} \frac{Km+m}{M+m} - 1 & \text{with prob } m/(m+M) \\ \frac{Km+m}{M+m} & \text{with prob } M/(m+M) \end{cases} \qquad \text{if } \frac{Km+m}{M+m} \in \mathbb{Z}$$

Proof. We establish the proposition by proceeding in three steps.

1. First, we note that the identity of purpose between party leaders and voters implies that in all party-optimal equilibria we can think of the party leaders as controlling not only the number of party candidates but also the distribution of votes cast by their party voters.

2. Second, we show that the proposition identifies the number of seats won by the minority when both parties follow maximin strategies.

(i) Suppose first that $m < M/K$. Then the M party can guarantee itself all K seats by dividing its votes equally over K candidates, and the m party cannot win any seat.

(ii) Suppose then $m > M/K$. For any x_{M}, party m maximizes the probability of winning z seats by dividing its votes equally over z candidates, and guarantees itself z seats if $mK/z > MK/(K-z+1)$, or $z < (Km+m)/(M+m)$. At the same time, party M maximizes the probability of winning $(K-z)$ seats by dividing its votes equally over $K-z$ candidates, and guarantees itself $K-z$ seats if $MK/(K-z) > mK/(z+1)$, or $z > (Km-M)/(M+m)$. We require z to be an integer. Note that $(Km-M)/(M+m) = (Km+m)/(M+m) - 1$. Hence either both $(Km+m)/(M+m)$ and $(Km-M)/(M+m)$ are integers, or neither one is an integer.

(ii.a) Suppose first that $(Km+m)/(M+m)$ is not an integer. Party m guarantees itself $\left\lfloor \frac{Km+m}{M+m} \right\rfloor$ seats, and party M guarantees itself $K - \left\lfloor \frac{Km+m}{M+m} \right\rfloor$ seats.

(ii.b and iii) Finally, suppose that either $m > M/K$ and $(Km+m)/(M+m)$ is an integer, or $m = M/K$ (and thus $(Km+m)/(M+m) = 1$). Then the m party can guarantee itself $(Km+m)/(M+m) - 1 \equiv \underline{z}$ seats, but can do better by spreading votes equally over $(Km+m)/(M+m) \equiv \overline{z}$ candidates. Similarly, the M party can guarantee itself $K - \overline{z}$ seats, but can do better by spreading votes equally over $K - \overline{z} + 1 = K - \underline{z}$ candidates. In equilibrium then, party m (M) spreads its votes equally over $\overline{z}$ $(K - \underline{z})$ candidates; a total of $K+1$ candidates receive votes, and all are tied with $[K/(K+1)](M+m)$ votes each. The tie-break rule selects K winners randomly from the $K+1$ candidates. It then follows that:

$$prob(z = \overline{z}) = \frac{\binom{K-\underline{z}}{K-\overline{z}}}{\binom{K+1}{K}} = 1 - \frac{\overline{z}}{K+1} = \frac{M}{m+M}$$

$$prob(z = \underline{z}) = 1 - prob(z = \overline{z}) = \frac{m}{m+M}$$

3. Because $u(k)$ is linear in k, the game is constant-sum. It follows that party-optimal equilibria are equilibria of a constant sum, two-player game. Hence, if equilibria exist, they must all yield maximin payoffs. It is not difficult to verify that the strategies described above are equilibria: neither party has a profitable deviation. Thus equilibria exist and all yield z minority victories. □

We complement the proof with two observations. First, as noted in the text, for given K, m, and M, in general multiple party-optimal equilibria exist, with different numbers of candidates and/or distributions of votes. For example, suppose $K = 4$, $m = 3$, $M = 6$, and describe an equilibrium by a vector $\{g, G, \{x_{\mathrm{m}}^k\}, \{x_{\mathrm{M}}^k\}\}$. Then $\{1, 3, \{12\}, \{8, 8, 8\}\}$ is an equilibrium. But so are $\{1, 4, \{12\}, \{6, 6, 6, 6\}\}$; $\{1, 3, \{12\}, \{10, 7, 7\}\}$; $\{1, 3, \{12\}, \{9, 8, 7\}\}$; $\{2, 3, \{8, 4\}, \{8, 8, 8\}\}$; $\{2, 3, \{8, 4\}, \{10, 7, 7\}\}$;

$\{1, 3, \{8, 4\}, \{9, 8, 7\}\}$, and there are many others. In all party-optimal equilibria, however, as the proposition states, $z = 1$ in this example.[37]

Second, to clarify the logic of the CV game, it is useful to differentiate it from a Colonel Blotto game, adapted to the parameters used here. In the Blotto game, two players, with Km and KM tokens respectively, simultaneously distribute them over K boxes; each player earns one point for each box in which the player's tokens are more numerous than the opponent's. In the CV game, each of the two players, again endowed with Km and KM tokens respectively, has a separate set of K boxes over which to distribute the tokens; the K boxes with most tokens are chosen, out of the total $2K$ boxes, and each player earns 1 point for each box chosen out of the player's own set of K. The two games are different. For example, in the Blotto game, the equilibrium typically requires mixed strategies, and the player with fewer tokens cannot be guaranteed any points; neither statement applies to the CV game.

A.2. Costly voting

A.2.1. Multi-winner plurality (MP). Pivot probabilities and probabilities of winning seats

We report here the binomial formulas for the pivot probabilities. Under MP, such formulas are well-known (see for example Levine and Palfrey, 2007).

$$\pi_{\mathrm{m}}^{T-1} = \sum_{x=0}^{m-1} \binom{m-1}{x} \binom{M}{x+1} F(c_{\mathrm{m}})^x [1 - F(c_{\mathrm{m}})]^{m-1-x} F(c_{\mathrm{M}})^{x+1} [1 - F(c_{\mathrm{M}})]^{M-(x+1)}$$

$$\pi_{\mathrm{m}}^{T} = \sum_{x=0}^{m-1} \binom{m-1}{x} \binom{M}{x} F(c_{\mathrm{m}})^x [1 - F(c_{\mathrm{m}})]^{m-1-x} F(c_{\mathrm{M}})^{x} [1 - F(c_{\mathrm{M}})]^{M-x}$$

and:

$$\pi_{\mathrm{M}}^{T-1} = \sum_{x=1}^{m} \binom{m}{x} \binom{M-1}{x-1} F(c_{\mathrm{m}})^x [1 - F(c_{\mathrm{m}})]^{m-x} F(c_{\mathrm{M}})^{x-1} [1 - F(c_{\mathrm{M}})]^{M-1-(x-1)}$$

$$\pi_{\mathrm{M}}^{T} = \sum_{x=0}^{m} \binom{m}{x} \binom{M-1}{x} F(c_{\mathrm{m}})^x [1 - F(c_{\mathrm{m}})]^{m-x} F(c_{\mathrm{M}})^{x} [1 - F(c_{\mathrm{M}})]^{M-1-x}$$

The frequency of minority victories is sensitive to the relative turnout rates of the two parties, captured by the two cutpoints c_{m} and c_{M}. Although the study of costly voting models has identified an "underdog effect"—the tendency for the minority's turnout rate to be higher than the majority's, or $c_{\mathrm{m}} > c_{\mathrm{M}}$—the existence of such an effect is sensitive to the exact specification of the model. It has been proven in a number of scenarios: when the voting cost is fixed and equal for all (Taylor and Yildirim, 2010a); when voters' direction of preferences is randomly drawn (Ledyard, 1984; Taylor and Yildirim, 2010b); when the size of the electorate is uncertain (Herrera et al., 2014; Krishna and Morgan, 2015). The specification used here differs from these models, and relative turnout under MP depends on V, the value of winning all seats. Because the model is widely used but this observation is missing from the literature, we make it explicit in the following remark.

Remark. For any finite $M \geq m$, and F continuous and atomless over support $[\underline{c}, \overline{c}]$, with $\underline{c} \geq 0$, there exists a finite $\widehat{V}(M, m)$ such that if $V = \widehat{V}$, then there exists an equilibrium with $c_{\mathrm{m}} = c_{\mathrm{M}}$.

Proof. Call $\widehat{c}$ the median of $F(c)$. Straightforward manipulations of the pivot probabilities show that if $c_{\mathrm{m}} = c_{\mathrm{M}} = \widehat{c}$, and thus $F(c_{\mathrm{m}}) = 1 - F(c_{\mathrm{M}}) = 1/2$, then $(\pi_{\mathrm{m}}^T + \pi_{\mathrm{m}}^{T-1}) = (\pi_{\mathrm{M}}^T + \pi_{\mathrm{M}}^{T-1}) = (1/2)^{M+m-1}\binom{M+m}{m}$. Hence for any M and m, $c_{\mathrm{m}} = c_{\mathrm{M}} = \widehat{c}$ is an equilibrium as long as $\widehat{c} = (V/2)(1/2)^{M+m-1}\binom{M+m}{m}$, or $V = \widehat{c}\left(2^{(M+m)}/\binom{M+m}{m}\right) = \widehat{V}$. $\square$

The derivation of the probabilities of winning different numbers of seats is straightforward. Consider the problem from the perspective of a minority voter. Begin with the probability of losing all positions, $\Pr(W_{\mathrm{m}} = 0)$. Such probability equals the probability that either all minority candidates receive strictly fewer votes than the majority candidates, or that all candidates are tied but minority candidates lose all tie-breaks. Or, $\Pr(W_{\mathrm{m}} = 0) = \Pr(S_{\mathrm{m}} < S_{\mathrm{M}}) + \Pr[(S_{\mathrm{m}} = S_{\mathrm{M}}) \cap (\mathrm{m} \text{ loses all tie-breaks})]$. That is:

[37] Other equilibria exist that are not party-optimal, where the lack of coordination by the voters of one of the parties prevents it from winning all the seats it could win. In the example above, $\{2, 3, \{6, 6\}, \{12, 12, 0\}\}$ is an equilibrium: majority voters fail to coordinate and because each only holds 4 votes, no profitable individual deviation exists. Each party wins two seats.

$$\Pr(W_{\mathrm{m}}=0)=$$
$$=\sum_{S_{\mathrm{M}}=1}^{M}\binom{M}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}\sum_{S_{\mathrm{m}}=0}^{S_{\mathrm{M}}-1}\binom{m}{S_{\mathrm{m}}}F(c_{\mathrm{m}})^{S_{\mathrm{m}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{m}}}+$$
$$+\sum_{S_{\mathrm{M}}=0}^{M}\binom{M}{S_{\mathrm{M}}}\binom{m}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}\times$$
$$\times F(c_{\mathrm{m}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{M}}}\left(1/\binom{2K}{K}\right)$$

Similarly, the probability that m wins all positions, $\Pr(W_{\mathrm{m}}=K)$ equals the probability that either all minority candidates receive strictly more votes than the majority candidates, or that all candidates are tied but minority candidates win all tie-breaks. Or, $\Pr(W_{\mathrm{m}}=K)=\Pr(S_{\mathrm{m}}>S_{\mathrm{M}})+\Pr[(S_{\mathrm{m}}=S_{\mathrm{M}})\cap(\text{m wins all tie-breaks})]$. That is:

$$\Pr(W_{\mathrm{m}}=K)=$$
$$=\sum_{S_{\mathrm{M}}=0}^{m-1}\binom{M}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}\sum_{S_{\mathrm{m}}=S_{\mathrm{M}}+1}^{m}\binom{m}{S_{\mathrm{m}}}F(c_{\mathrm{m}})^{S_{\mathrm{m}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{m}}}+$$
$$+\sum_{S_{\mathrm{M}}=0}^{M}\binom{M}{S_{\mathrm{M}}}\binom{m}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}\times$$
$$\times F(c_{\mathrm{m}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{M}}}\left(1/\binom{2K}{K}\right)$$

The probabilities of other numbers of minority victories can be derived in the same fashion. The probability of electing w minority candidates, with $w\in(0,K)$ equals the probability that all candidates are tied and m wins w tie-breaks. Thus:

$$\Pr(W_{\mathrm{m}}=w)=$$
$$=\sum_{S_{\mathrm{M}}=0}^{M}\binom{m}{S_{\mathrm{M}}}F(c_{\mathrm{m}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{M}}}\times$$
$$\times\binom{M}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}\binom{K}{w}\binom{K}{K-w}/\binom{2K}{K}$$

For given M, m, K, $F(c)$, and $\{u(k)\}$, for $k\in\{0,1,..,K\}$, the equilibrium yields expected turnout rates for voters of the two parties, the probabilities of winning $0,1,..,K$ positions for each party, and ex ante expected utility for an M and an m voter.[38]

A.2.2. Cumulative Voting (CV). Pivot probabilities and probabilities of winning seats

Consider first the perspective of a majority voter. The pivot probabilities correspond to the probabilities of the three events described in the text: breaking a tie (if $(K/G)S_{\mathrm{M-i}}=(K/g)S_{\mathrm{m}}$), making a tie (if $(K/G)(S_{\mathrm{M-i}}+1)=(K/g)S_{\mathrm{m}}$), or moving the outcome from a loss to a win on all contested positions (if $S_{\mathrm{M-i}}\in(S_{\mathrm{m}}(G/g)-1,S_{\mathrm{m}}(G/g))$). Note that since $S_{\mathrm{M}-i}$ and S_{m} are non-negative integers, the first event is only possible if either G/g is an integer, or $S_{\mathrm{M}-i}=S_{\mathrm{m}}=0$; the second event is only possible if G/g is an integer, and the third event is only possible if G/g is not an integer.

The equations corresponding to the pivot probabilities are logically straightforward. Denoting by a tilde pivot probabilities under CV:

$$\widetilde{\pi}_{\mathrm{M}}^{T}=I_{Q}[(G/g)S_{\mathrm{m}}]\sum_{S_{\mathrm{m}}=0}^{m}\left\{\binom{m}{S_{\mathrm{m}}}F(c_{\mathrm{m}})^{S_{\mathrm{m}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{m}}}\right.$$
$$\left.\binom{M-1}{(G/g)S_{\mathrm{m}}}F(c_{\mathrm{M}})^{(G/g)S_{\mathrm{m}}}[1-F(c_{\mathrm{M}})]^{M-1-(G/g)S_{\mathrm{m}}}\right\}$$

[38] Note in particular that if $u(K)-Eu_{T}^{MP}=Eu_{T}^{MP}-u(0)$, or $Eu_{T}^{MP}=[u(K)-u(0)]/2$, the equilibrium cutpoints $\{c_{\mathrm{m}},c_{\mathrm{M}}\}$ are identical to the cutpoints that solve the corresponding costly voting problem with a single winner.

$$\widetilde{\pi}_{\mathrm{M}}^{T-1} = I_Q\left[(G/g)S_{\mathrm{m}}\right] \sum_{S_{\mathrm{m}}=1}^{m} \left\{ \binom{m}{S_{\mathrm{m}}} F(c_{\mathrm{m}})^{S_{\mathrm{m}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{m}}} \binom{M-1}{(G/g)S_{\mathrm{m}}-1} F(c_{\mathrm{M}})^{(G/g)S_{\mathrm{m}}-1}[1-F(c_{\mathrm{M}})]^{M-1-[(G/g)S_{\mathrm{m}}-1]} \right\}$$

and

$$\widetilde{\pi}_{\mathrm{M}}^{W} = \left(1 - I_Q\left[(G/g)S_{\mathrm{m}}\right]\right) \sum_{S_{\mathrm{m}}=0}^{m} \left\{ \binom{m}{S_{\mathrm{m}}} F(c_{\mathrm{m}})^{S_{\mathrm{m}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{m}}} \binom{M-1}{\lfloor (G/g)S_{\mathrm{m}} \rfloor} F(c_{\mathrm{M}})^{\lfloor (G/g)S_{\mathrm{m}} \rfloor}[1-F(c_{\mathrm{M}})]^{M-1-\lfloor (G/g)S_{\mathrm{m}} \rfloor} \right\}$$

where $I_Q[(G/g)S_{\mathrm{m}}] = 1$ if $(G/g)S_{\mathrm{m}}$ is an integer, and 0 otherwise, and $\lfloor x \rfloor$ is the floor function, denoting the greatest integer smaller or equal to x.[39]

The problem is analogous for a minority voter. The relevant equations are:

$$\widetilde{\pi}_{\mathrm{m}}^{T} = I_Q\left[(g/G)S_{\mathrm{M}}\right] \left\{ \sum_{S_{\mathrm{M}}=0}^{M} \binom{M}{S_{\mathrm{M}}} F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}} \binom{m-1}{(g/G)S_{\mathrm{M}}} F(c_{\mathrm{m}})^{(g/G)S_{\mathrm{M}}}[1-F(c_{\mathrm{m}})]^{m-1-(g/G)S_{\mathrm{M}}} \right\}$$

$$\widetilde{\pi}_{\mathrm{m}}^{T-1} = I_Q\left[(g/G)S_{\mathrm{M}}\right] \sum_{S_{\mathrm{M}}=1}^{M} \left\{ \binom{M}{S_{\mathrm{M}}} F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}} \binom{m-1}{(g/G)S_{\mathrm{M}}-1} F(c_{\mathrm{m}})^{(g/G)S_{\mathrm{M}}-1}[1-F(c_{\mathrm{m}})]^{m-1-[(g/G)S_{\mathrm{M}}-1]} \right\}$$

$$\widetilde{\pi}_{\mathrm{m}}^{W} = \left(1 - I_Q\left[(g/G)S_{\mathrm{M}}\right]\right) \sum_{S_{\mathrm{M}}=0}^{M} \left\{ \binom{M}{S_{\mathrm{M}}} F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}} \binom{m-1}{\lfloor (g/G)S_{\mathrm{M}} \rfloor} F(c_{\mathrm{m}})^{\lfloor (g/G)S_{\mathrm{M}} \rfloor}[1-F(c_{\mathrm{m}})]^{m-1-\lfloor (g/G)S_{\mathrm{M}} \rfloor} \right\}$$

The probabilities of the minority winning different numbers of positions can be derived as under MP, but taking into account that the number of candidates, in each party, now may differ from the number of seats. The probability of the minority losing all seats must be 0 if $G < K$; if instead $G \geq K$, then as before it equals the probability that either all minority candidates receive strictly lower votes than the majority candidates, or that all candidates are tied but minority candidates lose all tie-breaks. That is:

$$\begin{aligned} \Pr(W_{\mathrm{m}} = 0 | G \geq K) = \\ = \sum_{S_{\mathrm{M}}=1}^{M} \binom{M}{S_{\mathrm{M}}} F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}} \sum_{S_{\mathrm{m}}=0}^{X(S_{\mathrm{M}})} \binom{m}{S_{\mathrm{m}}} F(c_{\mathrm{m}})^{S_{\mathrm{m}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{m}}} + \\ + \sum_{S_{\mathrm{M}}=0}^{M} \binom{M}{S_{\mathrm{M}}} \binom{m}{(g/G)S_{\mathrm{M}}} F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}} \times \\ \times F(c_{\mathrm{m}})^{(g/G)S_{\mathrm{M}}}[1-F(c_{\mathrm{m}})]^{m-(g/G)S_{\mathrm{M}}} I_Q\left[(g/G)S_{\mathrm{M}}\right] \binom{G}{K} \Big/ \binom{G+g}{K} \end{aligned}$$

where:

$$X(S_{\mathrm{M}}) = \begin{cases} (g/G)S_{\mathrm{M}} - 1 & \text{if } (g/G)S_{\mathrm{M}} \text{ is an integer} \\ \lfloor (g/G)S_{\mathrm{M}} \rfloor & \text{otherwise} \end{cases}$$

The probability of electing some but not all minority candidates can be derived analogously. For any $w \in (0, g)$, the probability of electing w minority candidates is 0 if $K - G > w$; it equals the probability that all candidates are tied and m

[39] Recall that $\binom{r}{y} = 0$ if $y > r$.

wins w tie-breaks if $K-G<w$, and equals the probability either that all are tied and m loses all tie-breaks or that all m candidates receive fewer votes if $K-G=w$. Thus:

$$\begin{aligned}
&\Pr(W_{\mathrm{m}}=w|K-G\le w)=\\
&=\sum_{S_{\mathrm{M}}=0}^{M}\binom{m}{(g/G)S_{\mathrm{M}}}F(c_{\mathrm{m}})^{(g/G)S_{\mathrm{M}}}[1-F(c_{\mathrm{m}})]^{m-(g/G)S_{\mathrm{M}}}\times\\
&\times\binom{M}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}I_Q[(g/G)S_{\mathrm{M}}]\binom{g}{w}\binom{G}{K-w}/\binom{G+g}{K}+\\
&+I_{K-G=w}\sum_{S_{\mathrm{M}}=1}^{M}\binom{M}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}\times\\
&\left(\sum_{S_{\mathrm{m}}=0}^{X(S_{\mathrm{M}})}\binom{m}{S_{\mathrm{m}}}F(c_{\mathrm{m}})^{S_{\mathrm{m}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{m}}}\right)
\end{aligned}$$

where $I_Q[(g/G)S_{\mathrm{M}}]$ and $X(S_{\mathrm{M}})$ are defined as above, and $I_{K-G=w}$ is an indicator function taking value 1 if $K-G=w$ and 0 otherwise.

Finally, the probability of electing exactly g minority candidates equals 1 if $K-G\ge g$; otherwise, it equals the probability that either all minority candidates receive more votes or that all candidates are tied and the g minority candidates win all tie-breaks. That is:

$$\begin{aligned}
&\Pr(W_{\mathrm{m}}=g|K-G<g)=\\
&=\sum_{S_{\mathrm{M}}=0}^{M}\binom{m}{(g/G)S_{\mathrm{M}}}F(c_{\mathrm{m}})^{(g/G)S_{\mathrm{M}}}[1-F(c_{\mathrm{m}})]^{m-(g/G)S_{\mathrm{M}}}\times\\
&\times\binom{M}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}I_Q[(g/G)S_{\mathrm{M}}]\binom{G}{K-g}/\binom{G+g}{K}+\\
&\sum_{S_{\mathrm{m}}=1}^{m}\binom{m}{S_{\mathrm{m}}}F(c_{\mathrm{m}})^{S_{\mathrm{m}}}[1-F(c_{\mathrm{m}})]^{m-S_{\mathrm{m}}}\times\\
&\left(\sum_{S_{\mathrm{M}}=0}^{Y(S_{\mathrm{m}})}\binom{M}{S_{\mathrm{M}}}F(c_{\mathrm{M}})^{S_{\mathrm{M}}}[1-F(c_{\mathrm{M}})]^{M-S_{\mathrm{M}}}\right)
\end{aligned}$$

where:

$$Y(S_{\mathrm{m}})=\begin{cases}(G/g)S_{\mathrm{m}}-1 & \text{if } (G/g)S_{\mathrm{m}} \text{ is an integer}\\ \lfloor (G/g)S_{\mathrm{m}}\rfloor & \text{otherwise}\end{cases}$$

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.geb.2023.05.012.

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